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ON
USE OF MATHEMATICAL MODELS TO OPTIMIZE WATER QUALITY MANAGEMENT

KHARKOV and ROSTOV-ON-DON, USSR

December 9-16, 1975

ENVIRONMENTAL RESEARCH LABORATORY
OFFICE OF RESEARCH AND DEVELOPMENT
U.S. ENVIRONMENTAL PROTECTION AGENCY
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FOREWORD

The Joint American-Soviet Committee on Cooperation in the Field of Environmental Protection, established by an agreement signed May 23, 1972, in Moscow, identified eleven ecological problem areas for cooperative investigation and exchange of information. In accordance with these objectives, the Joint Symposium on Mathematical Models to Optimize Water Quality Management was convened December 9-16, 1975, in Kharkov and Rostov-on-Don in the USSR.

During the Kharkov portion of the symposium, participants exchanged experience on methodological questions related to planning in accordance with legal requirements and organizational units existing in each country. Discussions focused on technical processes connected primarily with questions of simulation and optimization modeling of processes that determine the dynamics of water quality within river basins.

The symposium noted a specific process in the work organization and in the development of mathematical models, and established grounds for further scientific-technical cooperation in the analysis of the actual water protection planning process and realization of these plans.

Papers presented in Rostov-on-Don described a wide range of methods for mathematical modeling and many specific models. These papers can be used to compare and evaluate techniques of mathematical modeling developed in the cooperating countries and will serve as the basis for further progress in this field.

Symposium participants revealed two main problems confronting scientists from both countries. The first difficulty is to determine various constants (coefficients) that describe water ecosystems. The second difficulty is the selection of water quality criteria and the complexity of defining boundary conditions of the model. Participants agreed that both questions cannot be resolved at the present time.

The symposium demonstrated great mutual interest in the exchange of scientific ideas and information. It actively involved the participants and confirmed the usefulness of joint discussions of actual water protection problems.

The symposium showed that the problems of modeling the ongoing processes in water bodies, and the problem of surface water quality management, are very complex. At present, the solution (development of a universal approach to the model) are a long way off. We hope that the joint efforts of the scientists from the USSR and USA will attain this goal.

The published materials reflect the original positions of both sides before the discussion developed at the Kharkov part of the symposium. Publication of the proceedings by the Environmental Research Laboratory, U.S. Environmental Protection Agency, Gulf Breeze, Florida, fulfills the protocol agreement for Project II.1 requiring simultaneous and independent publication in both countries after coordination of manuscripts.

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ABSTRACT

The American-Soviet Symposium on Use of Mathematical Models to Optimize Water Quality Management examines methodological questions related to simulation and optimization modeling of processes that determine water quality of river basins. Discussants describe the general state of development and application of mathematical models designed to predict and optimize water quality management in the USA and USSR. Subjects of papers presented by American and Soviet specialists include: planning comprehensive water quality protection systems; modeling water quality in river basins; geographic-economic aspects of pollution control systems; identification of ecosystem models by field data; management decisions for lake systems on a survey of trophic status, limiting nutrients, and nutrient loadings; a descriptive simulation model for forecasting the condition of a water system; mathematical ecosystem models and a description of the water quality in water bodies; mathematical modeling strategies applied to Saginaw Bay, Lake Huron, to eutrophical processes in Lake Ontario, to hydrodynamics and dispersion of contaminants in the nearshore, and to coastal currents in large lakes; and the construction of a model of Lake Baikal on the principles of self-organization. Publication of the proceedings held December 9-16, 1975, in Kharkov and Rostov-on-Don, USSR, is in compliance with the Memorandum from the Fourth Session of the Joint American-Soviet Committee on Cooperation in the Field of Environmental Research.

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PLANNING COMPREHENSIVE WATER QUALITY PROTECTION SYSTEMS

Daniel P. Loucks

INTRODUCTION

This paper will review a number of mathematical modeling components that can be combined and solved for the preliminary planning of economically efficient comprehensive water quality management programs for river systems. The selection of this topic for this symposium stems from the discussions with VNIIVO specialists that took place in the spring and summer of 1975 in the USSR and in the U.S. The modeling techniques to be reviewed in this paper hopefully reflect an approach to long-run water quality protection planning that might be appropriate for various rivers (e.g. the Severski Donets) in the USSR.

Water quality management models based on optimization techniques have been developed and used for the preliminary definition and evaluation of alternative designs and operating policies of a variety of water quality control options. These include, but are not limited to, (1) wastewater treatment facilities, (2) wastewater effluent storage lagoons, (3) land disposal sites for wastewater disposal and additional treatment, (4) flow augmentation reservoirs, and (5) instream aeration devices. Each of these options can influence the quality of water in river systems. While other engineering alternatives are available (such as bypass piping of wastewater effluent and regional treatment facilities), it is this set of five water quality protection alternatives that will be considered and modeled in this paper.

The simultaneous examination of all appropriate engineering measures available for water quality protection permits the identification of a more economically efficient or cost-effective solution to water quality protection problems than would be possible if only one or two such measures were considered. The purpose of developing and using optimization and simulation models for water quality planning, of course, is to identify those particular measures, their design specifications, and their operating policies, that will satisfy water quality, economic, and possibly other objectives.

In the USSR, one of the more important objectives is cost minimization. Therefore, the discussion in this paper will be oriented towards that single objective of cost minimization, while meeting various stream quality standards involving dissolved oxygen (DO), biochemical oxygen demand (BOD), conservative substances [e.g., chlorides (CI)], and nutrients (e.g., nitrates).

Before beginning this review of some optimization modeling techniques for water quality protection, it is important to understand how people in the U.S. view the role of water quality management optimization models, as opposed to ecosystem simulation models, and even the meaning of optimization in this context. In spite of the relatively powerful solution procedures (algorithms) and the computer speed and capacity available today for solving large complex optimization problems, these procedures and computers are inadequate for solving any reasonably complex water quality protection problem. Thus, some simplifying assumptions are necessary regarding the biological-chemical and physical phenomena that occur in a water system and also regarding the political or management objectives.

Even if one could be certain of all of the values of various model parameters, there is still considerable uncertainty concerning the most correct form of a model of a particular aquatic system. Because of limitations in our knowledge of aquatic ecosystems, even the relatively complex multispecies ecosystem models that are currently being used for predictions of the biological, chemical, and physical quality of water bodies are limited (Russell, 1975). These ecosystem models are very data-demanding, and hence very costly. They are also extremely non-linear and multidimensional, making it difficult to incorporate them into an optimization program for estimating least-cost management solutions.

Given these limitations of existing ecosystem simulation models, one could question the value of developing optimization models that require many more simplifying assumptions and approximations. The answer is that such models are useful if through their use one is able to reduce a large set of possible engineering design and operating policy alternatives to a relatively small number for more detailed (and expensive) examination with simulation procedures. Our experience suggests that optimization models of water quality protection systems cannot find the best or optimal solution, but, if applied correctly, they can be used to eliminate clearly inferior alternatives from further consideration. When there are many engineering or management alternatives available, the need for a preliminary screening technique is substantial. The cost and time required to simulate even a fraction of all alternatives could be prohibitive.

Water quality optimization models are also less data-demanding than the ecosystem simulation models. Thus, with the speed of their solution, these models are a potentially useful tool for preliminary screening of alternative designs and operating policies. The specific applications of water quality optimization models discussed by DeLucia and Chi (1975) at this symposium are examples that could be cited to illustrate this point.

The models to be outlined in this paper will be static models. They will not be able to answer such questions as when additional wastewater treatment or storage lagoon capacities should be made available, or to permit prediction of the quality profile in a water body over a daily cycle. These models will simply enable "snapshots" to be taken at particular future periods, corresponding to the exogenous values of the wastewater flows at each industrial or municipal site. They will only be able to estimate the average water quality values in rivers with specified flow characteristics. Such models are for long-term planning, not short-term control. For greater precision, simulation models are required, and in fact, such models should be used to check and to improve upon the results of preliminary screening models (such as those outlined in this paper) prior to the actual implementation of a particular water quality protection plan.

This paper will first discuss each component of a comprehensive water quality model. These modeling components will include a flow augmentation reservoir, a wastewater treatment facility, an effluent storage lagoon, a land disposal system, and instream aeration. These components can then be combined into a single comprehensive model of any particular river system, such as the Severski Donets. Finally, the paper will discuss procedures that can be used to solve any resulting comprehensive model.

MODEL COMPONENTS

Low Flow Augmentation

Changes in streamflow can influence the water quality that occurs at any particular location in a river. Augmenting the flow with relatively high quality water dilutes the waste load, increases the stream velocity, spreads the degradation reaction further downstream from the waste sources, and affects the atmospheric reaeration rate. Temperature changes, if any, may alter the biochemical processes. Colder augmenting flows tend to decrease the deoxygenation and reaeration rates, thereby extending oxygen deficits and BOD concentrations for longer distances downstream. Lower temperatures also increase the saturation concentrations of dissolved oxygen. Increased flows also result in increased scouring of bottom or benthal deposits, which in turn increases the waste load of the stream.

The net effect of all changes resulting from augmented streamflows may increase or decrease the concentrations of oxygen, BOD, conservative substances, and nutrients, depending upon the location of the waste sources and the particular sites where quality is to be maintained. Hence, to put it simply, flow augmentation may be good or bad, or good and bad simultaneously, depending on a particular location in the river basin, the extent of augmentation, and other circumstances. The only way to evaluate the advantages and disadvantages of this alternative for water quality protection is to include it with the overall comprehensive water quality modeling effort. Just how this might be done is the subject of this section of the paper.

Water quality standards are usually based on a design streamflow condition. For example, in the more humid portions of the U.S., this design flow is the particular yearly minimum average 7-day flow that is equalled or exceeded 90% of the time (9 years in 10 on the average). This is commonly called the 7-day, 10-year flow. In much of Eastern Europe and in the USSR, it is my understanding that this design flow is the minimum average monthly flow exceeded 19 years out of 20, or 95% of the time. Both flows are minimum flow conditions that are exceeded most of the time. They are based on the assumption, generally but not always correct, that low-flow, stream-flow conditions result in poorer stream qualities than those that exist during higher streamflow conditions.

Upstream reservoirs that are used, in part, to supply waters for low flow augmentation, must be designed and operated in such a way as to increase the design flow condition, i.e., to increase the flows that will be equalled or exceeded by the same probabilities as the original unaugmented design flow conditions. Hence, models that are appropriate for the definition and evaluation of flow augmentation policies must be able to predict the reservoir storage requirements and operating policies that will maintain the probabilities of exceedence of augmented design streamflows, i.e., those streamflows that are used as a reference for water quality standards.

Low-flow augmentation models may differ depending on how the design streamflows are defined (Loucks and Jacoby, 1972). However, each low-flow augmentation model is developed to estimate reservoir storage capacity requirements and operating policies associated with any particular set of augmented streamflows. These flows will later be used in a water quality model for estimating the appropriate capacity of wastewater treatment facilities, lagoon storage basins, and other water quality protection options needed to meet various stream quality standards.

Water allocated for low-flow augmentation is not usually available for other uses, and hence some opportunity cost or loss is incurred. Additional reservoir storage capacity may also be required to meet the low-flow augmentation requirements, and this too costs money. Thus, a portion of the overall economic efficiency or cost minimization objective for water quality protection will be related to the total cost of flow augmentation, $C_1^r(Q^r) + C_1^r(K^r)$, provided by each reservoir r having an annual flow augmentation release of at least Q^r at least 95% of the time, and a storage capacity of K^r. That portion of the overall least-cost objective related to low-flow augmentation can be written as:

$$\text{minimize} \sum_r C_1^r(Q^r) + C_2^r(K^r) \tag{1}$$

To determine the storage capacity, K^r, and operating policy needed to provide an annual augmenting release of Q^r or equivalently a series of monthly flows Q_t downstream of each reservoir site r that are equalled or exceeded 95% of the time, one can examine overyear and within-year storage capacity requirements separately. First, the overyear capacity requirements can be defined by a series of continuity equations for each year y of record, equating the initial storage volume S_y^r plus the annual inflow I_y^r, less the flow augmentation release Q^r, less any excess release E_y^r, to the final storage volume, S_{y+1}^r.

$$S_y^r + I_y^r - Q^r - E_y^r = S_{y+1}^r \quad \text{for each year } y \quad (\forall y) \tag{2}$$

To estimate the probability that a release of at least Q^r can be made from reservoir r each year, the concept of a mean probability of exceedence can be introduced. The mean probability that any particular streamflow will be equalled or exceeded is based on the assumption that any future flow has an equal probability of falling within any interval defined by the ordered sequence of historical (and/or synthetic) streamflows. Suppose, for example, that there exists a record of n unregulated annual streamflows, hence n+1 streamflow intervals. Ranking these flows so that the largest streamflow has the lowest rank, m=1, and the lowest streamflow has the highest rank, m-n, the mean probability that a future unregulated annual streamflow will equal or exceed a flow of rank m is m/n+1. The expected recurrence interval of an annual flow of rank m is n+1/m years, the reciprocal of the mean probability.

An estimate of the mean probability of a given unregulated streamflow makes it possible to define the mean probability of any particular reservoir release Q^r. If the release variable Q^r is contained in each of the n equations (2), then the probability of its being equalled or exceeded is n/n+1. If this Q^r variable is omitted from the equation 2 having the lowest annual inflow I_y, i.e., the year having an inflow of rank m=n, then the probability of its being equalled or exceeded is reduced to n-1/n+1.

Including the Q^r term in equation 2 only when inflows I_y have a rank of m less than or equal to n+1 times the desired mean probability of exceedence p (say 0.95) will insure that the release Q^r will be equalled or exceeded 100p percent of the time. If, for example, there exists an annual unregulated streamflow record of 19 years, the inclusion of a release variable Q^r in each of the 19 equations (2) will ensure that the flow augmentation release will be equalled or exceeded with a probability of 19/20 or 95% of the time.

Overyear reservoir storage capacity, S_o^r, required for any flow augmentation release Q^r is obtained from the maximum value of all the storage volumes S_y^r.

$$S_y^r \leq S_o^r \quad \forall y \tag{3}$$

To ensure a particular distribution of the annual flow augmentation release Q^r in each month t, within-year reservoir storage capacity may also be required. This can be obtained by another mass balance, equating the sum of the initial within-year storage volume, s_t^r, plus that average proportion β_t^r of the annual flow release Q^r that flows into the reservoir during month t, less the monthly flow augmentation release Q_t^r, to the final storage volume s_{t+1}^r, for each month t, (when t=12, t+1=1).

$$s_t^r + \beta_t Q^r - Q_t^r = s_{t+1}^r \quad \forall t \tag{4}$$

The fraction β_t can be determined from the average monthly uncontrolled inflows to the reservoir, $I_t^r = 1/n \sum_y^n I_{yt}$

$$\beta_t = I_t^r / \sum_t I_t^r \quad \forall t \tag{5}$$

The total reservoir capacity K^r required for each monthly flow augmentation release Q_t^r will be the smallest capacity, K^r, that contains each overyear storage, S_o^r, and within-year storage, s_t^r.

$$K^r \geq S_o^r + s_t^r \quad \forall t \tag{6}$$

The model just developed, equations 1 through 6, can be solved for various chosen values of Q_t^r. Each set of augmented flows Q_t^r can then be incorporated into a water quality management model, to be discussed shortly. Once particular vectors of Q_t^r have been selected as optimal, reservoir rule curves defining the releases that can be made as a function of calendar time and existing storage volumes can be established. Such rules enable the operators of reservoirs to meet the flow augmentation quality and reliability requirements. The formulation and use of these types of reservoir design and operating models, and how they can be used to derive reservoir rule curves, has been described (Loucks, 1976).

Wastewater Treatment

The concentration, CW_i^p, of any potential pollutant p, (e.g., BOD, chloride, nitrogen) in the wastewater produced at site i, can be reduced by various unit treatment processes, k. The quantity of wastewater, QW_i^k, that is routed through each unit treatment process k will determine the cost and removal efficiency of the overall wastewater treatment facility. To illustrate how this overall efficiency and cost can be estimated, assume there are K_i possible unit treatment operations at site i (e.g., grit chamber, primary and secondary sedimentation basins, activated sludge unit, chlorination, etc.), each costing $CT_i^k(QW_i^k)$, depending on the hydraulic capacity, QW_i^k, and each removing a known fraction, f_k^p, of pollutant p. The portion of the cost minimization objective related to wastewater treatment is

$$\text{minimize} \sum_i \sum_k^{K_i} CT_i^k(QW_i^k) \qquad (7)$$

The concentration, C_i^p, of each pollutant in the effluent of the treatment facility depends on the quantity routed through each unit treatment process, and the configuration of these processes. Mass balance equations for both flows and pollutant masses are easily written for any proposed configuration of unit treatment processes, and these will define the mass and concentration of each pollutant in the effluent. An example of how these conservation equations can be written is illustrated below in Figure 1. In Figure 1, process 3 might be considered an alternative to 2 and 4; the only difference perhaps would be the recycling of a fraction ρ of the flow between 2 and 4.

Note that each of the expressions in Figure 1 is linear with respect to the unknown flows, QW, in each link of the treatment system. Since the final effluent flow equals the known influent flow, the concentrations, C_i^p, are the unknowns. If certain alternative wastewater unit process configurations include a portion of the flow being returned to a previously considered unit process, e.g., a return flow as shown in unit process k=3 in Figure 1, the resulting expressions for masses of pollutants may be nonlinear unless the recycled portion ρ is known. These looped components (e.g., a and b in Figure 1) can be considered as a single component whose hydraulic capacity must include the quantity of the recycled flow.

The total fraction, X_i^p, of each pollutant p removed by the entire treatment facility can be determined by comparing the known influent concentration to the effluent concentration in the model solution.

$$X_i^p = C_i^p / CW_i^p \qquad \forall p \qquad (8)$$

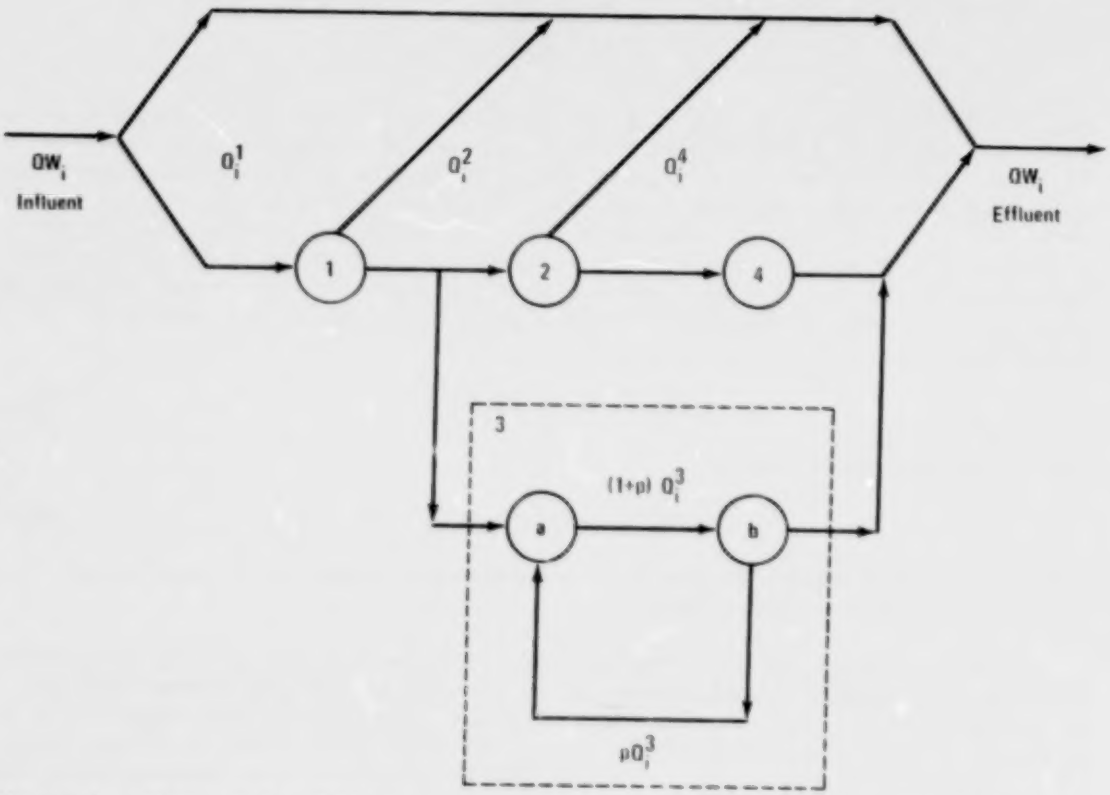
QW_i
Influent
Q_i^1
Q_i^2
Q_i^4
QW_i
Effluent
1
2
4
3
(1+ρ) Q_i^3
a
b
ρQ_i^3

Figure 1. Flows and pollutant concentrations throughout a potential wastewater treatment system.

Process k	Influent Flow	Effluent Concentration C^P_{ik}
1	$Q^1_i \leq QW_i$	$CW^P_i(1-f^P_1) = C^P_{i1}$
2	$Q^2_i \leq Q^1_i - Q^3_i$	$CW^P_i(1-f^P_1)(1-f^P_2) = C^P_{i2}$
3	$Q^3_i \leq Q^1_i - Q^2_i$	$CW^P_i(1-f^P_1) \left[\frac{(1-f^P_a)(1-f^P_b)}{1+\rho - \rho(1-f^P_a)(1-f^P_b)}\right] = C^P_{i3}$ *
4	$Q^4_i \leq Q^2_i$	$CW^P_i(1-f^P_2)(1-f^P_4) = C^P_{i4}$
SYSTEM	QW_i	$[CW^P_i(QW_i - Q^1_i) + C^P_{i1}\ (Q^1_i - Q^2_i - Q^3_i)$ $+ C^P_{i2}\ (Q^2_i - Q^4_i) + C^P_{i3}\ Q^3_i + C^P_{i4}Q^4_i]/QW_i = C^P_i$

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*Obtained by equating the recycled concentration to the effluent concentration.

Defining some constants, γ_i^p, such that for a variable X_i,

$$X_i = \gamma_i^p X_i^p \quad \forall p \tag{9}$$

and comparing the costs derived from equation 7 for various values of X_i, the cost of wastewater treatment at each site i can be expressed as a function $CT_i(X_i)$. This procedure is appropriate at each site i only where a particular pollutant p is dominant, i.e., where it will dictate the design of the treatment facility. If this is not the case, then each unit process must be included in the overall model. However, when a single pollutant p is dominant, the detailed analysis of each wastewater treatment component k can be omitted from the more comprehensive river basin model. The objective component for wastewater treatment,

$$\text{minimize} \sum_i CT_i(X_i) \tag{10}$$

and constraint equations

$$CW_i^p(1 - X_i/\gamma_i^p) = C_i^p \quad \forall p \tag{11}$$

are all that are needed in the more comprehensive model of a river basin water quality protection system.

What is needed for the remainder of the overall water quality protection model are the quantities or masses of pollutants in the wastewater effluent of the treatment facility. This effluent may either be stored temporarily in a lagoon or discharged directly into a water body or disposed of directly on land. Assuming lagoons are reasonable alternatives for the temporary storage of treated wastewater effluent during periods of low stream, or land assimilative capacity, they too can be modeled for the purpose of determining cost-effective lagoon capacities and operating policies.

Effluent Storage Lagoons

The ability to temporarily store treated wastewater effluent when the assimilative capacity of the receiving land or water is low, and release it when the assimilative capacity is high, may permit some reduction in the total cost of water quality protection. Effluent storage lagoons can be modeled and included in the overall planning model to determine whether or not any cost savings can be achieved by their installation and operation. The objective component of these storage lagoon submodels will be to minimize the total annual cost, $CL_i(KL_i)$, of constructing and operating each lagoon of capacity KL_i.

$$\text{minimize} \sum_i CL_i(KL_i) \tag{12}$$

The lagoon itself can achieve some removal of certain nonconservative pollutants. Relationships between pollutant removal efficiencies and effluent detention times in the storage lagoons are hard to obtain, especially in view of the possibility of relatively long detention times, incomplete mixing, and the varying temperatures and storage volumes. It appears, however, that removal rate constants can be estimated from the literature for long storage periods in temperate climates (Fisher, 1968; Renn, 1973). Assuming first order reactions for the removal of each pollutant p, a mass balance for each month can be written relating storage volumes, S_{ti} influent and effluent concentrations, C_i^p and $\overline{C}_{ti}^p$, and flows QW_{ti} and $\overline{QW}_{ti}$, in each month t at each site i.

$$\left(\frac{S_{t+1,i} + S_{ti}}{2}\right) \left(\overline{C}^p_{t+1,i} - \overline{C}^p_{ti}\right) = QW_{ti}C^p_i - \overline{QW}_{ti}\overline{C}^p_{ti} - k^p_t\overline{C}^p_{ti}\left(\frac{S_{t+1,i} + S_{ti}}{2}\right) \qquad (13)$$

$$t = 1,2,\ldots,12 \text{ (if } t = 12,\ t+1 = 1)$$

where:

c_i^p = concentration of pollutant p in the treated effluent entering the storage lagoon in month t.
$\overline{C}^p_{ti}$ = average concentration of pollutant p in the effluent released from the storage lagoon during month t.
k_t^p = pollutant p removal rate constant for the effluent in storage lagoon during month t (temperature effects included), month^{-1}.
QW_{ti} = volume of wastewater influent to storage lagoon in month t.
$\overline{QW}_{ti}$ = volume of wastewater effluent discharged from the storage lagoon to the land during month t.
S_{ti} = volume of wastewater in the storage lagoon at beginning of month t.

Often there exists a maximum limit of pollutant removal that can occur in a lagoon, i.e., after long detention times no more removal occurs. This can be stated, for example, as

$$\overline{C}^p_{ti} \geq \text{some fraction of the influent concentration, } C_i^p \quad \forall t,i \qquad (14)$$

The term $\left(\frac{S_{t+1,i} + S_{ti}}{2}\right)$ is the average volume of treated wastewater effluent in the storage lagoon at site i during time period t. The S_{ti}'s are obtained from the continuity equations for storage and may vary from period to period. The effects of evaporation in each month t from the surface of the lagoon can be approximated by the fraction ϵ_{ti}

$$\epsilon_{ti}S_{ti} + QW_{ti} - \overline{QW}_{ti} = S_{t+1,i} \quad \forall t,i \qquad (15)$$

where the lagoon capacity, KL_i, must be no less than each storage volume, S_{ti}.

$$KL_i \geq S_{ti} \quad \forall t,i \qquad (16)$$

In order to provide adequate detention times for the settling of suspended solids in the stored effluent, a minimum detention time, θ_{min}, may be required.

$$\frac{S_{ti} + S_{t+1,i}}{2QW_{ti}} \geq \theta_{min} \quad \forall t,i \qquad (17)$$

It is also important to ensure that the storage lagoon does not become anaerobic, especially during late winter and spring. This limits the depth of the lagoon and the BOD loading per unit area-day. The lagoon area, LA_i, is determined by the actual and maximum allowable daily BOD loadings.

$$LA_i \geq \frac{\text{maximum BOD loading/day}}{\text{allowable BOD/unit area-day}} = \max_t \left(\frac{\frac{1}{D_t} QW_{ti} C_t^P}{LR_h} \right) \qquad (18)$$

where:

LR_h = allowable daily BOD loading per unit area for storage lagoon of depth h.

D_t = the number of days in month t.

The area, LA_i, of the storage lagoon must also be no less than the maximum storage volume divided by the depth, h_i, at site i.

$$LA_i \geq S_{ti}/h_i \qquad \forall t \qquad (19)$$

The maximum of the areas, LA_i, found by equations 18 and 19, will be the area required for the storage lagoon. If the land area required for a lagoon costs money, then this annual cost should be included in objective function, equation 12.

The wastewater effluent in a storage lagoon can be either discharged into water or onto land, depending in part on the type of pollutants contained in the effluent, the availability of suitable land and its costs, and on the capacities of water and land to assimilate the wastes remaining in the effluent.

If land is available, then one alternative for additional wastewater treatment and stream quality protection is to discharge the wastes onto land. The extent to which this is possible can be estimated by including this alternative, where feasible, within the comprehensive water quality protection planning model.

Wastewater Disposal on Land

Land disposal of wastewater effluents can be an effective means of further removing various pollutants, such as BOD, nitrogen, phosphorus, pathogens (bacteria and viruses), heavy metals, and the like. This additional treatment of wastewater effluent is sometimes referred to as effluent polishing. In the U.S., land application of wastewater effluent is increasingly being considered as an important option in the protection of water quality. This is only true, of course, in situations where suitable land exists and when the continued application of effluent would not adversely affect the physical characteristics and productivity of the land.

If land disposal is to be considered, a model is needed to estimate the quantity of treated wastewater effluent that can be disposed on land without violating various maximum allowable concentrations of nutrients in the drainage water. Since any soil system has a limited capacity to "polish" secondary effluents, restrictions must be placed on the amount of concentration of pretreated wastewater applied per unit area.

The capacity of most soils (especially those found near the Severski Donets) to remove essentially all of the BOD, phosphorus, and heavy metals is quite satisfactory for average wastewaters. Hence, modeling the interaction of these pollutants in the soil is not necessary. While the accumulation of phosphorus and heavy metals cannot be dismissed in the long-run, especially when applying secondary effluents over a considerable period of time, the primary focus is usually on nitrogen. The nitrogen in the wastewater effluent is mostly of the NO_3-N form because of the storage detention times in the lagoon which allow almost complete nitrification (Renn, 1973).

Nitrogen is highly mobile in soils, is often associated with eutrophication, and can also be a public health problem. If the nitrogen concentration exceeds the drinking water standard (in the U.S. this equals 10 mg/ℓ NO_3-N) in recharged groundwater, its use for water supplies may be severely limited due to the lack of an economical treatment technology that can remove nitrate. Organics are not desired in water supplies because they often impair their taste and odor. By addition of organics (e.g., from sludge and wastewater effluent), the slow depletion of organic matter in the soil is reduced or prevented, but care is also needed to prevent excessive concentrations of organics in the runoff (Renn, 1973). Hence a method is needed to assist the environmental engineer in designing and operating economically efficient wastewater treatment and land disposal systems.

In this section a simple model is proposed which can assist in evaluating land disposal schemes of pretreated wastewater effluent on well-drained flat grassland for a variety of wastes and climatic conditions. The model can be used to estimate the maximum hydraulic and assimilative capacity of the soil and the capacity and operating policy of an effluent storage lagoon, if required, due to seasonal or monthly changes in the soil assimilative capacity. Since the costs, $CI_i(IA_i)$, for irrigation and drainage systems can be expressed as a function of the irrigation area, IA_i, required, the objective function of this portion of the overall comprehensive model can be written

$$\text{minimize} \sum_i CI_i(IA_i) \qquad (20)$$

The land which receives the lagoon effluent from the treatment plant or the storage lagoon must be relatively level and well-drained. It may or may not be underlain by an impermeable layer. The drainage capacity of such soils that are underlain by an impermeable layer depends on the hydraulic conductivity coefficient of the soil and the spacing of the tilage drains.

The soil drainage capacity, d, is dependent on the horizontal distance between tiles, s, the vertical distance between the impermeable layer and the tile drain, h_o, the depth of the water table above the impermeable layer, H, and the hydraulic conductivity of the soil, k.

$$d = \frac{2k}{s^2}(H^2 - h_o^2) \qquad (21)$$

The upper layer of the soil is kept aerobic by maintaining the water table below the root zone. This permits normal plant growth. Hence, the application rate of wastewater effluent must be limited so as not to exceed the drainage capacity of the soil. Application rates can be calculated from a soil-water balance which includes precipitation and evapotranspiration.

$$M_{t+1,i} = M_{ti} + IR_{ti} + P_{ti} - E_{ti} - d_{ti} \qquad \forall t,i \tag{22}$$

where at each site i,

$$M_{ti} \leq M_i \qquad \forall t,i \tag{23}$$

$$d_{ti} \leq d_i \qquad \forall t,i \tag{24}$$

and

d_i = drainage capacity of soil (equation 21)
d_{ti} = drainage rate during month t
E_{ti} = average evapotranspiration during month t
IR_{ti} = application rate of wastewater effluent during month t
M_i = moisture content in the soil at saturation
M_{ti} = moisture content in the soil at the beginning of month t
P_{ti} = average precipitation during month t

The relationship between that portion, LQ_{ti}, of the lagoon effluent, $\overline{QW}_{ti}$, discharged on land, the application rate, IR_{ti}, and the land area, IA_i, used for disposal of wastewater effluent, is given by

$$LQ_{ti} \leq IR_{ti}(IA_i) \tag{25}$$

For most locations, average monthly values for precipitation, P_{ti}, and evapotranspiration, E_{ti}, can be obtained from climatological summaries. If E_t is not readily available, it may be estimated for certain crops from handbooks (Agricultural Engineers Handbook, 1961), or with the aid of some empirical formula (Hamon, 1961).

If the average monthly temperature falls below 0° Celsius, the soil may be frozen. When this occurs, no wastewater effluent may be discharged. Where snow accumulates, soils may be frozen even after the average monthly temperature rises above 0° Celsius.

The application rate of wastewater decreases if soil clogging occurs due to the suspended solids content in the wastewater effluent. By ensuring that the application rate, IR_{ti}, does not exceed a safe limit, $IR_{max,i}$, favorable infiltration characteristics can be maintained over long periods of time.

$$IR_{ti} \leq IR_{max,i} \qquad \forall t \tag{26}$$

Having a model of the hydraulic and physical aspects of the wastewater land disposal system, it is now necessary to model the soil nitrogen balance. Haith (1973) has presented a concise summary of monthly inventory equations describing the soil organic and inorganic nitrogen. For convenience, the site subscript i will be dropped here, but would be needed in the overall model if more than one land site was being considered for wastewater disposal.

$$\text{organic N} : O_{t+1} = (1-m_t)O_t + XO_t \qquad \forall t \tag{27}$$

$$\text{inorganic N} : I_{t+1} = I_t + m_tO_t + XI_t - L_t - N_t \qquad \forall t \tag{28}$$

where:

I_t = soil inorganic nitrogen at beginning of month t
L_t = inorganic nitrogen lost due to drainage during month t
m_t = fraction of soil organic nitrogen converted (mineralized) to inorganic nitrogen during month t
N_t = inorganic nitrogen removed from the soil by plant growth during month t
O_t = soil organic nitrogen at beginning of month t
XI_t = inorganic nitrogen added to the soil due to wastewater irrigation during month t
XO_t = organic nitrogen added to the soil due to wastewater irrigation during month t

The quantity of organic nitrogen, XO_t, and inorganic nitrogen, XI_t, added to the soil can be found from the wastewater application rate, IR_t, and the concentration of nitrogen, $\bar{C}^p_{ti}$, in the wastewater effluent.

$$XO_t = (\alpha)\bar{C}^p_{ti}\ IR_t \qquad \forall t \tag{29}$$

$$XI_t = (1-\alpha)\bar{C}^p_{ti}\ IR_t \qquad \forall t \tag{30}$$

In the above equations, the parameter α is the fraction of organic nitrogen in the total nitrogen of the applied wastewater effluent, and the pollutant p represents nitrogen.

The mineralization rate of nitrogen depends, in part, on soil temperature. If the temperature drops below 0^o Celsius, no mineralization is assumed to take place. The yearly mineralization rate can be estimated from a knowledge of the fraction of organic matter, m_o, annually mineralized in the soil. The monthly fraction mineralized, m_t, can be estimated with a linear relationship.

$$m_t = m_oT_t / \sum_{t \ni T_t > 0} T_t \qquad \forall t \tag{31}$$

where T_t is average temperature during month t in ^oC. The sum includes only those months t whose average temperature is greater than 0^oC.

If no nitrate adsorption in the soil is assumed for month t, the NO_3-N concentration, n_t, in the soil solution is simply the average quantity of inorganic nitrogen in the soil, $(I_t + I_{t+1})/2$, divided by the average soil moisture content, $(M_t + M_{t+1})/2$, in month t.

$$n'_t = \frac{I_t + I_{t+1}}{M_t + M_{t+1}} \qquad \forall t \qquad (32)$$

Drainage occurs when the soil is saturated, i.e., when $M_t = M$. Hence, the NO_3-N concentration, n_t, in the drainage water is

$$n_t = \frac{(I_t + I_{t+1})}{2M} \qquad \forall t \qquad (33)$$

The amount of NO_3-N lost, L_t, in the drainage water is equal to the average concentration of NO_3-N in the drainage water, n_t, times the quantity, d_t, of water drained.

$$L_t = d_t n_t \qquad \forall t \qquad (34)$$

Substituting equation 33 into 34 yields

$$L_t = d_t(I_t + I_{t+1})/2M \qquad \forall t \qquad (35)$$

Grass or pasture land has a high uptake of nitrogen compared to crops such as corn or wheat. Therefore, grass appears to be an ideal cover crop for wastewater disposal sites, at least for nitrogen control. Among other advantages are reduced erosion and runoff, maintenance of satisfactory infiltration capacity, fairly even nitrogen removal over the whole land area in contrast to "point uptake" by corn, for example, and fairly uniform growth rates for most of the year. Moreover, grasses grow well in wet soils, a common characteristic of many potential land sites used for wastewater disposal. Cooke (1967) estimates that grass takes up about 2/3 to 3/4 of the applied nitrogen. Sopper (1973) found an uptake efficiency over 97.3% when canary grass was irrigated with secondary wastewater effluent. Thus a reasonable, and conservative, estimate of the maximum nitrogen uptake, N_t, for grasses could be assumed to equal 70% of the available nitrogen in the soil.

$$N_t \leq 0.7(m_t O_t + XI_t + I_t) \qquad \forall t \qquad (36)$$

On the other hand, the uptake of nitrogen usually does not exceed some maximum value, N_t^{max}, depending on the plant type (Overman, 1975).

$$N_t \leq N_t^{max} \qquad \forall t \qquad (37)$$

If the yearly potential nitrogen uptake is known, monthly uptake rates for the growing months (e.g., months with average temperatures, $T_t > 0$) can be estimated. For example, a nitrogen removal of 420 kg/ha per year is not unusual when grass is heavily fertilized. If we assume a maximum annual uptake of 420 kg/ha and a growing season of April to October, with April and October removing only one-half the amount of the other months, the maximum monthly uptakes, N_t^{max}, would be 35 kg/ha for April and October and 70 kg/ha for May through September.

Constraints limiting the nitrogen content of the drainage water can be formulated in at least two ways. One way is to limit the total annual quantity of nitrogen loss in the drainage water per unit area, $\sum_t L_t$, to a certain maximum amount, N^{max},

$$\sum_t L_t \leq N^{max} \tag{38}$$

Another way is to restrict the concentration of nitrogen in the drainage water. For example, constraints could be written ensuring that the NO_3-N concentration of the drainage water, n_t, does not exceed the permissible nitrate-N concentration for drinking water, e.g., 10 mg/ℓ.

$$n_t \leq 10 \qquad \forall t \tag{39}$$

This constraint can be modified to require that only the average NO_3-N concentration of the drainage water be no greater than the permissible nitrate standard.

The total organic carbon applied per unit area should not be in excess of the assimilative capacity of the soil. Overloading will cause anaerobic conditions and insufficient removal efficiencies for COD. About 4.5 to 11.2 metric tons/ha-year of organic carbon are needed to maintain a static organic carbon content in the soil. Most of the carbon will come from plant residues. Sandy soil can assimilate loadings of up to 67 metric tons/ha-year of organic carbon without detrimental effects (Thomas and Bendixen, 1969). Such high loading rates are not likely to be achieved by applying storage lagoon effluents; yet they may be obtained by spraying sewage sludge. Defining C_{max} as the maximum permissible organic carbon load, the constraint limiting the application of organic carbon

$$\sum_t C_t^p \, IR_t \leq C_{max} \qquad p = \text{organic carbon} \tag{40}$$

will rarely be limiting for wastewater effluent disposal on land.

Stream Water Quality

So far various land based wastewater treatment and disposal alternatives have been modeled. The resulting effluent from these facilities will be discharged into the stream and affect its quality. The extent to which all discharges affect water quality will determine, along with each facility's cost, the extent to which such facilities will be built and operated.

Recall that the effluent from the storage lagoon, $\overline{QW}_{ti}$, is divided into that which is discharged into land, LQ_{ti}, and that which is discharged into the river or stream, RQ_{ti}, at each site i and month t.

$$\overline{QW}_{ti} = LQ_{ti} + RQ_{ti} \qquad \forall i,t \tag{41}$$

If land is used for wastewater disposal, essentially all the pollutants are removed except nitrogen, and the quantity, $L_{ti}IA_i$, or concentration, n_{ti}, of nitrogen may be limited by constraint equation 38 or 39 if these are included in the model. The quantity of drainage water that eventually enters the stream from the land application site is difficult to predict and depends considerably on the particular site and its relation to the stream. Whatever it is, it will usually be negligible in comparison to the total streamflow and hence can be ignored.

The quality of each pollutant p discharged to the stream directly from the storage lagoon at each site i, as obtained from equations 13 and 14, equals the lagoon effluent concentration $\overline{C}^p_{ti}$ times the effluent flow, RQ_{ti}. As previously indicated, nitrogen is perhaps the only significant pollutant contained in the drainage water of many land application sites, and its quantity, from equation 35, is L_{ti}. Hence, the total mass, m^p_{ti}, of each pollutant entering the stream equals

$$m^p_{ti} = \begin{cases} \overline{C}^p_{ti}\ RQ_{ti} + L_{ti} & p = \text{nitrogen} \quad (42) \\ \overline{C}^p_{ti}\ RQ_{ti} & \forall p \neq \text{nitrogen} \quad (43) \end{cases}$$

The total mass and concentration of each pollutant p at any particular site in a stream will depend on the quantities of waste discharged upstream and on numerous factors such as the stream flow, velocity, temperature, depth, decay rates, and the like. A variety of equations have been developed and used to predict, for given stream flow and wastewater effluent conditions, the average mass of pollutant p that will exist at a site j in the stream in month t resulting from a discharge of one unit mass of pollutant p at site i upstream. These coefficients can be denoted as d^p_{ijt}, and are functions of the design stream flow conditions in period t. Hence for each discrete flow augmentation level [$\sum_r Q^r_t$, (equation 4), plus the appropriate interflow between upstream reservoirs r and site j], there will be a set of coefficients, d^p_{ijt}. The total mass, M^p_{tj}, of pollutant p at each site j will equal that from uncontrolled sources, μ^p_{tj}, plus that resulting from the controlled discharges.

$$M^p_{tj} = \sum_i d^p_{ijt}\ m^p_{ti} + \mu^p_{tj} \qquad \forall t,j \qquad (44)$$

For wastewater effluents having a biochemical oxygen demand, BOD, the coefficients d^p_{ijt} for estimating both BOD and dissolved oxygen deficits are described in the symposium paper by DeLucia and Chi (1975). These are just some of the types of equations developed to predict, in a very simplified manner, stream water quality.

Constraints limiting the concentration of pollutants at any particular site j would have the general form:

$$M^p_{tj}/F_{tj} \leq \text{Maximum allowable concentration at site j in month t} \qquad \forall t,j \qquad (45)$$

where F_{tj} is the design streamflow at site j in month t.

Artificial Instream Aeration

Artificial instream aeration is another method of increasing the dissolved oxygen concentration of streams. This is usually accomplished by injecting oxygen or air into water through a network of perforated pipes or by rotating devices that cause surface turbulence, thereby increasing the area over which oxygen transfer can occur. These methods may be particularly efficient for the temporary improvement of near anaerobic conditions, i.e., at sites where the dissolved oxygen deficits are relatively high.

The oxygen transfer rate due to aeration devices varies directly with the oxygen deficit, the water quality, temperature, and with the flow. Denoting H_{ti} as the rate of oxygen transfer per unit of power input under actual conditions, H_s as the rate of oxygen transfer under standard conditions (i.e., tap water quality at 20°C, and at one atmosphere of pressure), DOD_{ti} as the dissolved oxygen deficit concentration at site i in period t (i.e., a particular value of the left-hand side of one of equation 44), and K_{ti} a parameter related to the actual streamflow conditions at site i in period t, then

$$H_{ti} = H_s K_{ti} \, (DOD_{ti}) \qquad \forall i,t \qquad (46)$$

The oxygen transfer rate, OR_{ti}, at site i from artificial aeration is the product of the weight of oxygen per power input unit, H_{ti}, the total power input per unit of time, W_{ti}, and a coefficient β needed for unit conversion.

$$OR_{ti} = \beta H_{ti} \, W_{ti} \qquad (47)$$

The oxygen transferred per unit of time, OR_{ti}, at each site i cannot exceed the aerator capacity KA_i times the total weight transferred per unit power input, H_{ti},

$$OR_{ti} \leq \beta H_{ti} \, KA_i \qquad (48)$$

which is equivalent to specifying that the power input per unit of time, W_i, cannot exceed the capacity, A_i.

Assuming that the oxygen transfer per unit of time, H_{ti}, is an average value throughout period t and that the aerators are operating at a constant power input W_{ti}, the estimation of the minimum cost combination of capacities, KA_i, and power inputs, W_{ti}, can be simplified. Since there is no need for (and higher costs associated with) extra capacity, power input per unit of time can be equated to capacity KA_i. Hence from equations 46 and 47

$$OR_{ti} = \beta H_s \, K_{ti} \, (DOD_{ti})(KA_i) \qquad (49)$$

in which both DOD_{ti} and KA_i are unknowns. The total annual cost $CA_i(KA_i)$ is now a function of only the capacity, KA_i, and this portion of the overall objective can be written

$$\text{minimize} \; \sum_i CA_i(KA_i) \qquad (50)$$

MODEL SYNTHESIS

To illustrate how these model components can be combined into a more comprehensive model, it will be sufficient to consider the simple river basin system shown in Figure 2. This system includes two reservoirs (sites 1 and 2), a source of BOD which can be treated and then stored in a lagoon for later discharge onto land and/or into the water (site 3), the river system itself, which contains one point at which artificial aeration may be feasible (site 5), and a water quality site (site 6) in which standards exist restricting the chloride, BOD, DO, nitrogen, and phosphorus concentrations.

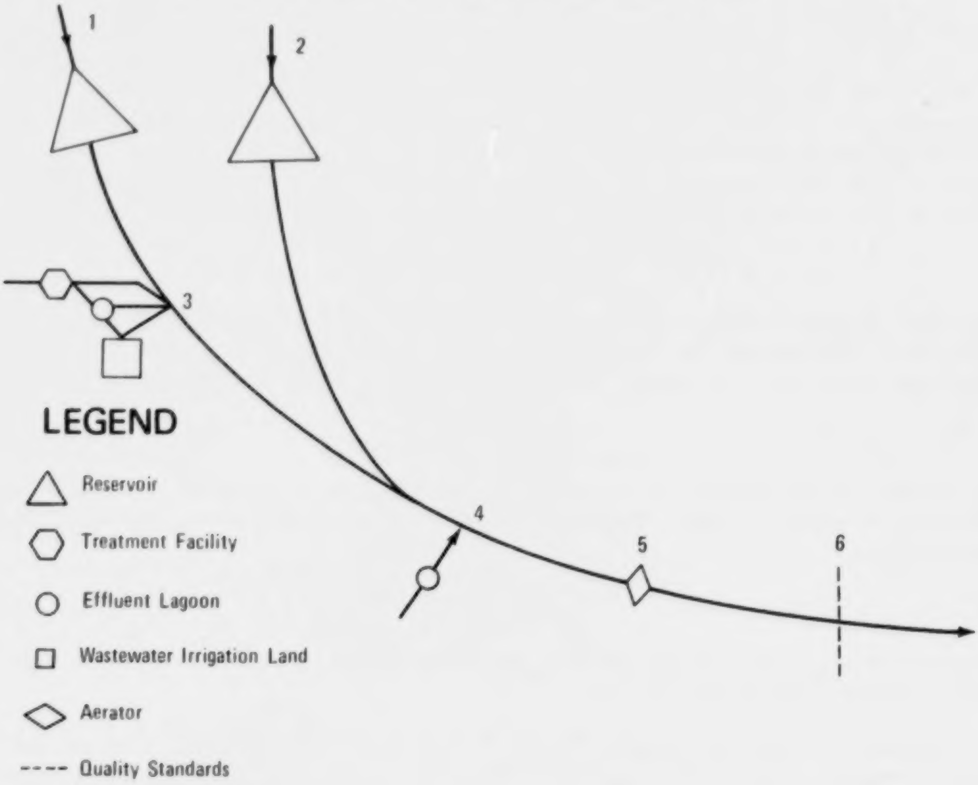
1
2
3
4
5
6
LEGEND
Reservoir
Treatment Facility
Effluent Lagoon
Wastewater Irrigation Land
Aerator
---- Quality Standards

Figure 2. A hypothetical water quality management problem.

Flow Augmentation from Sites 1 and 2

At reservoir sites 1 and 2, reservoir 1 has a capacity of K^1. The existing capacity K^2 at reservoir 2 can be increased, $\hat{K}^2$, at a cost $C_2^2(\hat{K}^2)$. The costs for providing a 95% firm flow of Q^r at each reservoir site $(r=1,2)$ are the loss of benefits or opportunity costs $C_1^1(Q^1)$ and $C_1^2(Q^2)$ of using that firm flow elsewhere. Hence, the total reservoir costs equal

$$CR = C_1^1(Q^1) + C_1^2(Q^2) + C_2^2(\hat{K}^2) \tag{51}$$

where for each reservoir site r, overyear continuity requires

$$S^r_y + I^r_y - \alpha_y Q^r - E^r_y = S^r_{y+1} \qquad \forall y \qquad (52)$$

for each year y of record and

$$\alpha y = \begin{cases} 1 \text{ if rank m of } I_y \text{ is } \leq (n+1)(.95) \\ <1 \text{ otherwise, depending on what is considered} \\ \text{reasonable in years that a failure occurs} \end{cases} \qquad (53)$$

(If y = the last year of record, then y+1 = the first year of record.)

Within-year continuity requires

$$s^r + \frac{I^r_t}{\sum_t I^r_t} Q^r - Q^r = s^r_{t+1} \qquad \forall t,r \qquad (54)$$

$$\text{if } t = 12,\ t+1 = 1$$

The storage capacity, K^r, necessary to release a 95% firm yield of Q^r_t is estimated from

$$S^r_o \leq S^r_y \qquad \forall y,r \qquad (55)$$

and

$$S^r_o + s^r_t \leq \begin{cases} K^r & r = 1 \\ \\ K^r + \hat{K}^r & r = 2 \end{cases} \qquad \forall t \qquad (56)$$

Waste Treatment at Site 3

Since carbonaceous BOD is the dominant waste at site 3, a cost effective analysis can be made of potential wastewater treatment facilities at site 3, each associated with a particular removal fraction, X_3. Once this has been done, by using procedures already discussed, the treatment cost CT_3 can be expressed as a function of the removal fraction X_3.

$$CT_3 = CT_3(X_3) \qquad (57)$$

The effluent concentration of carbonaceous BOD equals

$$CW^p_3\ (1-X_3) = C^p_3 \qquad p = \text{carbonaceous BOD} \qquad (58)$$

The effluent concentration of all other pollutants, including nitrogenous BOD, nitrogen, and phosphorus, is

$$CW^p_3(1-X_3/\gamma^p_3) = C^p_3 \qquad (59)$$

Storage Lagoon at Site 3

The equations for the lagoon at site 3 are those required for the continuity of storage and pollutant concentration.

$$\epsilon_{t3} S_{t3} + QW_{t3} - \overline{QW}_{t3} = S_{t+1,3} \qquad \forall t \qquad (60)$$

$$KL_3 \geq S_{t3} \qquad \forall t \qquad (61)$$

$$\left(\frac{S_{t3} + S_{t+1,3}}{2}\right) \left(\overline{C}^P_{t+1,3} - \overline{C}^P_{t3}\right) = QW_{t3} C^P_3 - \overline{QW}_{t3} \overline{C}^P_{t3}$$

$$- k^P_{t3} \overline{C}^P_{t3} \left(\frac{S_{t3} + S_{t+1,3}}{2}\right) \qquad \forall t \qquad (62)$$

Additional constraints may be required to define appropriate detention times, loading rates, and lower bounds on concentration reductions in the storage lagoon.

$$\frac{S_{t3} + S_{t+1,3}}{2\ QW_{t3}} \geq \text{minimum detention time} \qquad \forall t \qquad (63)$$

$$KL_3 \geq \frac{h_3}{D_t} (QW_{t3}\ C^P_3)/LR_h \qquad p = BOD \qquad \forall t \qquad (64)$$

$$\overline{C}^P_{t3} \geq \text{some minimum fraction of influent concentration, } C^P_3 \qquad \forall t \qquad (65)$$

The cost of the storage lagoon is

$$CL_3 = CL_3(KL_3) \qquad (66)$$

The lagoon effluent can be released to either the land or directly into the river.

$$\overline{QW}_{t3} = LQ_{t3} + RQ_{t3} \qquad (67)$$

Wastewater Disposal on Land at Site 3

Constraint equations are needed to define the soil moisture balance and ensure that the application of wastewater does not exceed the soil drainage capacity.

$$M_{t+1,3} = M_{t3} + IR_{t3} + P_{t3} - E_{t3} - d_{t3} \qquad \forall t \qquad (68)$$

$$M_{t3} \leq M_3 \qquad \forall t \qquad (69)$$

$$d_{t3} \leq D_3 \qquad \forall t \qquad (70)$$

The land area required depends on the application rate and the quantity released to the land

$$LQ_{t3} = IR_{t3}\ IA_3 \qquad \forall t \qquad (71)$$

The application rate in each month should not exceed some maximum safe sustained value.

$$IR_{t3} \leq IR_{max,3} \qquad \forall t \qquad (72)$$

Also required are the inventory equations for soil organic and inorganic nitrogen.

$$Q_{t+1,3} = (1-m_{t3})\ O_{t3} + XO_{t3} \qquad \forall t \qquad (73)$$

$$I_{t+1,3} = I_{t3} + m_{t3}O_{t3} + XI_{t3} - L_{t3} - N_{t3} \qquad \forall t \qquad (74)$$

where

$$XO_{t3} = \alpha\ \overline{C}^p_{t3}\ IR_{t3} \qquad p = \text{nitrogen} \qquad \forall t \qquad (75)$$

$$XI_{t3} = (1-\alpha)\ \overline{C}^p_{t3}\ IR_{t3} \qquad p = \text{nitrogen} \qquad \forall t \qquad (76)$$

The quantity of nitrogen in the drainage water equals

$$L_{t3} = d_{t3}(I_{t3} + I_{t+1,3})/2M_3 \qquad \forall t \qquad (77)$$

The cost of the irrigation land area required for wastewater disposal is

$$CI_3 = CI_3(IA_3) \qquad (78)$$

Finally, the total quantity of each pollutant p entering the stream at site 3 equals that from the irrigation area, if the pollutant is nitrogen, and that discharged directly from the lagoon, if any.

$$m^p_{t3} = \overline{C}^p_{t3}\ RQ_{t3} + \begin{cases} L_{t3} & \text{if } p = \text{nitrogen} \\ 0 & \text{if } p \neq \text{nitrogen} \end{cases} \qquad \forall t \qquad (79)$$

Lagoon Storage at Site 4

For a known monthly flow QW_{t4} of wastewater containing a chloride concentration of C_4, continuity of storage requires that

$$\varepsilon_{t3}S_{t4} + QW_{t4} - \overline{QW}_{t4} = S_{t+1,4} \qquad \forall t \qquad (80)$$

where

$$KL_4 \geq S_{t4} \qquad \forall t \qquad (81)$$

The chloride concentration $\bar{C}_{t4}$ in the effluent, $\overline{QW}_{t4}$, discharged into the stream, is obtained from the mass balance:

$$\left(\frac{S_{t4} + S_{t+1,4}}{2}\right) (\bar{C}_{t+1,4} - \bar{C}_{t4}) = QW_{t4}\, C_4 - \overline{QW}_{t4}\, \bar{C}_{t4} \qquad \forall t \qquad (82)$$

The above equation assumes no loss of chloride in storage. The last term represents the mass of chloride, m^p_{t4} (p = chloride), released to the stream at site 4 in each period t.

The cost of the lagoon equals

$$CL_4 = CL_4(KL_4) \qquad (83)$$

Dissolved Oxygen Deficit at Site 5

The dissolved oxygen deficit concentration at site 5 results from the controlled BOD source at site 3 and all uncontrolled sources. For any particular design flow condition in each month t, the dissolved oxygen deficit at site 3 equals

$$DOD_{t5} = m^{p^c}_{t3} \cdot \bar{d}^{p^c}_{3jt} + m^{p^n}_{t3} \cdot \bar{d}^{p^n}_{3jt} + \mu'_{tj} \qquad \forall t \qquad (84)$$

where p^c and p^n are the carbonaceous and nitrogenous BOD indicators and μ' refers to the DO deficit from uncontrolled upstream sources.

Artification Aeration at Site 5

The reduction of dissolved oxygen deficit at site 5 is a function of the existing deficit, the capacity of the aerator and some known coefficients for each design flow condition.

$$OR_{t5} = \beta H_s K_1 (DOD_{t5})\ (KA_5) \qquad \forall t \qquad (85)$$

The dissolved oxygen deficit immediately downstream of the aerator is

$$DOD_{t5} - OR_{t5} = DOD^{+}_{t5} \qquad \forall t \qquad (86)$$

The cost of the aerator is

$$CA_5 = CA_5(KA_5) \qquad (87)$$

Stream Quality Standards at Site 6

For any particular design flow condition, the dissolved oxygen deficit, DOD_{t6}, at site 6 is simply a constant [θ_{56t} = exp(-reaeration rate constant times time of flow from site 5 to site 6)] times the deficit at site 5, plus the additional deficit, μ_{t56}, caused by uncontrolled BOD sources between sites 5 and 6.

$$DOD_{t6} = (DOD_{t5})(\theta_{56t}) + \mu'_{t56} \qquad \forall t \qquad (88)$$

The dissolved oxygen at site 6 will equal the saturation quantity of oxygen for the design flow condition, DOS_{t6}, less the deficit quantity DOD_{t6}. This quantity of DO can be converted to a concentration which may be required to be no less than some minimum allowable concentration. Letting F_{t6} be the design flow at site 6 in month t,

$$F_{t6} = Q_t^1 + Q_t^2 + \text{(interflow between sites 1, 2, and 6 exceeded 95\% of the time)} \qquad \forall t \qquad (89)$$

The dissolved oxygen standard can be written:

$$(DOS_{t6} - DOD_{t6})/F_{t6} \geq \text{minimum allowable DO concentration in month t at site 6.} \qquad \forall t \qquad (90)$$

Constraints ensuring that other pollutant concentrations are no greater than their maximum allowable concentrations can be expressed as:

$$(\sum_i d^p_{ijt}\, m^p_{ti} + \mu^p_{t6}) / F_{t6} \leq \text{maximum allowable concentration of pollutant p at site 6 in month t} \qquad \forall t,p \qquad (91)$$

Model Objective

The overall objective of this cost-effective model is to minimize the total annual cost, i.e., the sum of equations E1, E7, E16, E28, E33, and E37.

$$\text{minimize } CR + CT_3 + CL_3 + CI_3 + CL_4 + CA_5 \qquad (92)$$

MODEL SOLUTION

There are, no doubt, numerous ways to solve the model developed in the proceeding section, or any similar water quality protection model. Since there are some nonlinear terms in the objective and constraints, it is certainly possible to obtain at least locally optimal solutions with a nonlinear programming algorithm, e.g., the sequential unconstrained minimization technique (SUMT) developed by Fiacco and McCormick (1968). While this and other nonlinear programming techniques are capable of solving water quality protection models similar to those described in this paper, they are usually not as readily available or operational in the U.S. as are linear, separable, mixed integer, or dynamic programming algorithms, some of which can result in globally optimal solutions.

Note that the objective function, while nonlinear, is separable. Hence each separate function can be approximated by linear segments if desired. Figure 3 reviews three procedures for doing this. Linear programming, using approximation methods (a) or (b), can be employed if the function F(X) in Figure 3 is to be maximized. Note method (b) requires no additional constraints for increased accuracy, only an increase in the number of variables w_ℓ. Yet method (b) cannot be used if the concave function F(X) is to be minimized. For minimization of concave functions (or maximization of convex functions) separable programming applied to method (a) will ensure that each allocation x_ℓ equals its upper bound, $a_\ell - a_{\ell-1}$, before $x_{\ell+1} > 0$, but there is no guarantee that a global minimum will be obtained. To guarantee a global minimum, the mixed integer programming method (c) can be used as a solution technique. Note that any function of the form $\sum_\ell F_\ell x_\ell$ using method (a) is equivalent to $\sum_\ell \sum_j^\ell F_j (a_j - a_{j-1}) w_\ell$ using method (b) and $\sum_\ell (b_\ell + F_\ell \hat{x}_\ell) z_\ell$ using method (c). Details of these linearization and solution procedures can be found in most texts in operations research.

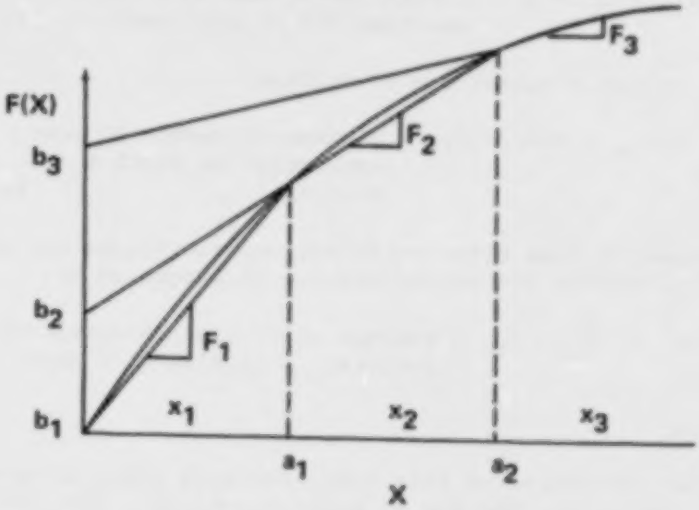

Figure 3. Piecewise linearization procedures.

Method (a)

Optimize $F(X) = F_1 x_1 + F_2 x_2 + F_3 x_3 + \ldots$
Subject to $X = x_1 + x_2 + x_3 + \ldots$
$x_1 \leq a_1$
$x_2 \leq a_2 - a_1$
$x_3 \leq a_3 - a_2$
.
.
.

Method (b)

Maximize $F(X) = F(a_1) w_1 + F(a_2) w_2 + F(a_3) w_3 + \ldots$
Subject to $X = a_1 w_1 + a_2 w_2 + a_3 w_3 + \ldots$
$w_1 + w_2 + w_3 + \ldots \leq 1$

Method (c)

Minimize $F(X) = b_1 z_1 + F_1 \hat{x}_1 + b_2 z_2 + F_2 \hat{x}_2 + b_3 z_3 + f_3 \hat{x}_3 + \ldots$
Subject to $X = \hat{x}_1 + \hat{x}_2 + \hat{x}_3 + \ldots$
$\hat{x}_1 \leq a_1 z_1 \quad \hat{x}_2 \leq a_2 z_2 \quad \hat{x}_3 \leq a_3 x_3 \ldots$
$z_1 + z_2 + z_3 + \ldots \leq 1$
all z_i integer

The products of quotients of two or more decision variables found in the constraint set pose a more difficult problem in model solution. At each wastewater treatment site these nonlinear expressions result from a consideration of effluent storage and land application of wastewater effluent. If only one, or perhaps two, pollutants are of concern at each site, then each treatment/disposal site can be examined separately, by dynamic programming or by gradient techniques, to develop a cost function for the mass of pollutant discharged into the stream at that site (Koenig and Loucks, 1975). This permits much of the detail of each treatment/disposal site to be omitted from the overall river basin model.

If a variety of pollutants are discharged into the receiving water at each wastewater treatment site, then it is necessary to include the model components of each treatment site in the overall river basin model. This may pose a problem if computer capacity is limited, especially if any linear programming algorithm or its extensions are used.

Models similar to, but larger than that constructed in the previous section have been solved at Cornell University by using separable and mixed integer algorithms. Separation of product terms has been accomplished by use of logarithms (to the base two) and piecewise linearization procedures. The separation approach is illustrated below for a product of two nonnegative unknowns, X and Y having maximum values of X_{max} and Y_{max}.

$$\text{Let } cz = XY$$

Assume $X, Y \geq 1$ and $ax = X$, $by = Y$.

The constants a, b, and c are chosen so as to ensure that x, y, and z do not exceed 10. It is in the range from 1 to 10 that the piecewise linearization of the $\log_2$ function can be relatively accurate.

Next, define variables $\log_2 x$, $\log_2 y$, and $\log_2 z$ such that

$$\log_2 c + \log_2 z = \log_2 a + \log_2 x + \log_2 b + \log_2 y$$

or

$$\log_2 z - \log_2 x - \log_2 y = \log \left(\frac{ab}{c}\right)$$

Now all that is needed is to relate each variable x, y, and z to its $\log_2$, e.g.,

$$X = \sum_{i=1}^{n} x_i$$

$$0 \leq x_i \leq \frac{10}{n} \qquad \forall i$$

$$\log_2 x = \sum_{i=1}^{n} \alpha_i x_i - \text{constant}$$

where α_i is the average slope of the logarithmic function in interval i. For example, if the interval of each x_i is 1 (n=10), then the values of α_i, beginning at i=2 (i.e., $X \geq 1$) are 1.0, 0.58, 0.42, 0.32, 0.26, 0.22, 0.18, 0.17, and 0.15.

Assuming $X_1 \leq 1$, and that $\alpha_1 = 2$ (instead of ∞), then the constant in the above equation is 2.

If the α_i's associated with each variable (e.g., x, y, and z) are the same, then separable programming will yield global solutions, albeit approximate ones to the original nonlinear problem. If a small value is assigned to any variable in the solution of the model, it is usually a signal that the value should be 0, and the model can be resolved after setting it equal to 0.

Admittedly this logarithmic separation is a relatively crude approach to solving the model and it takes a few iterations to determine the appropriate constants a, b, and c and zero values variables that are contained in nonlinear expressions. But the method works and the solutions obtained so far have been sufficiently accurate for the preliminary screening of design and operating policy alternatives for water quality protection.

Undoubtedly the most difficult nonlinearity encountered in any water quality protection model that includes both the river flows and wastewater effluent discharges as endogenous or unknown variables is the dependence of the pollutant transfer functions, d^p_{ijt}, on streamflow. From a practical point of view, it seems wise to separate the analysis of design streamflows, Q^r_t, from the remainder of the problem. Before solving the water quality portion of the problem, a variety of reasonable flow vectors $Q^r = \{Q^r_t\}$ can be assumed, and their minimum cost, CR, can be determined by solving equations E1 through E6. Then for each Q^r_t, the appropriate transfer coefficient, d^p_{ijt}, can be derived for use in the water quality model, equations E7 through E42, where the cost, CR, is now known. The vector Q^r that minimizes equation E42 will be the most cost effective of all those assumed flow vectors Q^r. This approach is discussed in considerable detail elsewhere (Loucks and Jacoby, 1972).

CONCLUSIONS

Rather than a comprehensive state-of-the-art report on water quality protection modeling, this paper has attempted to briefly outline several approaches to water quality protection modeling. Emphasis has been placed on problem definition and model development, not on model solution techniques. The models presented are not intended to be definitive; they only illustrate how various water quality control subsystems can be modeled and analyzed.

Problem definition, modeling, and analysis involve considerable judgment in addition to some mathematical and computational skills. How this is accomplished in any particular situation depends not only on the problem itself, but also on the skill of the systems analyst(s), and on the available data, programming algorithms, and computational facilities. Clearly, the systems models and techniques illustrated in this paper are not substitutes for judgment, but are only a means of enhancing this judgment by providing information and opportunities that otherwise might not be available.

Experience to date has demonstrated that some of the more obvious types of information and opportunities provided by systems approaches to water quality problems include: 1) an increased capability for defining and evaluating possible alternatives and for keeping a wider range of options open and available for analysis at each level of decisionmaking; 2) an improved capacity for testing assumptions and data to estimate the effects of economic, hydrologic, political, and technological uncertainties; 3) a method whereby all assumptions and judgments, and the consequences of these assumptions and judgments, are made explicit and available for all to see and question, if so desired (This feature is viewed as a distinct disadvantage by some observers.); and 4) a means of communication between all the participants in the planning and evaluation stages.

The analysis of water pollution control problems, like any public policy planning process in a dynamic environment, is never completed. It is an iterative process, one that involves continual updating of information and techniques of analysis. Since this activity costs money, the extent of any increase in the types and precision of the data collected or any increase in modeling sophistication depends on the additional benefits to be achieved. These costs and benefits are often difficult to assess, especially when environmental quality problems are involved. Needless to say, however, if the systems analysis studies are done well at each level of decisionmaking, the increased data base should enhance the content of the debate over what decision to make--a debate that can center on which assumptions are best rather than on whether the best solution is given a particular set of assumptions.

As the art and science of environmental systems analysis mature and as more qualified analysts become available and involved in environmental quality problems, the contributions from quantitative systems analyses, together with the more traditional planning and economic-engineering studies, should have a substantial influence on environmental quality policymaking processes.

NOTATION

The following notation is listed in the order presented in the paper. Physical units are expressed in terms of length, L, mass, M, and time, T. Depending on the actual units used, many of the equations in the paper may require a constant for unit conversion, e.g., from mg to kg or from days to months.

Variables, Functions, and Parameters

$C_1^r(Q^r)$ = annual opportunity cost of a release Q^r from reservoir r, \$.

$C_2^r(K^r)$ = annual cost of a capacity K^r at a reservoir site r, \$.

S_y^r = initial storage volume in year at reservoir r, L^3.

I_y^r = net inflow to reservoir r in year y, L^3T^{-1}.

E_y^r = excess release from reservoir r in year y, L^3T^{-1}.

S^r_o = overyear storage capacity of reservoir r, L^3.

β_t = proportion of average inflow in month t to total annual average inflow.

I^r_{yt} = inflow in month t in year y, L^3T^{-1}.

I^r_t = average inflow in month t at reservoir r, L^3T^{-1}.

s^r_t = initial within-year storage in month t in reservoir r, L^3.

CW^p_i = concentration of pollutant p in wastewater influent at site i, ML^{-3}.

QW^k_i = quantity of wastewater routed through treatment process k at site i, L^3T^{-1}.

K^i = total number of treatment processes k at site i.

$CT^i_i(QW^k_i)$ = annual cost of treatment process k at site i having a hydraulic capacity of QW^k_i, \$.

f^p_k = fraction of concentration of pollutant p removed by process k.

C^p_i = concentration of pollutant p in the wastewater effluent of the treatment facility at site i, ML^{-3}.

X^p_i = fraction of pollutant p removed by treatment facility at site i.

$CT_i(X_i)$ = annual cost of treatment facility at site i as a function of its "design" efficiency X_i, \$.

$CL_i(KL_i)$ = annual cost of effluent storage lagoon of capacity KL_i at site i, \$.

KL_i = capacity of effluent storage lagoon at site i, L^3.

S_{ti} = initial storage volume in lagoon in month t at site i, L^3.

$\overline{C}^p_{ti}$ = average concentration of pollutant p in the effluent lagoon during month t at site i.

k^p_t = removal rate constant for pollutant p in month t, T^{-1}.

$\overline{QW}_{ti}$ = volume of wastewater effluent discharged in month t from storage lagoon at site i, L^3T^{-1}.

ϵ_{ti} = evaporation rate fraction.

θ_{min} = minimum detention time, T.

LA_i = lagoon area at site i, L^2.

D_t = number of days in month t.

LR_h = allowable pollutant loading rate for storage lagoon of depth h, $ML^{-2}T^{-1}$.

h_i = maximum depth of storage lagoon volume at site i.

$CI_i(IA_i)$ = annual cost of land area, IA_i, used for wastewater disposal at site i, \$.

d = soil drainage capacity, LT^{-1}.

h_o = vertical distance between impermeable layer and tile drain, L.

s = horizontal distance between tiles, L.

H = depth of water table above impermeable layer, L.

k = hydraulic conductivity of soil, LT^{-1}.

M_{ti} = moisture content of soil at beginning of month t at site i, L.

d_{ti} = drainage rate during month t at site i, LT^{-1}.

E_{ti} = average evapotranspiration during month t at site i, LT^{-1}.

P_{ti} = average precipitation during month t at site i, LT^{-1}.

IR_{ti} = application rate of wastewater effluent in month t at site i, LT^{-1}.

LQ_{ti} = quantity of wastewater effluent discharged to land in month t at site i, L^3T^{-1}.

$IR_{max,i}$ = maximum wastewater application rate at site i, L,T^{-1}.

I_t = soil inorganic nitrogen at beginning of month t, ML^{-2}.

m_t = fraction of soil organic nitrogen converted to inorganic nitrogen during month t.

N_t = inorganic nitrogen removed from the soil by plant growth during month t, ML^{-2}.

O_t = soil organic nitrogen at beginning of month t, ML^{-2}.

XI_t = inorganic nitrogen added to soil by wastewater application during month t, ML^{-2}.

XO_t = organic nitrogen added to soil by wastewater application during month t, ML^{-2}.

α = fraction of organic nitrogen in the total nitrogen content of the wastewater applied to land.

n_t = concentration of NO_3-N in the drainage water, ML^{-3}.

C_{max} = maximum permissible organic carbon loading rate, $ML^{-2}T^{-1}$.

RQ_{ti} = quantity of wastewater effluent discharged to the stream at site i during month t, L^3T^{-1}.

m^p_{ti} = mass of pollutant p entering stream at site i during month t, MT^{-1}.

d^p_{ijt} = mass of pollutant p at a downstream site j resulting from the discharge of one unit of pollutant p at site i upstream, in month t, M.

M^p_{tj} = total mass of pollutant p at site j in month t, M.

μ^p_{tj} = quantity of pollutant p at site j in month t resulting from uncontrolled sources, M.

F_{jt} = design streamflow at site j in month t, L^3.

H_{ti} = rate of oxygen transfer per unit of power input under actual conditions L^{-1}.

H_s = rate of oxygen transfer per unit of power input under standard conditions, L^{-1}.

DOD_{ti} = dissolved oxygen deficit concentration in month t at site i, ML^{-3}.

OR_{ti} = oxygen transfer rate, MT^{-1}.

W_{ti} = power input per unit of time, LMT^{-2}.

KA_i = aerator capacity, LMT^{-1}.

$CA_i(KA_i)$ = annual cost of aerator of capacity KA_i at site i, \$.

Indices

n = number of years (or segments)
m = rank of an annual flow
y = index for year
r = index for reservoir site
i = index for wastewater source or treatment site (or for segment of a nonlinear function)
t = index for month of year
p = index for a type of pollutant

Operators

$\sum$ = sum
$\forall$ = for all
$\ni$ = such that

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DISCUSSION

KUZIN: In this paper, the current approach to solving complex problems of planning technical water protection measures for water basins has been described. The principles for creating an economic-mathematical model for optimizing a system of activities has been reported in detail.

1. The proposed model for determining minimum cost for a system of measures is static and is intended for a fixed planning period. Therefore, with each essential change in source conditions, the model must be reconstructed, and the earlier accepted solutions are corrected. This procedure can be considered as a process of uninterrupted planning.

2. A description of non-point source pollution control in cost-effect terms is considerably more complex than measures for point sources of water pollution. Therefore, when the influence of surface flow on water quality must be taken into consideration, it is necessary to consider the essential complication of the global model.

3. It is not clear what type of model is being used to calculate pollutant transformation in the water body. Simplified models based on instantaneous mixing of wastewaters with river water and numerical solutions by a computer do not present any particular difficulties. However, dilution in rivers is calculated with a model based on the turbulent diffusion differential equation, the complexity of solving optimization problems increased, especially in the case of a subdivided grid system.

4. When the purpose function for solving optimization problems for the system of water protection measures presented generally in the report are minimized, the parameters for each water body are determined.

The approach to the development of models is also followed in the USSR, and offers a perspective method for solving the complex problems of planning water protection. It is necessary to show that the application of methods for optimizing and planning water protection in the USSR can be broader because of the possibility for manipulating the treatment level of wastewaters from industrial establishments. As you know, stringent limits have been placed on industrial discharges by a 1972 law enacted in the US.

STANISHEVSKIY: Dr. Louck's report contains information on economic-mathematical modeling of the following water protection measures: (1) flow augmentation; (2) wastewater treatment; (3) construction of wastewater lagoons for preliminary treatment; (4) irrigation by wastewaters for preliminary treatment; and (5) artificial aeration. These measures can be used jointly or examined as alternatives for water protection.

It is necessary to note that in most previous U.S. efforts related to optimization problems, water quality was monitored only for one parameter--dissolved oxygen concentration. In this report, four indicators were used--dissolved oxygen concentration, biochemical demand in oxygen, content of chlorides, and nitrates.

As a whole, the material presented in the report is of scientific interest and confirms that research approaches for development of models for water protection measures and their optimization being carried out in the U.S. and USSR have been chosen wisely. An exchange of scientific information on the problem will be beneficial to both countries.

THOMANN: I would just like to indicate that it is no more difficult to include diffusion in any of these optimzation models. In fact, the inclusion of a turbulent and tidal dispersion and diffusion in estuaries in our country goes back ten years to the Delaware estuary. Optimization includes multidimensional, (two- or three-dimensional steady-state diffusional problem structures), as well as multivariable feedback and feed-forward systems. These extensions do not necessarily contribute to the complexity of the problem providing all extensions are linear.

Also it is getting easier to write equations but not so easy to apply those equations to the real world. It seems that once the door was opened, our ability to write equations increased dramatically, but I really believe the problems we face are the successful application of those equations to actual planning problems and situations.

OPTIMIZATION OF WATER QUALITY CONTROL IN A RIVER BASIN

A.K. Kuzin

INTRODUCTION

In the USSR, a comprehensive approach to the development of water pollution control is represented in the general plan for use and protection of water resources, especially when capital investments for the development of sewer systems under the plan have been determined. The structure, composition, and effectiveness of water protection measures are determined during the development of long-term regional and river basin utilization plans. Broad use and protection of water resources should be in accord with the long-term plans for the national economy. The methodology for the development of such schemes, however, has not been perfected. In the majority of cases, only a few sites for effluent treatment plants have been evaluated. The application of methods of optimization of technical solutions is limited.

The implementation of these schemes within the long-term plans of national economic development can lead to rigid requirements for water quality protection construction projects which, at present costs, would mean an annual increase in expenditures of millions of rubles. Evaluation of the numbers of variants of future water pollution control systems, and decisions on optimum solutions to problems by a specified evaluation criterion, will be possible only through a wide application of methods of mathematical modeling with modern computers for analyzing very complex models.

One main difficulty in developing an acceptable technique for optimizing anti-pollution measures within the limits of a river basin is the exceptional complexity of analyzing the interrelationships of water quality, expenditures for protection, and economic profits. Even for a small section of a river, under the conditions of a probabilistic river flow availability, the complexity of chemical and biological processes of degradation, discharge of effluents into the water body, and the connection between the foregoing, can be only approximately established. Another serious difficulty is the multiplicity of possible combinations of control actions on water quality. It is obvious that a special scientific approach, such as systems analysis, is needed to define the optimum combination of questions related to the complex problem. The process of developing a model for optimizing water protection measures over the entire basin, classifying optimization problems, selecting basic principles to use for solution, and considering certain types of problems in detail is described for systems analysis.

Constructing Large-Scale Models of Water Quality Control

In a broad sense, water quality control is considered a system of effects on the basin and its environment. These effects are projected both for the distant future and for perturbation that may be caused by accidents. In the first case, long-term strategy can be developed by considering statistical objectives and corresponding capital investment policies. In the second case, it is only possible to prepare a response plan because it is impossible to forecast breakdowns at effluent treatment installations and oil pipelines, leaks from gas storage tanks, tanker accidents, and so forth. The combination of problems connected with the functioning of the river basin and its environment are considered in developing a strategy for water quality control.

First, we shall consider the presence of cause and effect links between the input, as an object of effect on the environment, and output, as a transformation resulting from different effects. The input is determined as a measure of the influence of controllable or non-controllable impulses that can be evaluated quantitatively. The output is determined as the state of water quality at a given point of the river basin at a corresponding moment of time. If we represent the water quality as a vector $\vec{U}_i(t)$ where $\vec{U}_i$ is the water quality in i-th part of the river basin at the period of time t, and the state of factors acting upon the system as a vector $\vec{H}_i(t)$, where $\vec{H}_i$ are the factors acting upon the i-th part of the river basin, then the relationship between the input and output can be determined with the help of a certain transition matrix F_i:

$$F_i\vec{U}_i(t) = \vec{H}_i(t) \qquad (1)$$

Equation 1 is the most general form that can be applied over a wide range of conditions. Indeed, the water state vector allows us to characterize its quality at any point in the basin at the required moment of time. The vector of effect factors can reflect all elements of man's economic activity affecting the state of the basin. The transition matrix, for different specific applications of this equation, show the change in the vector of the first type, depending on the value of the second one. In water quality control, during long-term changes, we are not interested in the state of water quality at each point or at any moment, but rather the average monthly or average annual indices for separate cross-sections of the basin. Therefore, detailed description of processes of transformation of water quality are not necessary. In practice, the periods of the effective functioning of water protection measures are assumed to be indefinitely long $[H_i(t)]$. This is due to the long period of construction, operation of water protection installations, and the considerable inertia of their effect.

In addition to the consideration of the limits of a river basin as a single hydrographical system (in a physical sense), socio-economic considerations also exist. These are revealed, in particular, when basin water quality goals, as well as technical water protection measures, are planned. Indeed, when water quality goals are established for the limits of the basin, the specific conditions affecting water quality can be fully evaluated, and the economic profits and losses, corresponding to different water quality goals, can be statistically evaluated. In planning for technical water quality protection through a complex study of the main sources of water pollution in the given basin, a wide application of methods for optimization and mathematical modeling is possible with modern computers to provide the required water goals for minimum resource expenditure.

Optimization of water protection measures over the entire basin is difficult because of the complexity of mathematical description, the diversity of environmental effects on the water quality state of the basin, and the voluminous initial information requiring preliminary mathematical treatment. Two approaches exist for modeling complex processes: to construct a gigantic "super-model" with the development of a complex algorithm of optimization, or to create a system of several small separately optimized models. The main advantage of the so-called "super-model" is the possibility of a totally integrated and centralized control of the system. Such models, however, have very serious disadvantages which make them inefficient when modeling complex manufacturing and/or economic processes.

For solving optimization problems in the field of water protection, the "super-model" cannot be recommended for the following reasons: Such a model would not be reliable, because admissible simplifications in small-scale systems (accepted during the construction of physical-biological-mathematical models of effluent transformation in a water medium relationship), characteristics of the existing and forecasting states of the basin, and the quality of effluents can essentially distort both the input and output of the global model of the basin and its environment. It is also known that the pollutant transformation equations are suitable for describing local processes and cannot be modeled for the entire basin with any degree of authenticity.

The modification and updating of "super-models" is complex. The realization of basin water protection programs is affected by a number of economic, technical, and social factors; therefore, the once-developed program would undergo constant specifications and modifications. In addition, it is practically impossible to simultaneously complete the whole complex of planned measures. Their realization, as a rule, is piecemeal. The global model requires time-consuming analyses, because obstacles can interfere with the process of perfecting programs for river basin water protection.

Thus, the construction of the "super-model" does not allow us to balance the effects and efforts in separate parts of the program or to investigate their influence on social transformations, which is necessary when ranking program components and the initial allocation of resources for completion is increased or decreased.

Further, the use of the global model in planning of water protection measures hinders the "man-machine" dialogue, which limits creative intervention in the decision-making process. The links between exact algorithmic methods of problem solution and non-mathematical ones and, in particular, heuristic methods, are excluded. Their application allows us to reject unnecessary calculations and obtain results with less expenditure of machine time.

Finally, the computational abilities of general-purpose computers are limited both in memory size and time required for calculations. Experience in solving water protection optimization problems within the limits of a river basin, by using the "M-222" computer, has shown that such problems can be programmed only if subdivided into less complex subproblems.

These considerations favor an alternative approach to the economic-mathematical modeling of river basins--a system of mutually coordinated optimization models. Effluent discharges, concentrated in small stretches of water bodies (within the limits of large towns or industrial centers), facilitate the creation of separate water protection installations, definable by optimization models. Within the limits of a river basin, hundreds of water protection installations can be organized. These are intended to maintain the required water quality in the adjacent sections with minimum capital outlay.

It is not obligatory to match the optimization models throughout the entire basin. It is frequently more expedient to match only certain models chosen by heuristic methods. In this case, a relationship between the optimization models in the evaluated stretches is established: The models of two neighboring stretches are connected in sequence through a transformation model which has a feedback link through an analyzer (Figure 1). The role of the transformation model is to transform the outputs from one model into the inputs of the other by an algorithm. The analyzer (or a group of individuals) creates the model on the basis of engineering intuition and determines if the input-output of joined models should be changed. Since the feedback process between models can be repeated a number of times, the analyzer must decide when to stop the iterative process of matching models. The water quality at the inflow and outflow of each evaluation stretch is thus established.

Classes of Optimization Models

Optimization problems can be solved by three classes of models: 1) optimization models with functionally different technical methods; 2) models with differing interactions of technical methods and types; 3) optimization models with operatively different technical solutions.

The optimum structure of water protection measures is determined by Class I models. The measures are divided into three groups depending on the stage of the water pollution process. The first group is related to the production process of the main product, such as transition to new raw materials or equipment that reduce the volume of effluents or improve its qualitative composition by recycling water or utilizing effluents, etc. The second group acts on effluents prior to discharge into water bodies: effluent treatment, or accumulation, burning, pumping into underground voids, etc. Finally, the third group utilizes measures that are implemented in the water bodies: dilution of effluents by water releases, water aeration, biological reclamation, and so forth.

The first group of pollution preventative measures is the most effective and reliable. Plans for water pollution management are developed within possibilities of advanced technology, as specified in the fundamentals of the water laws of the USSR (1971) and by Cherkinskiy (1971). Currently, rates and concentration of effluent discharge are defined relative to the units of production for particular branches of industry with some allowance for advanced technology developed within the framework of the CMEA (1973). Measures in the second and third group are alternatives only.

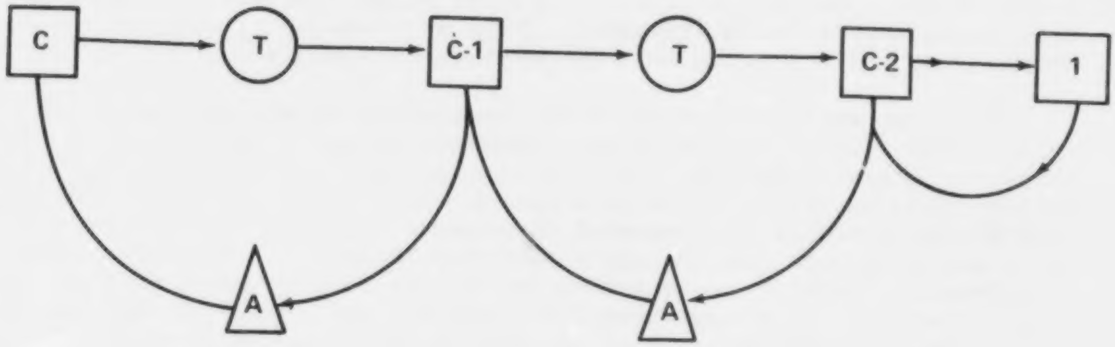

Figure 1. Diagram of links of models of one type when water protection measures are otpimized within the limits of a basin. C - number of models of one type, T - transformation model, A - analyzer.

The degree of efficiency of the measures with differing functions is estimated by the decrease in pollutant concentrations at the end of the river section under sutyd. When evaluating this type of problem, it is impossible to evaluate all the available technological solutions; generalized relationships of cost effectiveness can be derived from optimization models of class 2 and 3 solutions.

The class 2 models allow us to determine the optimum siting capacity and the number of waste treatment systems on the basis of information from class 1 models related to the amount of treatment required. Characteristics of the technical efficiency of alternative treatment methods, which can be derived from class 3 model output, must be known from an exact solution. Approximate solutions require only the generalized characteristics of alternative technologies.

The links existing among the models of the above classes are shown in Figure 2. The investigation of these links allows us to expose the characteristic features of a multilevel hierarchical system of models by a successive vertical arrangement of subsystems, interdependence of actions of subsystems of different levels, and the ability of top level subsystems to interfere at lower levels (Mesarovich et al., 1973).

The successive vertical arrangement of models results from the level of their abstraction. Higher level models are less structured and are difficult to formalize quantitatively. Their behavior is related to large subsystems which have broad strategic possibilities. Decision-making on the top level is the most complex. Models of a lower level describe the specific aspects of the system's behavior and are easily formalized. These problems have numerical solutions.

The interdependence of actions of the subsystems is the existence of two kinds of links between them: downward, to successively define the problem, and upward to specify the characteristics of technical and economical effectiveness of the solutions, i.e., the higher level defines the problems at the lower level. The higher level subsystems are assigned priorities so that they can interfere with and control subsystem actions at the lower level.

The interference takes place both prior to the moment of decision-making at the lower levels, and afterwards. In the first case, the action regions of the lower subsystems are specified (e.g., the degree of technical efficiency of the measures) on the condition that the efficiency of the whole system would be ensured by the chosen criterion. In the second case, previous decisions are corrected if the basic assumptions were erroneous.

As shown, the problem of optimizing water protection measures within the evaluation stretch has all the distinctive features of a complex multilevel problem. For a solution, it is expedient to use a multilevel approach. The experience of realizing complex large-scale problems shows that with the multilevel approach, the available resources are used more efficiently. The problem is considerably simplified by organizing the hierarchical structure.

Construction of a multipurpose system of models begins with development of a higher level of hierarchy. Information on the lower level can be obtained by aggregating the variables of this level, since this effects the efficiency of decision-making on the higher level. Aggregation allows the production of a simplified description of the lower-level system. Thus, for the first level subsystems of a hierarchical system of models for optimizing water protection measures over a river basin stretch, the information is represented as a simplified "costs-effectiveness" relationship for each measure. The information on each separate object in the particular measure is not considered.

The development of subsequent levels of a model's system is also based on a simplified description of the lower level by aggregating the variables. A certain mutual autonomy of the decision-making process exists on each level in relation to other levels. This makes it possible to pass a considerable amount of the work of finding an optimum plan for the entire system to the lower levels. The optimization of subsystems of each level is carried out by using appropriate mathematical methods and algorithms.

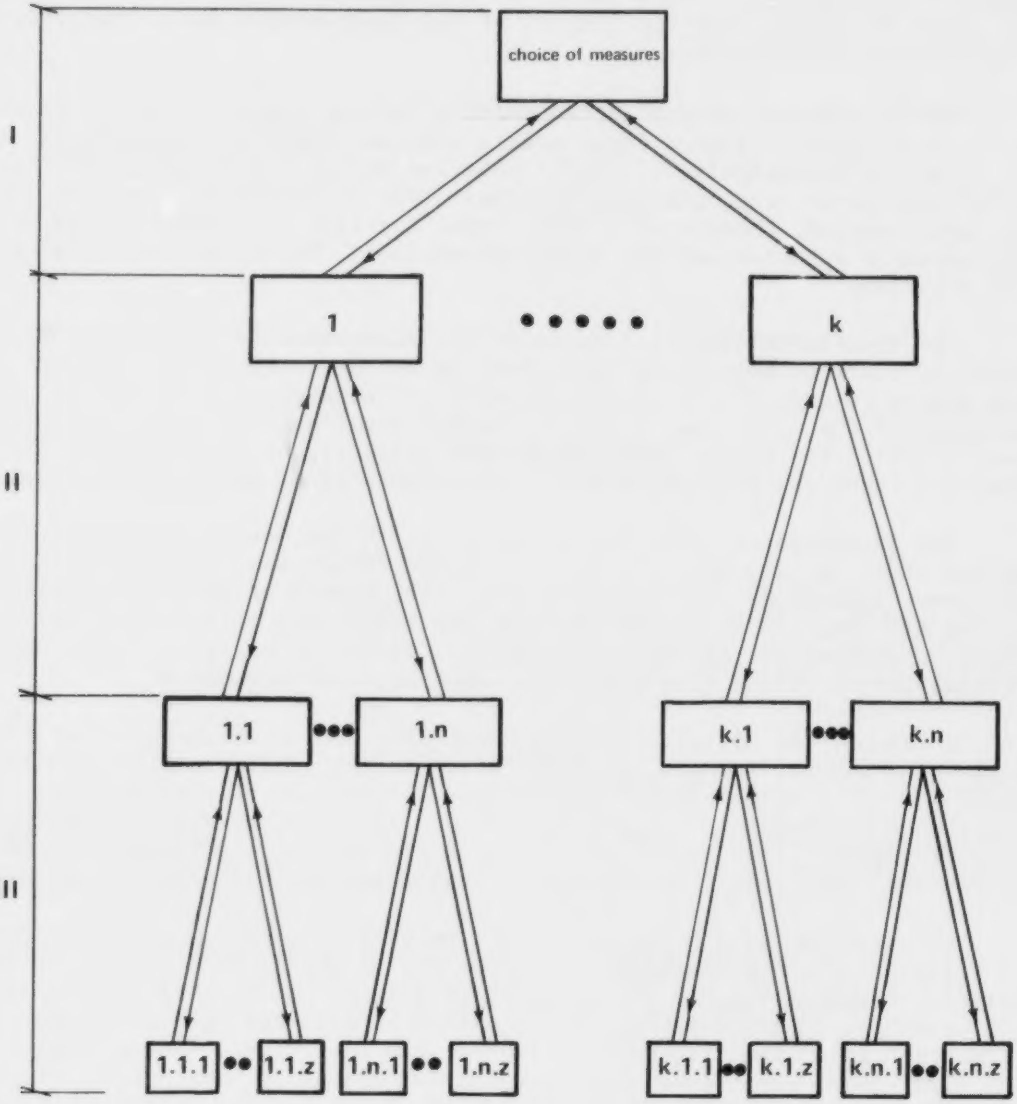

Figure 2. Diagram of choosing the type of measure and its parameters. I - choice of the type of measure; II - choice of capacity of objects entering the measure; III - choice of technical effectiveness of objects; k - number of possible measures; n - number of objects entering the measure; z - number of variants of the technical effectiveness of the object.

The construction of a three-level hierarchical system of models for optimizing water protection measures over a section of a basin is preferable in the most complex cases, particularly for evaluating sections with many water pollution sources that can be ameliorated by a variety of pollution control methods. A one or two-level system can be used successfully. The investigator decides on the number of hierarchical levels required. The decision is governed by the necessity to match model complexity with the complexity of the problem considered.

The Stages of Solving Optimization Problems

In addition to different classes and types of optimization problems for water protection measures, there are general methods for simulation. The development of an optimization model to protect water quality requires such stages as: defining the objective function of the problem, establishing limitations, choosing a suitable hydrological model and a model to simulate mixing and self-purification processes, analyzing and statistically treating technical-economical indices, choosing the numerical method, and developing a computer program framework.

Problem Formulization--

The initial step in a problem of any class or type is the formulation of objective function. One fundamental requirement is to minimize resources required for desired results.

The objective of the optimization procedures is to maintain the water quality standards for a specific body of water. These standards are defined as the maximum permissible concentrations (M.P.C.) of pollutants in water. The M.P.C. standards are defined in terms of the protection of human health (drinking water standards), protection of fishing resources, and the preservation of ecological systems. Water quality standards for municipal, recreational, and fishing waters have been established; additional standards exist to cover unique water bodies. Standards for industrial and irrigation water sources are being developed.

The effectiveness of the pollution control system is evaluated in terms of the total cost for the pollution control complex to achieve the required water quality standards.

For the most general case, this can be formulated as

$$\min S = \min_{P_l} \sum_{l=1}^{k} S_l(P_l), \quad (l = 1,2,\ldots,k) \tag{2}$$

provided

$$U^B(U^F, \sum_{l=1}^{k} P_l, \gamma, \eta) \leq U^{\partial on}, \tag{3}$$

where S, S_l are the total costs of the complex and l-th measure, correspondingly;
P_l - efficiency of the l-th measure;
$U^{\partial on}$ - M.P.C. of the pollutant in a water body;
U^B - calculated pollutant concentration, depending on the background

concentration;

U^F - efficiency of water protection measures, and also the intensity of mixing γ and self-purification η processes.

Modeling Processes in a Water Body--

The generalized linkage between water quality objectives and available resources is described in equations 2 and 3 for a water body. It is possible to link "process" models of self-purification, hydrology, and hydrography into economic mathematical models.

The discharge model is required to determine the hydraulic flow in the river system, particularly to determine the minimum mean monthly discharge per year with 95% confidence. This rated discharge is derived from the projected annual discharge curve, which should be based on long-term observational data (at least a 60-year period). If the discharge records are for shorter periods than derived statistically with defined confidence, limits can be developed. Binomial distribution curves are the most suitable for the annual discharge values. To establish the theoretical probability curve from observed discharge, the coefficient of variation, which is the standard deviation, is expressed as a percentage of the arithmetic mean discharge. The values for the coefficients and the computational formulas are given in the Hydrographic Manual (1972).

The variation within annual discharge patterns can be defined by developing discharge graphs (hydrographs) q=f(t). The discharge hydrograph for a river with regulated flows in particular stretches is controlled by the flow control strategy. The discharges can be determined from a model.

The so-called "mixing model" allows us to determine, first, the distance at which a complete mixing of the effluents with the water of the receiving body takes place, and second, the maximum pollutant concentration at any distance from the place of its discharge into the water body. The values of pollutant concentration in natural water and in effluents, the discharge flow and the discharge conditions of the effluents, and the various characteristics of the water body are required initial inputs to the models.

Selection of the different mixing models are outlined in the manual describing rated hydrographic characteristics (G.G.I., Leningrad, 1973). Finite difference models, in which the finite increments are substituted for the differentials in the steady turbulent diffusion differential equation, are recommended as the most suitable method for evaluating steady dilution in rivers. The equation in the finite difference form is reduced to an evaluation relationship to calculate the concentration's cross-section of the stream. A plane and spatial solution scheme exists for solving this problem; the spatial scheme provides a more exact solution. The accuracy of the calculation also increases if the transverse circulation and the irregularity of depth along the stream are considered.

If it is necessary to include self-purification processes, the one-dimensional water quality model based on the following equations is accepted as the most suitable solution by Vasilyev, Temnoyeva, and Shugrin (1965)

$$\frac{\partial O_2}{\partial t} + v\frac{\partial O_2}{\partial x} = \frac{1}{W}\frac{\partial}{\partial x}\left(DW\frac{\partial O_2}{\partial x}\right) + k_z(O_2^{np} - O_2) - kL + f_{O_2} \tag{4}$$

$$\frac{\partial L}{\partial t} + v\frac{\partial L}{\partial t} = \frac{1}{W}\frac{\partial}{\partial x}\left(DW\frac{\partial L}{\partial x}\right) - k_1 L + f_L \tag{5}$$

where v is the flow velocity;
W - flow cross-sectional area;
O_2^{np} - dissolved oxygen at saturation;
k_1 - BOD rate coefficient;
k_z - reaeration coefficient;
z - free surface water level;
f_{O_2}, f_L - mass of oxygen and organic substances delivered per unit of time, respectively, characterized by BOD.

The integration of equations 4 and 5 is performed with numerical methods of implicit and explicit difference schemes.

The appropriate type of pollutant transformation model for a water body depends on many factors (availability of initial data, required accuracy of calculation, computer resources, etc.). Therefore, the choice of model is significant for solving optimization problems.

Building "Cost Effectiveness" Relationships--

To develop and complete economic-mathematical models, it is necessary to describe water pollution prevention measures in terms of costs and effects. To build the relationship S(P), a statistical treatment of the technical-economic effectiveness of advanced technology is needed. Stanishevskiy and Kuzin (1974) have described a least square method which allowed the development of empirical formulae that have a high correlation coefficient.

In constructing relationships between costs and specific water protection parameters of a technological process, several variables must be analyzed simultaneously by multiple correlation statistics. For effluent treatment sites, a formula has been developed which relates costs for effluent treatment S^{or} to the treatment efficiency P, pollution concentration in untreated effluent U^{cm} and the capacity of the treatment process q.

$$S^{or} = \eta(P, U^{cm}, q) \tag{6}$$

For the majority of effluent categories, the relationship between costs and untreated effluent quality indices has a discrete character and cannot be described analytically. A considerable increase in pollutant concentration in effluents leads to a change in the treatment technology, since the "costs-quality" relation within the framework of existing technology is rather weak. For example, to decrease BOD_{20} in municipal effluents, where the index is within $100 \leq BOD_{20} < 250$ mg/ℓ, a common biological treatment is used, for which the cost indices differ by 10 to 15%; at BOD_{20} of effluents exceeding 250 mg/ℓ, preaerators are additionally provided, and instead of the common aerotanks, two-stage aerotanks with regenerators are used. This immediately increases the cost by 30 to 40%. Therefore, it is expedient to average technical-economic indices for separate gradations of effluent quality.

When this concept is incorporated into equation 6, it then becomes

$$S_r^{or} = \eta_r(P,q), \quad (r=1,2,\ldots\mu) \tag{7}$$

where μ is the number of water quality levels for which the cost indices are averaged.

In particular, given a BOD_{20} of untreated effluents up to 250 mg/ℓ and their discharge within $10 \leq q \leq 200{,}000$ m^3/day, the following formula for determining the expenditures for a decrease in oxidizability S^{oy} (kopeks/m^3) (Stanishevskiy and Kuzin, 1974) is derived:

$$S^{or} = \begin{cases} (15.103\,P^3 + 25.881 + 4.74)q^{(0.015P-0.4122)} & \text{given } 0.2 \leq P \leq 0.85 \\ (406.80\,P - 308.13)q^{(0.684P-0.9847)} & \text{given } 0.85 \leq P \leq 0.99 \end{cases} \tag{8}$$

The form of the cost-effectiveness relationship in conjunction with other factors, determines the choice of the numerical method for the optimization problem; therefore, the problems of constructing the relationships and the choice of numerical methods are directly interrelated.

Choice of Numerical Optimization Methods--

There is no formalized method for selecting numerical methods to solve optimization problems. However, the requirements for an efficient method are known: its complexity should not exceed the complexity of the formulated problem; the accuracy of the solution must be adequate; and the computer program should run on general purpose machines with minimum computing time.

Optimization problems of various classes and types can be solved by integer, linear, dynamic programming, and steepest gradient descent methods and some of their combinations. These methods have no essential advantage, however, for solving simple problems when the measures within a non-ramified stretch of the river are optimized and a small number of water protection installations (up to ten) is examined. However, relatively greater calculation accuracy can be derived from integer or dynamic programming.

When a medium class computer such as the M-222 is used for calculations and the number of installations in a pollution control system is increased by 50 or more, then efficacy of the various numerical methods described differs. The steepest gradient descent method is the most efficient for defining an approximate solution to the extremes of a function with several variables. More accuracy can be obtained from the combined method which is a synthesis of dynamic programming and relaxation research. Integer linear programming and dynamic programming are limited by computer memory.

During the formulation of optimization problems for small ramified stretches of a river basin, dynamic programming and the steepest gradient descent method are accepted methods. The choice of the numerical method depends on the character of the problem solved, the type of model of pollutant transformation in water bodies, the form of cost effectiveness relationships, and numerous other factors. Therefore, the choice of the numerical optimization method requires an individual approach which considers specific features of the formulated problem.

Typical Optimization Problems for Water Protection Measures

This section describes the most common types of optimization problems for water protection measures solved individually and examined in the structure of complex optimization models. The problems are related to different classes of the technical solution optimization problems in the field of water protection.

Optimizing Effluent Treatment--

The treatment of effluent discharged into water bodies is one of the most important measures preventing water pollution. Considering the treatment plants as a complex, a considerable economy between 10 to 30% can be achieved by optimizing the degree of effluent treatment. It is assumed that the role of treatment in reducing the water body pollution has been determined, and the treatment installation sites and, consequently, their capacity, have been established. Therefore, the analysis becomes a comparison by the optimum criterion, of operationally different alternatives, i.e., in determining such levels of effluent P treatment for which the total costs of treatment S^{or} at the n effluent treatment plant would be minimum, i.e.,

$$\min S^{or} = \min_{P_i} \sum_{i=1}^{n} S_i^{or}(P_i), \quad (i=1,2,\ldots,n), \tag{9}$$

given the limitations

$$\sum_{i=1}^{n} U_i^{or}(P_i) \geq U^{or}, \tag{10}$$

$$P_i^{min} \leq P_i \leq P_i^{max} \tag{11}$$

where U^{or} is the specified decrease of measured pollutant concentration in the water body as a result of effluent treatments;

U_i^{or} is the decrease of measured pollutant concentration by the i-th effluent treatment plant;

P_i^{min}, P_i^{max} is the minimum permissible and maximum possible effluent treatment level, respectively.

Further model development requires interaction with the hydrological model of the water basin, with the model of the process of mixing effluents with river water, and also with the models of biochemical demand for dissolved oxygen and reaeration of river water. This is required if the pollutants delivered into the water body are capable of biochemical oxidation.

Depending on the availability of initial data and corresponding accuracy of calculations, the solution of equations 9-11 can be achieved with the help of various models and numerical methods. It is during the forecast of necessary water pollution control measures and long-term planning that the hydraulic characteristics of the river are only approximately determined and the possible technological schemes of treatment cannot be exactly specified. As a consequence, the number of river water quality parameters taken into account in the calculation is limited, simplified process models are constructed, and numerical methods yielding approximate results are chosen. During short-term planning of water protection measures, when the characteristics necessary for evaluation are specified, it is possible to use models and calculation methods with relatively exact solutions. For example, the use of models of pollutant transformation in river water, constructed on the basis of the turbulent diffusion equation and dynamic programming, can be used (Kuzin and Stanishevskiy, 1974).

Optimizing the Dimensions of Effluent Lagoons--

In the absence of economically substantiated methods for effluents treatment, the accumulation and discharge of effluents during periods of high flow are permitted, providing the dilution capacity of the rivers is adequate.

Where effluent lagoons are permitted, their efficiency is determined by the decrease in pollutant concentration at water intake during effluent discharge periods. If the discharge from the lagoon is well organized, then their efficiency depends on the capacity W^H which is related to their total cost S^H by the relationship $S^H = f(W^H)$.

In the case where several lagoons are constructed within a river stretch, and the effluents are characterized by the same limiting pollution index, the problem of choosing the optimum lagoon dimensions arises, i.e.,

$$\min S^H = \min \sum_{i=1}^{n} S_i^H (W_i^H), \tag{12}$$

given the limitations

$$\sum_{i=1}^{n} U_i^H (W_i^H) \leq U^{H\partial on}, \tag{13}$$

$$W_i^{Hmin} \leq W_i^H \leq W_i^{Hmax}, \tag{14}$$

where S^H is the total cost of all the effluent lagoons considered as a complex;

U_i^H is the increase of measured pollutant concentration in a water body due to functioning of the i-th lagoon;

$U^{H\partial on}$ is the maximum permissible increase of measured pollutant concentration in a water body as a result of effluents discharge from all lagoons in the system.

W_i^{Hmin} is the minimum capacity of the i-th lagoon depending on the sanitary rules for effluents discharged into water bodies;

W_i^{Hmax} is the maximum capacity of the i-th lagoon depending on the availability of sites suitable for construction.

The quantity $U^{H\partial on}$ is found when the problems of the higher levels of the hierarchical system are solved; that is, when part of the various measures for protecting the water body from pollution are determined.

In contrast to the hydrological model used for determining the level of effluent treatment, when the minimum average monthly river flow of 95%

frequency is taken as the rated value, models of the intraannual and long-term flow distribution are constructed when the lagoon dimensions are determined.

The discrete dynamic programming method is most effective as a numerical model for solving equations 12 to 14. The solution begins with effluents discharge at the existing hydrological regime of the river. Then the effluent discharge from the lagoons is determined, and then the regulating volume, a sum of the seasonal and long-term components, is found for each lagoon. This problem is considered in detail by Mukhopad (1975).

Optimizing the Volume of Acceptance Releases--

As a result of the fast growth in the volume of industrial and municipal effluents in the last decades, the natural flow of rivers in many water basins is insufficient to dilute thoroughly treated effluents. In such cases, either special releases to increase the dilution capacity of a water body or further decreases in the quantity of pollutants in discharged effluents are required. Decisions on the level of effluent treatment, and the volumes of effluent discharge, should consider these measures by methods of mathematical simulation and economic optimization. Parameters which yield the required water quality indices in the monitoring sections at a minimum total cost should be chosen.

An important feature of flow augmentation as a pollution prevention measure is the ability to dilute the effluents of all point sources located in the release zone, and so reduce concentrations of different kinds of pollutants. Therefore, when developing an objective function for cost minimization of a system of water protection measures by using flow augmentation and effluent treatment, it is necessary to provide for the possibility of optimizing expenditures for effluent treatment of differentiated categories, i.e., the objective function of the problem can be presented as:

$$\min S = \min_{W,P_i} \left[W^n S^n(W^n) + 365 \sum_{i=1}^{n} \sum_{i=1}^{m} q_{ij} S_{ij}^{or} (P_{ij})\right] \qquad (15)$$

given limitations on:

the maximum rated river flow Q_r providing for flow augmentation

$$Q_r \leq Q_r^{max}, \qquad (16)$$

W^n is the maximum size of the water reservoir and the volume allocated for flow augmentation

$$W^n \leq W^{nmax}, \qquad (17)$$

the concentration of limiting substances in the monitoring section of the water body

$$U_j^B \leq U_j^{\partial on}, \quad (j=1,2,\ldots,m), \qquad (18)$$

the degree of effluents treatment P

$$P_{ij}^{min} \leq P_{ij} \leq P_{ij}^{max}, \qquad (19)$$

where S is the total cost for flow augmentation and treatment of effluents from point sources located in the zone effected by flow augmentation;
S^n, S^{or}_{ij} is the total specific costs for flow augmentation and effluent treatment of the i-th source, containing the j pollutant correspondingly;
m is the number of kinds of limiting pollutants taken into account.

The equations 15-19 present a multistep problem. After choosing the river flow with some provision for flow augmentation, depending on the accuracy of the calculations required and the limitations 16 and 17, each step of the corresponding capacity of the river reservoir W^n is calculated, and the problem of optimizing the level of effluent treatment is solved. As a numerical method for solving the problem, it is expedient to use the steepest gradient descent, since this method provides a maximally quick determination of the extreme function from several variables. This is necessary during the multistep process to determine optimum solutions (Kuzin and Osmachko, 1975).

Problems of determining optimal capacities of treatment plants, optimizing the distribution of aeration structures on rivers and reservoirs, optimizing measures for prevention pollution of point sources by surface flow from cities and agricultural areas, and several other factors are not examined in this report. Mutually matched solutions to various optimization problems during the development of basin water pollution prevention programs will significantly decrease economical costs.

CONCLUSION

In the context of this paper, water quality control has been considered in the development of a long-term optimum strategy determining capital investment policy. The basin water protection programs, part of the nation's overall water protection program, are based on strategic objectives for 10 to 15 years.

Weighty socio-economic prerequisites favor the basin approach to the control of water quality-effect factors in developing water quality goals and water protection measures.

There are at least two ways of constructing an economic-mathematical model for optimization of anti-pollution measures on a drainage basin scale: construction of a united "super-model" or a system of small mutually matched models. Although a totally integrated and centralized control of the system is possible, a number of serious shortcomings of a practical nature limit the use of "super-models."

Optimization problems in the field of water protection are divided into three classes: Class 1 - problems of optimizing functionally different technical facilities; Class 2 - problems of interaction of differing facilities; and Class 3 - problems of optimizing operationally different technical solutions. The investigation of relations among them has shown that during the optimization of water protection measures within the limits of a basin or basin section, a multilevel system of models can be developed. This system requires a multilevel approach.

The solution of optimization problems follows the sequence of formalization. This involves modeling processes in a water body, working out the relationships reflecting the link of cost indices to parameters of the technological processes, choosing numerical solution methods, and developing algorithms for computerization.

The most widespread types of optimization problems are the problems of optimizing the level of effluent treatment, the value of sanitary releases, dimensions of effluent treatment installations, and parameters of aeration devices for rivers and water reservoirs.

The decision-making methodology outlined here, based on modeling and optimization, is a powerful analytical tool for complex problems. Its widespread use in the development of basin water protection programs (given the present dimension of the problem) will yield high economic profits to the national economy.

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DISCUSSION

DeLUCIA: First, I want to comment on Dr. Kuzin's system using smaller interrelated models rather than super models and to point out the analogy to work being done in the U.S. Recently partially decoupled systems have been used by the U.S. In these decoupling or decomposition systems, total decomposition and solution take place, either by heuristics or specific algorithms.

My second general observation pertains to Dr. Kuzin's suggestion that smaller interrelated models allow easier intervention in the man-machine interface. I would agree, but caution not to forget there are many means for intervention when we use large optimization models. First, it is most important that our younger generation of applied mathematicians who work with these problems do not ignore that experience and intuition of our older colleagues. Second is the use of heuristics to do site analysis that you then incorporate into a larger model; namely, for considering the cost of treatment or removal at a particular point. One can enter into a model, a super model, a functional form that represents prior analysis, looking at a whole range of technologies--whether it be treatment, land disposal, etc.

My third comment concerns whether super models necessarily require assumptions in the transformation equations relevant to the physical system that question their validity. My comments are twofold: I believe Dr. Loucks wanted to point out that one never suggests that super models (mathematical programming models) be used without correlation with simulation models and linear assumptions.

GORTSKO: In my opinion, optimization models are not the only instruments for solving questions of water resource management. Optimization is, without doubt, necessary, but if we are talking about optimum water quality within the limits of a river basin, then the question of an optimal goal arises. If we optimize, developing a system not based on the best alternative, the results will not be good enough.

The approach for developing an analogous model is different. First, it is necessary to construct a simulation model of water resource use within the limits of the region. This model can be less refined, but serves as the base for optimization models. A simulation model is necessary to forecast what will occur within a specific time period. A simulation model of an aquaculture complex is being applied in Rostov, USSR.

KUZIN: By "global model," I imply a model of a whole river basin system. I have not selected all indices, only those accepted for modeling a given river basin. When verifying the "inflow-outflow" model, we compared the economic indices to confirm that the expenditures decreased by 10-30%.

Joint examination of our models will allow us to enrich the methodology being used in both countries. Creation of models for specific objectives should be the final stage of cooperation.

MODELING WATER QUALITY IN RIVER BASINS

Ye.V. Yeremenko

INTRODUCTION

The suitability of specific waters for particular uses is determined by the combination of physical, chemical, and biologic criteria; however, current mathematical models of water quality use mainly physical criteria. The bases for mathematical models are hydrodynamic or hydraulic equations, and the turbulent diffusion equations for calculating pollutant concentrations with some allowance for non-conservative properties.

Depending on whether the size of a river basin section or the whole river basin is considered, different data needs on pollutant concentration in the river are required. If a long stretch of watercourse is considered, it is expedient to use one-dimensional solutions; otherwise a three-dimensional or two-dimensional solution should be used. The zone within which the pollutant concentration is not completely mixed in the cross-section is called the three-dimensional diffusion zone. We shall consider the calculation of the pollutant concentration in this zone.

Calculation of Pollutant Concentrations in the Three-Dimensional Diffusion Zone

We shall consider whether the concentration in this zone can change not only in the three coordinate directions, but in a relatively shallow flow--only in the plan. Since the three-dimensional diffusion zone is small, the pollutant is considered to act as a conservative substance within the zone.

In the USSR, it is common practice to follow the method described by Karaushev (1969) to discuss three-dimensional mixing zones. This method describes two and three-dimensional fields of pollutant concentration in prismatic channels using the difference method based on the turbulent diffusion equation.

$$V\frac{\partial C}{\partial X_1} = D_m\left(\frac{\partial^2 C}{\partial X_2^2} + \frac{\partial^2 C}{\partial X_3^2}\right) \tag{1}$$

By using the explicit difference scheme for a square grid, this equation is reduced to a simple formula for calculating the concentration

$$C_{k+1,n,m} = \frac{1}{4}(C_{k,n+1,m} + C_{k,n-1,m} + C_{k,n,m+1} + C_{k,n,m-1}) \tag{2}$$

in the next section. The distance between the sections is

$$\Delta X_1 = \frac{V(\Delta X_2)^2}{4Dm} \tag{3}$$

There is no diffusion of the pollutant through the boundary surfaces $\partial C/\partial n=0$. Similar relations were obtained for solving the two-dimensional plan problem, where instead of equation 2, we have

$$C_{k+1,m} = \frac{1}{2}(C_{k,m+1} + C_{k,m-1}) \tag{4}$$

and instead of equation 3, the relation

$$\Delta X_1 = \frac{V(\Delta X_3)^2}{2Dm} \tag{5}$$

In the above relations, V is the mean cross-section velocity, Dm the turbulent diffusion coefficient.

The above formulas make it possible to calculate the concentrations in steady-state flows. The diffusion coefficient Dm is assumed to be identical in each direction and equal to the coefficient in the vertical direction

$$D_2 = \frac{gHV}{M\tilde{C}} \tag{6}$$

where H is the mean depth of the channel,
$\tilde{C}$ is the Chezy coefficient, and
M is the Chezy coefficient functions.

However, in equation 2, the difference between diffusion coefficients in different directions in real channels has been indicated. The essential influence of this circumstance is described by Ye.V. Yeremenko (1975).

On the basis of the solution of equation

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial X_1} = D\frac{\partial^2 C}{\partial X_1^2} + D_2\frac{\partial^2 C}{\partial X_2^2} + D_3\frac{\partial^2 C}{\partial X_3^2} + f \tag{7}$$

with the initial condition $C(\vec{X},0) = \eta(\vec{X})$ and the boundary conditions $\partial C/\partial X_2 = \partial C/\partial X_3 = 0$ (applied to a rectangular channel with a steady power source) of the form

$$C^* = \frac{\gamma\Delta}{8\P}\sqrt{\frac{\beta_2\beta_3}{\P}}\int_0^\infty \frac{\exp[-(\frac{X_1}{\sqrt{\tau}} - \frac{\gamma}{2}\sqrt{\tau})^2]}{\sqrt{\tau^3}} \cdot \sum_{n=-\infty}^{\infty}\left\{\exp[-\frac{(X_2 - X_2^0 + n)^2}{\tau}\Delta^2\beta_2]\right.$$

$$\left. + \exp[-\frac{(X_2 + X_2^0 + n)^2}{\tau}\Delta^2\beta_2]\right\} \cdot \sum_{\hat{X}=-\infty}^{\infty}\left\{\exp[-\frac{(\overline{X}_3 - \overline{X}_3^0 + K)^2}{\overline{\tau}}\beta_3] + \right.$$

$$+ \exp\left[-\frac{(\bar{X}_3 + \bar{X}_3^0 + K)^2}{\bar{t}}\beta_3\right]\Big\} d\bar{t} \qquad (8)$$

where $\bar{X}_1=X_1/2B$; $\bar{X}_2=X_2/2H$; $\bar{X}_3=X_3/2B$; $(0,X_2^0,X_3^0)$ are the source coordinates, $\Delta=H/B$ $\gamma=VB/D_1$; $\bar{t}=D_1t/B^2$; $C^*=C/C_F$; $\beta_2=D_1/D_2$; $\beta_3= D_1/D_3$; $C_F=Q/VBH$ is the mean cross-section concentration. B is the width of the channel, and Q is the water discharge.

The dimensions of a three-dimensional diffusion zone were established, given different relations D_3/D_2. The distance between the source and the section, where $C^*_{max}=C_{max}/C_F=1.05$, was accepted as the length of the three-dimensional diffusion zone l.

Figure 1 illustrates the results of the evaluation of zone l with different D_3/D_2 and Chezy coefficients $\tilde{C}$, which indicate the essential influence of the relation D_3/D_2. Given the known relation between coefficients $D_3/D_2=3.4$, obtained by Elder (1959), l decreases in comparison with the case $D_3/D_2=1$ by a factor of more than three.

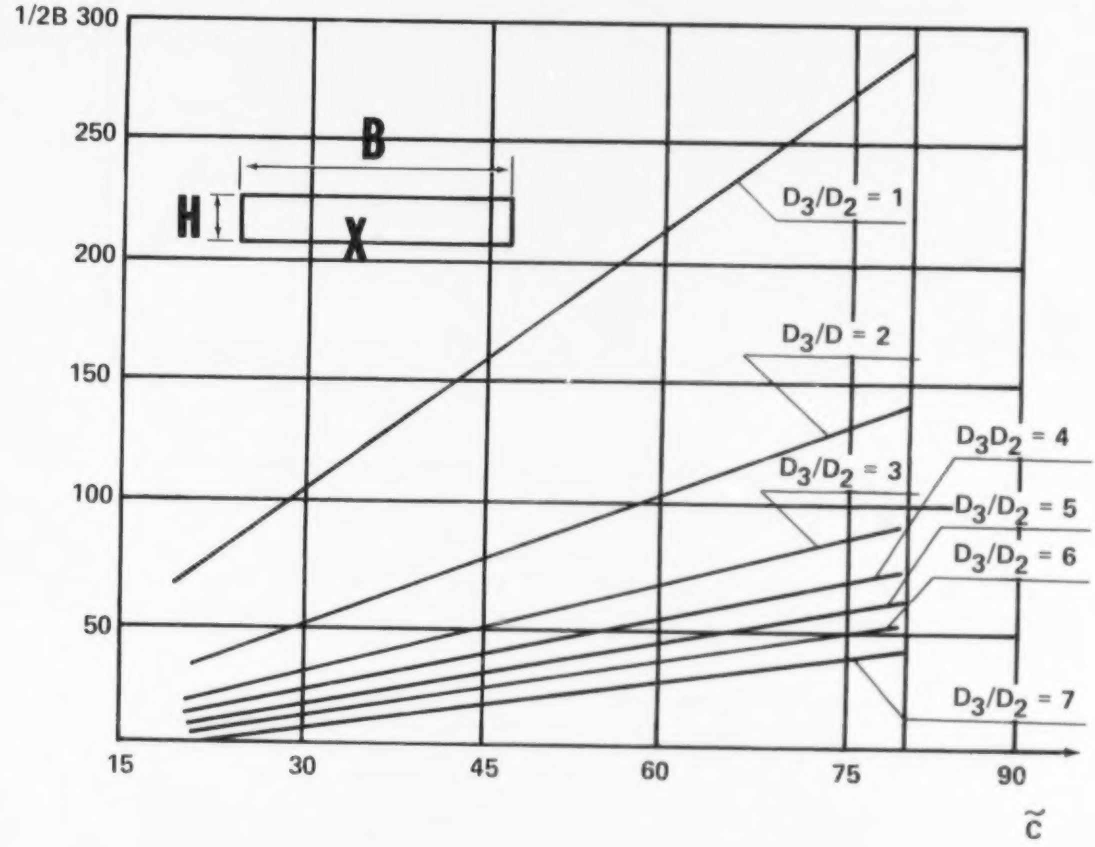


Figure 1. Influence of the ratio D_3/D_2 on the length of the three-dimensional diffusion zone (location of the source is indicated in the figure).

Yeremenko, Kolpak, and Selyuk (1975), using this and the solution of equation 8 for a rectangular channel, the difference method, the diffusion equation, calculations, and generalization of results, constructed nomograms to determine $l(C^*_{max}=1.05)$ in channels of rectangular and parabolic cross-sections of a sector type, with different positions of the source in the cross-section and relations B/H. An example of such a nomogram for a channel of a rectangular cross-section is given in Figure 2. Since at $C^* \to 1$ the slopes of the concentration variation curves are very steep, then the values of zone l essentially decrease at $C^* < 1.05$. This can be described with diagrams that show the decrease of the maximum value (C^*_{max}) along the channel. Such diagrams, generalized for different Chezy coefficients $\breve{C}$, are also obtained for three different kinds of cross-sections, depending on the relation H/B and the location of the source (Figure 3).

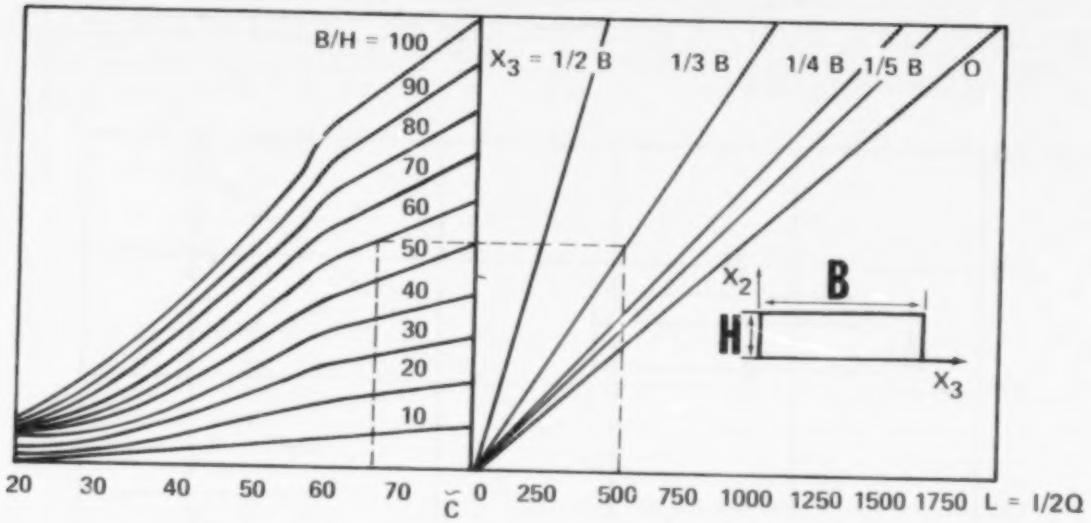


Figure 2. Nomogram for evaluating the length of the three-dimensional diffusion zone.

The dimensions of the three-dimensional diffusion zone can be essentially affected by the transverse circulation that, in a number of cases, exceeds the coefficient anisotropy effect, and so special methods of calculation are being developed.

Karaushev (1969), in the case when the transverse velocity components are considerably smaller than the longitudinal component, suggests evaluations by calculating the partical trajectories determined by the equation

$$\frac{dx}{V_1(X_1,X_2,X_3)} = \frac{dy}{V_2(X_1,X_2,X_3)} = \frac{dz}{V_3(X_1,X_2,X_3)} \qquad (9)$$

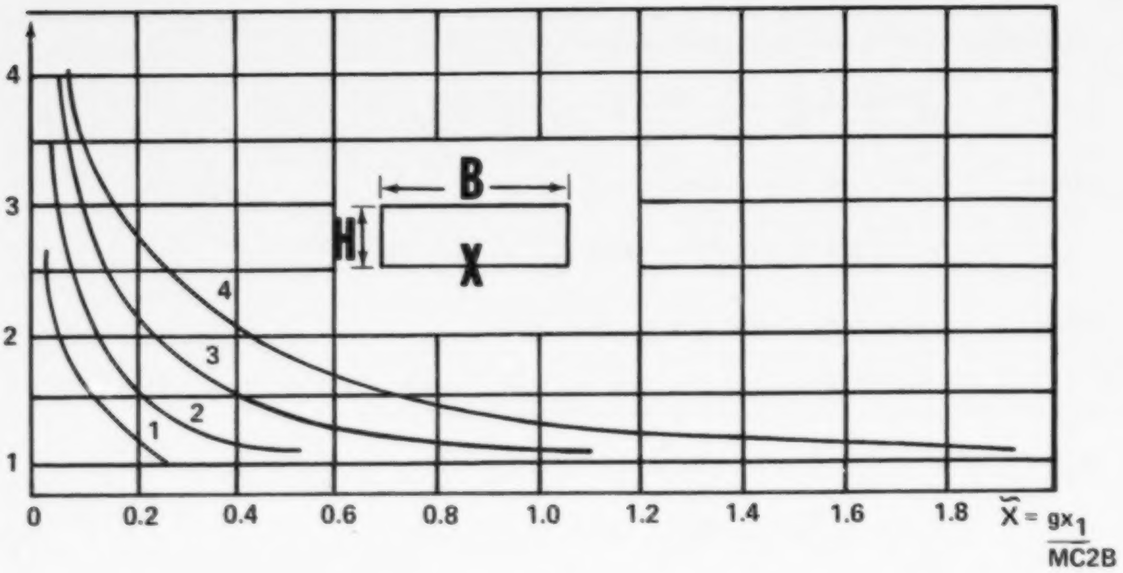


Figure 3. Variation of the maximum pollutant concentration along the flow. Conventional designations 1-H/B=1/10; 2-H/B=1/20; 3-H/B=1/40; 4-H/B =1/70; (the location of the source is indicated in the figure).

Therefore, it is understood that changes in the pollutant concentration at any point in the grid area during transition from one section to another are due not only to transfer and diffusion according to equation 1, but also to displacement from transverse velocities. The evaluation is also carried out by formula 3, and the displacement, according to equation 9.

The appearance of the transverse circulation is usually accounted for by the curvilinear plan contour of the channel. In this case, a method of evaluating the three-dimensional pollutant concentration field in the channel of a rectangular cross-section is considered by Fedorov, Lapshev, and Bezobrazov (1975). They proceed from the equation in cylindrical coordinates

$$\frac{U_n}{r}\frac{\partial C}{\partial n} + U_r\frac{\partial C}{\partial r} + U_z\frac{\partial C}{\partial z} = D_z\left(\frac{\partial^2 C}{\partial r^2} + \frac{1}{r}\frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2}\right) \qquad (10)$$

where U_n, U_r, and U_z are the velocity components in the coordinate directions. $D_z = gHV/M\tilde{C}$ (g is the acceleration due to gravity).

The boundary and initial conditions of the solution of equation 10 are

$$\left.\frac{\partial C}{\partial r}\right|_{r=R\pm B/2} = 0; \quad \left.\frac{\partial C}{\partial z}\right|_{z=\pm H/2} = 0; \quad C\big|_{n=n_0} = C_0(r,z), \qquad (11)$$

where R is the radius of curvature of the channel axis.

The velocity components were determined from work by Rozovskiy (1957) in which the equation 10 was reduced to the form

$$\frac{a_1}{R+y}\frac{\partial C}{\partial \eta} + a_2\frac{\partial C}{\partial y} = \frac{gH}{M\tilde{C}}\left(\frac{\partial^2 C}{\partial y^2} + \frac{1}{R+y}\frac{\partial C}{\partial y} + \frac{\partial^2 C}{\partial z^2}\right), \tag{12}$$

where $a_1 = 1.17 - 0.5(z/H)^2;$

$$a_2 = \frac{0.09H}{R+y}\left[2205\left(\frac{z}{H}\right)^{\eta} - 3363\left(\frac{z}{H}\right)^2 + 680\right]\left[1 - \left(\frac{2y}{B}\right)^2\right];$$

with $0 \leq z \leq H, \quad -B/2 \leq y \leq B/2, \quad \dot{\eta} > 0.$

To integrate equation 12, the explicit scheme of the finite-difference method was used, thus the main recurrence formula is

$$C_{i+1,j,k} = \tau\left\{C_{i,j,k}\left[\frac{1}{\tau} - 2A_1\left(\frac{1}{\gamma^2} + \frac{1}{\mu^2}\right) - \frac{B_1}{\gamma}\right] + C_{i,j+1,k}\left(\frac{A_1}{\gamma^2} + \frac{B_1}{\gamma}\right) + \right.$$

$$\left. + C_{i,j-1,k}\frac{A_1}{\gamma^2} + \frac{A_1}{\mu^2}\left(C_{i,j,k+1} - C_{i,j,k-1}\right)\right\},$$

where i,j,k is the numbering of the grid nodes correspondingly along η, y, z;
τ is the grid spacing along the coordinate η;
γ and μ are grid parameters along y and z, correspondingly;

$$A_1 = \frac{gH(R+y)}{M\tilde{C}a_1}; \quad B_1 = \frac{gH}{M\tilde{C}a_1} - \frac{a_2}{a_1}(R+y).$$

The stability of the solution by the explicit scheme is achieved by choosing the parameters τ, γ, and μ according to the conditions

$$\frac{A_1}{\gamma^2} + \frac{B_1}{\gamma} > 0, \quad \frac{1}{\tau} - 2A_1\left(\frac{1}{\gamma^2} + \frac{1}{\mu^2}\right) - \frac{B_1}{\gamma} > 0$$

Thus, the choice of the spacing and the grid parameters τ, γ, and μ depend essentially on the initial parameters of the channel specified by the relations R/B and B/H.

The influence of the boundary conditions is achieved by extending the matrix $(C_{i,j,k})$ by the two last rows and the two last columns, setting their elements equal to the corresponding neighboring elements of the original matrix.

The computation is usually carried out for relative or conditional concentrations obtained by assuming the area of the constant value of concentration equal to the area of one segment. To pass from these conditional concentrations, it is necessary to multiply the results by the coefficient ε, depending on the discharges of effluents and the flow, and the dimensions of the accepted grid.

$$\varepsilon = \frac{Q_0(j-1)(k-1)}{VBH} \tag{13}$$

According to the algorithm constructed on the basis of these expressions, it is possible to compute the concentration distribution in any cross-section located at the angular distance η from the place of discharge.

In the course of preliminary estimates, it is sometimes important to evaluate only the distribution of maximum concentrations along the channel. This problem is solved by the method of multiple evaluations of conditional variants, which are characterized by relative parameters R/B and B/H, and by subsequent statistical treatment of the series of concentration determined as functions of angular length. As a result, an estimation formula was obtained:

$$C_{max} = (C_0 - C_F) \exp(-\beta \frac{H}{B} \eta^{0.25}) + C_F, \qquad (14)$$

where $\beta = f(R/B;\ B/H)$.

By solving the plan problems, it is also possible to determine the equalization zone of the concentration in the flow section in channels of great width. Sherenkov (1975) considered pollutant distribution in a meandering channel, using a two-dimensional model. The equations for calculating flow are written in the curvilinear system of orthogonal coordinates parallel to the dynamic axis of the flow. These steady-state motion equations, after discarding the terms of the highest order of smallness, can be presented in the form

$$\frac{\partial V_s}{\partial t} + \alpha[\frac{1}{\kappa} \frac{\partial V_s^2}{\partial S} + \frac{\partial}{\partial n} V_s V_n - \frac{\sigma}{\kappa} V_s V_n] = -\frac{g}{\kappa} \frac{\partial H}{\partial S} + \frac{\partial}{\partial n} \tau_{sn} + \frac{\sigma}{\kappa} \tau_{sn} - \frac{\lambda V_s^2}{2h} + \frac{T_s}{\beta h}; \qquad (15)$$

$$\alpha \frac{\sigma}{\kappa} V_s^2 = -g \frac{\partial H}{\partial n} + \frac{Tn}{\rho h}; \qquad (16)$$

$$\frac{\partial h}{\partial t} + \frac{1}{\kappa} \frac{\partial}{\partial S} V_s h + \frac{\partial}{\partial n} V_n h - \frac{\sigma}{\kappa} V_n h = 0 \qquad (17)$$

Here V_s, V_n are the values of the mean depth longitudinal S, where S and transverse n are the coordinate velocity components;

α is the coefficient for the irregularity of velocity distribution over the depth of the channel;

H is the free surface water level;

h is the depth of the channel; curvilinear system (metric) coefficients $\kappa=1-\sigma n$, $\kappa_h=\kappa_z=1$;

T_s, T_n are the tangential stresses of the wind acting on the free surface of the channel;

τ_{sn} are the mean depth turbulent tangential stresses caused by the horizontal turbulent momentum transfer;

λ is the coefficient of hydraulic friction on the bottom; and

ρ is the density of water.

The system of equations is closed using the Reichardt-Konovalov hypothesis

$$V_n S_n - \frac{\tau_{sn}}{\rho} = A \frac{\partial V_s^2}{\partial n}, \qquad (18)$$

which gives good correspondence with the experiment, with Λ being determined for uniform flow conditions at a specified water discharge Q and slope T from equations

$$Q = gT + \frac{\partial}{\partial n} \Lambda \frac{\partial V_s^2}{\partial n} - \frac{\lambda V_s^2}{2h}, \qquad Q = \int_0^B V_s h dn \tag{19}$$

The calculation of the distribution of velocities and free surface levels of steady flow is carried out on the basis of equations 15-17 with boundary conditions at

$$S = 0;\ V_s = V_{so}(n);\ \ V_n = V_{no}(n);\ \ H = H_o(n), \tag{20}$$

and at $S>0$; $V_s=V_n=0$ along the bank line by successive approximations with transition to new variables

$$S_\star \equiv S; \quad \imath = \frac{B\Lambda - n}{B}, \tag{21}$$

where $B_\Lambda(S)$ is the coordinate of the left bank, $B(S)$ is the width of the river at the section; the solution region is transformed into a semi-strip of unit width, and equations 15-16 are written in the general form

$$\frac{\partial V_s^2}{\partial S_\star} + a_1 \frac{\partial^2 V_s^2}{\partial \imath^2} + a_2 \frac{\partial V_s^2}{\partial \imath} + a_3 V_s^2 + a_4 \frac{\partial H}{\partial S_\star} + a_5 \frac{\partial H}{\partial \imath} = f_1(S_\star, \imath), \tag{22}$$

$$a_6 V_s^2 + \frac{\partial H}{\partial \imath} = f_2(S_\star, \imath), \tag{23}$$

where $a_1(S_\star,\imath) = -\frac{\Lambda\kappa}{\alpha B^2}$; $a_2(S_\star,\imath) = \frac{1}{B}\left(\frac{\partial B\Lambda}{\partial S} - \imath\frac{\partial B}{\partial S}\right) - 2\alpha\sigma\Lambda$; $a_3(S_\star,\imath) = \frac{\kappa\lambda}{2\alpha h}$;

$a_4 = \frac{1}{\alpha g}$; $a_5 = \frac{1}{\alpha g B}\left(\frac{\partial B_\Lambda}{\partial S} - \imath\frac{\partial B}{\partial S}\right)$; $a_6(S_\star,\imath) = \frac{\alpha\sigma g B}{\kappa}$; $f_1(S_\star,\imath) = \frac{\kappa T_s}{\alpha g h}$;

$$f_2(S_\star,\imath) = \frac{\alpha B T_n}{\rho h}. \tag{24}$$

The character of the boundary conditions (equation 20) in the new coordinates does not change. The free surface $H(S_\star,\imath)$ is presented as a sum of the mean section surface $H_F(S)$ and the deviation from the mean surface $\zeta(S_\star,\imath)$

$$H = H_F + \zeta(S_\star,\imath); \quad \frac{\partial H}{\partial \imath} = \frac{\partial \zeta}{\partial \imath}. \tag{25}$$

In solving the first approximation, it is assumed that

$$\frac{\partial H_F}{\partial S_\star} \gg \frac{\partial \zeta}{\partial S} \quad \text{and} \quad \frac{\partial \zeta}{\partial S} = 0 \tag{26}$$

and equation 26, after substituting into it equation 23, is written in the form

$$\frac{\partial V_s^2}{\partial S_\star} + a_1 \frac{\partial^2 V_s^2}{\partial \imath^2} + a_2 \frac{\partial V_s^2}{\partial \imath} + (a_3 - a_5 - a_6)V_s^2 + a_4 \frac{\partial H_F}{\partial S_\star} = f_1(S_\star,\imath) - a_5 f_2(S_\star,\imath); \tag{27}$$

The quantity $\partial H_F/\partial S_*$ enters equation 27 as a parameter and is unknown beforehand.

The parabolic equation 27 is solved numerically by the standard algorithm at different values $\partial H_F/\partial S_*$ and by the calculated values V_s, the integral condition of continuity is checked:

$$Q = B\int_0^1 V_s(\iota)h(\iota)d\iota \tag{28}$$

where Q is the specified discharge in the river channel.

The value of $\partial H_F/\partial S_*$ and $V_s(\iota)$, when the equation (28) is valid, is the required result. Substituting the solution of the first approximation $V_s^{(I)}$ into equation 23, we obtain the value $\zeta^{(I)}$ for the first approximation.

$$\zeta = B\left\{\int_0^1 [a_6(V_s^I)^2 - f_2(S_*,\iota)]\, d\iota - \int_0^1 d\iota\, [a_6(V_s^I)^2 - f_2(S_*,\iota)]\, d\iota\right\} \tag{29}$$

Substitution into equation 22, using equation 25 allows the values V_s^{II} of the second approximation to be determined.

Calculations have shown that the second approximation is necessary only when

$$\sigma B \simeq 1 \quad \text{and} \quad \sigma B > 0.3 \tag{30}$$

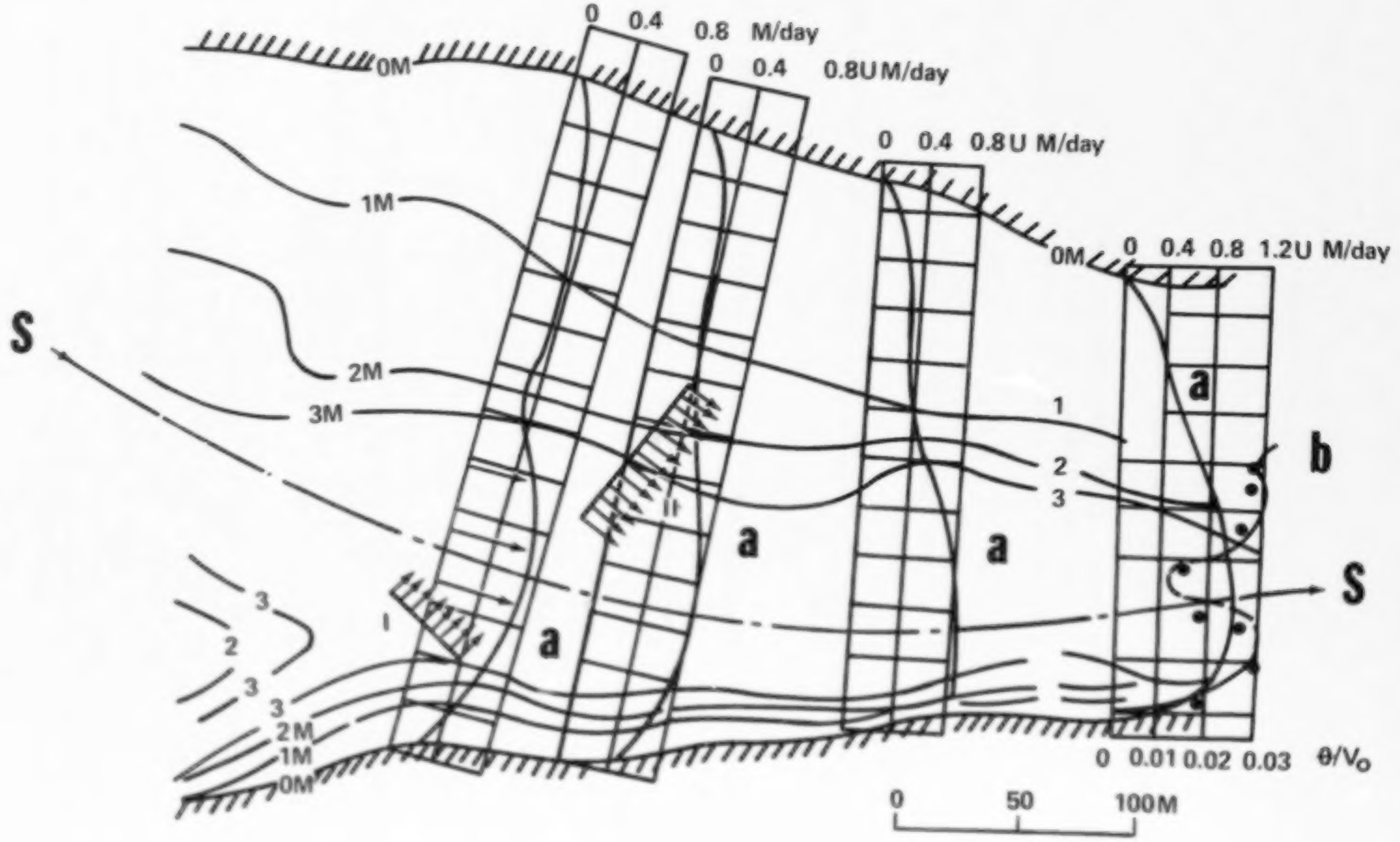
The determination of values V_s and H makes it possible to find V_n from the continuity equation 17.

The semi-empirical pollutant transfer equation in the plan in the curvilinear orthogonal coordinate system, parallel to the flow dynamic axis, is written in the form:

$$\frac{\partial\theta}{\partial t} + \frac{V_s}{\kappa}\frac{\partial\theta}{\partial S} + V_n\frac{\partial\theta}{\partial n} = \frac{1}{\kappa h}\left[\frac{\partial}{\partial S}\frac{h}{\kappa}D_{ss}\frac{\partial\theta}{\partial S} + \frac{\partial}{\partial S}hD_{sn}\frac{\partial\theta}{\partial n} + \frac{\partial}{\partial n}hD_{ns}\frac{\partial\theta}{\partial S} + \frac{\partial}{\partial n}h\kappa D_{nn}\frac{\partial\theta}{\partial n}\right] + \frac{G_n - G_o}{h} + f, \tag{31}$$

where θ is the mean depth pollutant concentration value, G_n and G_o are the pollutant flow through the free surface and into the bottom, and f is the rate of generation or decay of the pollutant (due to biochemical transformations) at a given point of the flow plan.

The value D forms the matrix of values of effective diffusion coefficients that take into account both turbulent and convective diffusion caused by the heterogeneity of the velocity field and by secondary flows. To determine D, the approach used by Maron (1971) to solve the eigenvalue problem, is generalized by Sherenkov (1975) for a complex flow. Sherenkov describes the relationships for determining D; equation 31 is integrated by the difference method. An evaluation of the distribution of velocities and concentrations in dissipative discharge of effluents on one of our rivers is illustrated in Figure 4.



0 0.4 0.8 M/day
0 0.4 0.8U M/day
0 0.4 0.8U M/day
0 0.4 0.8 1.2U M/day
0M
1M
2M
3M
S
1
2
3
b
a
a
a
a
II
I
S
3
2
3
3
2M
1M
0M
0 0.01 0.02 0.03 θ/V₀
0 50 100M

Figure 4. Results of the evaluation of the flow plan and the pollutant concentration in a river channel: a is the contour; b is the contour of the relative concentration θ/V_o according to the evaluation; dots show the values of θ/V_o according to the results of measurements; arrows show the values of V according to field measurements; I and II are dissipative discharges through which the pollutants are delivered; V_o is the initial pollutant concentration.

Paal (1975) describes a pollutant diffusion problem in which the pollutant is a non-conservative substance. The initial equation used is:

$$V\frac{\partial \theta}{\partial X_1} - \bar{D}_3 \frac{\partial^2 \theta}{\partial X_3^2} - K_1 \theta = 0 \qquad (32)$$

In the solution, the limitation of the function is assumed at $X \to \infty$.

If there is a discharge of effluents, the solution of equation 32 for determining BOD is presented in the form

$$L = \frac{L_{CT} g}{H\sqrt{\pi V \bar{D}_3 X_1}} \frac{1}{\mathrm{erf}\left(\frac{B\sqrt{V}}{\sqrt{\bar{D}_3 X_1}}\right)} \exp\left(-\frac{(X_3 + \frac{B^2}{2})V}{4\bar{D}_3 X_1} - K_1\frac{X_1}{V}\right), \qquad (33)$$

where L_{CT} and g are correspondingly BOD and the discharge of a liquid effluent.

To determine the distribution of the oxygen deficit, the following formula has been obtained:

$$\bar{S} = \left\{\bar{S}_{CT}\exp\left(-K_2\frac{X_1}{V}\right) + L_{CT} \frac{K_1}{K_2 - K_1} \left[\exp\left(-K_1\frac{X_1}{V}\right) - \exp\left(-K_2\frac{X_1}{V}\right)\right]\right\} \cdot \frac{g}{H\sqrt{\pi V \bar{D}_3 X_1}}$$

$$\cdot \frac{1}{\mathrm{erf}\left(\frac{B\sqrt{V}}{2\sqrt{\bar{D}_3 X_1}}\right)} \exp\left(-\frac{(X_3 + \frac{B}{2})^2 V}{4\bar{D}_3 X_1}\right) \qquad (34)$$

The coefficient $\bar{D}_3$ is determined from experimental data (Paal) for smooth channels by the formula

$$\frac{D_3}{RU_*} = 41.6\ Re^{-0.5} \qquad (35)$$

Evaluation of Pollution Concentration in One-Dimensional Problems

One-dimensional solutions are necessary for long channels when the three-dimensional diffusion zone only occupies a small section of the channel. The evaluation of concentrations of non-conservative pollutants in non-steady-state stream flows is of general interest. If we assume that water bodies are vertically well mixed (no density stratification), the most complete system of equations for evaluating the pollution concentrations affecting water quality is described by Yeremenko (1975). The equations are:

$$\frac{\partial S}{\partial t} + V\frac{\partial S}{\partial X} = \frac{1}{W}\frac{\partial}{\partial X}\left(WD\frac{\partial S}{\partial X}\right) + K_2(S_1 - S) - K_1 L - K_n N + f_s + P - P_P - P_M;$$

$$\frac{\partial L}{\partial t} + V\frac{\partial L}{\partial X} = \frac{1}{W}\frac{\partial}{\partial X}\left(WD\frac{\partial L}{\partial X}\right) - K_1 L + f_L;$$

$$\frac{\partial N}{\partial t} + V\frac{\partial N}{\partial X} = \frac{1}{W}\frac{\partial}{\partial X}\left(WD\frac{\partial N}{\partial X}\right) - K_n N + f_N;$$

$$\frac{\partial C_i}{\partial t} + V\frac{\partial C_i}{\partial X} = \frac{1}{W}\ \frac{\partial}{\partial X}\ (WD\frac{\partial C_i}{\partial X}) - K_iC_i + f_C;$$

$$\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial X} = \frac{1}{W}\frac{\partial}{\partial X}\ (WD\frac{\partial T}{\partial X}) - \frac{B}{W}\left\{A_1[e_m(T_B) - e] - B_1T - \Delta R + \Delta T\right\} + f_T;$$

$$\frac{\partial Q}{\partial t} + 2V\frac{\partial Q}{\partial X} + B(C^2-V^2)\ \frac{\partial Z}{\partial X} = gW[(i + \frac{1}{B}\ \frac{\partial W}{\partial X})F_r - \frac{Q|Q|}{K^2}];$$

$$B\frac{\partial Z}{\partial t} + \frac{\partial Q}{\partial X} = g, \qquad (36)$$

where S, L, and N are the respective concentrations of dissolved oxygen, BOD is due to carbonic and nitrogenous organic compounds; C_i is the concentration of the i-th ingredient present in waste water; T is the water temperature; f is the capacity of the sources of pollutant; Q is the water discharge; W is the cross-sectional area of the channel; V=Q/W is the mean flow velocity; Z is the free surface water level; F_r is the Froude number; B is the width of the channel at water level Z; g is the influx of water per unit length of channel; K_1,K_n, K_i are the coefficients of ingredient decay rates; D is the longitudinal dispersion coefficient; P is the photosynthetic production of oxygen; P_P is the oxygen consumption by water plants; P_M, the oxygen consumption by bottom sediments; A_1 and B_1 are the coefficients of heat transfer by evaporation and convection, respectively; T_B is the air temperature; all other designations which characterize temperature balance are contained in the regulations operating in the USSR (Fedorov, Lapshev, and Bezobrazov).

The first five equations are the transfer equations, and the last two are Saint-Venant equations evaluating the non-steady flow motion.

A similar system of equations for evaluating pollutant concentration was used by Vasiliyev and Voyevodin (1975).

The values of the coefficients D, K_1, K_n, K_i, K_2, and also the values P, P_P, P_M are required to integrate the transfer equations. Almost all the quantities can be determined by investigations of specific water bodies or from published values. In the papers presented, special attention is given to the determination of the longitudinal dispersion coefficient.

In Paal's paper (1975), the following relations were obtained from laboratory investigations of regularities of variation of coefficients D for different regions of resistance (Figure 5).

$$\frac{D}{RU_*} = 3400Re^{-0.5}, \qquad (37)$$

$$\frac{D}{RU_*} = f(Re\ \frac{RU_*}{\nu}), \qquad (38)$$

$$\frac{D}{RU_*} = 16(\frac{V}{U_*})^{1.25} \qquad (39)$$

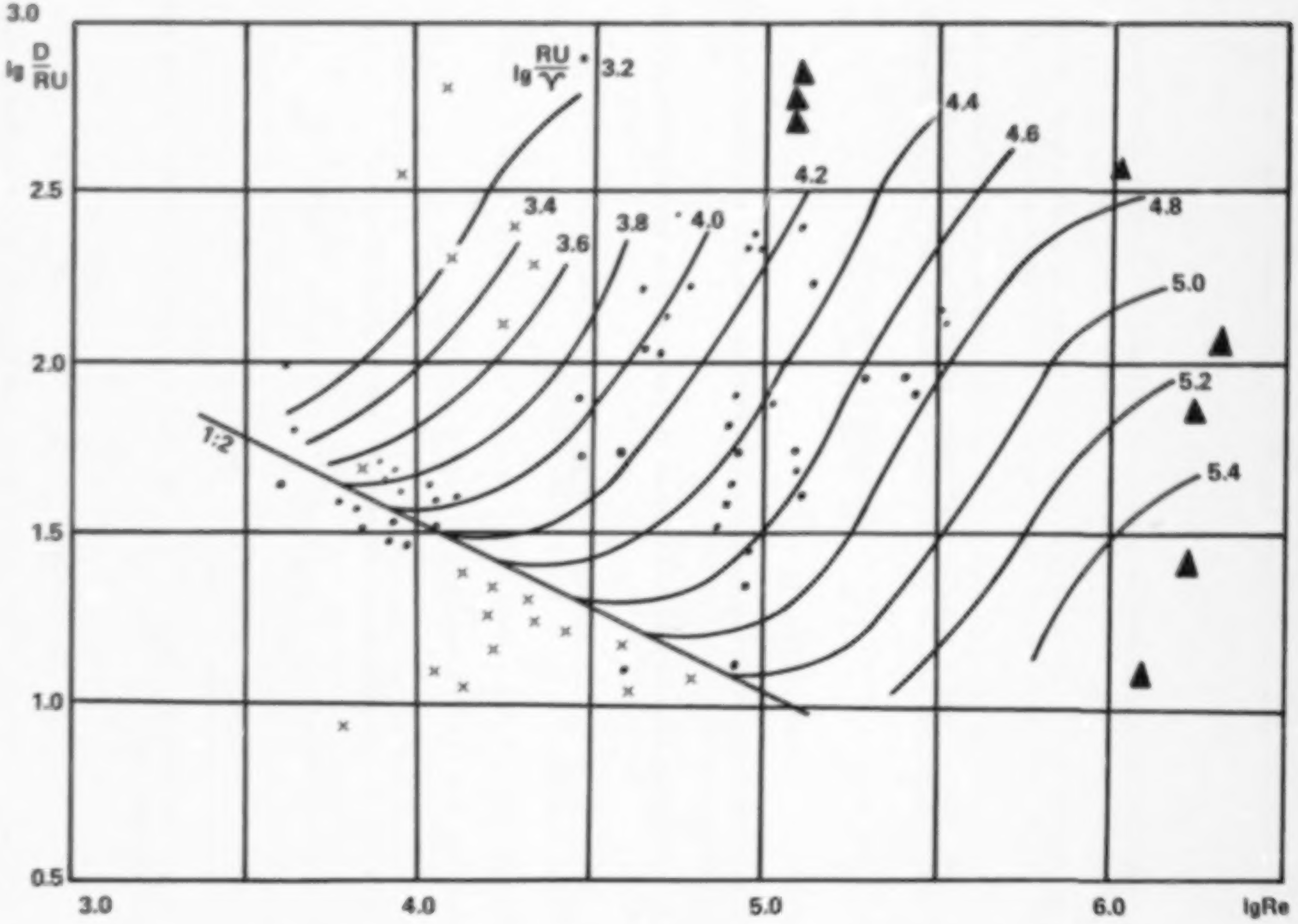


Figure 5. Diagram of the variation of values $\lg\frac{D}{RU*}$ depending on Reynold's number Re.

Vasiliyev and Voyevodin (1975), using Harlimann's work, determined the longitudinal dispersion coefficient for an open prismatic channel by the formula

$$D = 20.2 \sqrt{g}|V|R/\tilde{C}. \tag{40}$$

For an arbitrary channel, however, the longitudinal dispersion coefficient can be an order of magnitude higher than for a prismatic channel.

The determination of the dispersion coefficient in a non-steady-state flow in channels of great width is considered in Yeremenko's paper (1975).

Taylor's assumption that the effect of the longitudinal diffusion is negligible, when compared with the diffusion in directions transverse to the direction of flow, is used as the initial three-dimensional diffusion equation

$$\frac{\partial C}{\partial t} + V_h \frac{\partial C}{\partial X_1} + V_2 \frac{\partial C}{\partial X_2} + V_h(F-1)\frac{\partial C}{\partial X_1} = \frac{\partial}{\partial X_2}\left(D_2 \frac{\partial C}{\partial X_2}\right) + \frac{\partial}{\partial X_3}\left(D_3 \frac{\partial C}{\partial X_3}\right), \tag{41}$$

where $F(X_2,t) = V_1(X_2,t)/V_h$, $X_2 = \frac{X_2}{h}$,

V_h is the mean depth velocity; D_2, D_3 are constant in the cross-section diffusion coefficients.

By averaging equation 41 with depth in respect to a rectangular prismatic channel

$$\frac{\partial \overline{C}}{\partial t} + V_h \frac{\partial \overline{C}}{\partial X_1} + \frac{1}{h}\frac{\partial}{\partial X_1} hV_h \int_0^1 (F-1)\overline{c}dx_2 = \frac{\partial}{\partial X_3}\left(D_3 \frac{\partial \overline{C}}{\partial X^3}\right), \tag{42}$$

is obtained where $\overline{C} = \frac{1}{h}\int_0^h cdx$.

By inserting the trivial result $\frac{1}{h}\frac{\partial}{\partial X_1} hV_h \int_0^1 (F-1)\ \overline{c}\overline{dx}_2 = 0$

into equation 42, we obtain, after subtracting 41 from 42:

$$\frac{\partial(\overline{C}-C)}{\partial t} + V_h \frac{\partial(\overline{C}-C)}{\partial X_1} - V_2 \frac{\partial C}{\partial X_2} + \frac{1}{h}\frac{\partial}{\partial X_1} V_h \int_0^1 (F-1)(\overline{C}-C)\, dx_2 - V_h(F-1)\frac{\partial C}{\partial X_1}$$

$$= \frac{\partial}{\partial X_3}\left[D_3 \frac{\partial(\overline{C}-C)}{\partial X_3}\right] - \frac{\partial}{\partial X_2}\left(D_2 \frac{\partial C}{\partial X_2}\right). \tag{43}$$

Using Taylor's approximation ($C \rightarrow \overline{C}$) from equation 43, we obtain the equation for determining $C = f(D_2, V_2, \overline{C})$

$$\frac{\partial}{\partial X_2} D \frac{\partial C}{\partial X_2} - V_2 \frac{\partial C}{\partial X_2} = V_h(F-1)\frac{\partial \overline{C}}{\partial X_1} \tag{44}$$

If C is known, it is possible to determine the longitudinal dispersion coefficient E_{12} in a vertical plane, because

$$\frac{\partial}{\partial X_1} hV_h\int_0^1 (F-1)\ c\,dx_2 = -\frac{\partial}{\partial X_1} (hE_{12}\frac{\partial \overline{C}}{\partial X_1})$$

Then substituting $V_h = V\eta(\overline{X}_3,t)$, where $X_3 = \frac{X_3}{B}$; V is the average velocity by active section, equation 42 can be written as

$$\frac{\partial \overline{C}}{\partial t} + V\frac{\partial \overline{C}}{\partial X_1} + V(\eta-1)\frac{\partial \overline{C}}{\partial X_1} = \frac{1}{h}\frac{\partial}{\partial X}(hE_{12}\frac{\partial \overline{C}}{\partial X_1}) + \frac{1}{h}\frac{\partial}{\partial X_3}(D_3 h\frac{\partial \overline{C}}{\partial X_3}). \tag{45}$$

Averaging equation 45 by width and assuming no loss of the pollutant into the banks of the channel, we obtain

$$\frac{\partial G}{\partial t} + V\frac{\partial G}{\partial X_1} + \frac{1}{Bh}\ \frac{\partial}{\partial X_1} hBV\int_0^1 (\Theta-1)\ \overline{c}\,dx_3 = \frac{1}{h}\frac{\partial}{\partial X_1}(hE_{12}\frac{\partial G}{\partial X_1}), \tag{46}$$

where $G=1/B \int_0^B \overline{c}\,dx_3$. Subtracting as before, equation 45 from 46, and fixing $\overline{C}\to G$, we obtain an equation for defining $\overline{C}$.

$$\frac{1}{h}\frac{\partial}{\partial X_3}(hD_3\frac{\partial \overline{C}}{\partial X_3}) = V(\Theta-1)\frac{\partial G}{\partial X_1} \tag{47}$$

The solution for this equation, as is generally known, offers the possibility for finding the horizontal longitudinal dispersion coefficient of E_{13} because

$$\frac{\partial}{\partial X_1} WV\int_0^1 (\Theta-1)\ \overline{c}\,dx_3 = -\frac{\partial}{\partial X_1}(WE_{13}\frac{\partial G}{\partial X_1}). \tag{48}$$

By substituting equation 48 into equation 46, it is possible to substitute longitudinal dispersion coefficients in horizontal and vertical planes, because

$$\frac{\partial G}{\partial t} + V\frac{\partial G}{\partial X_1} = \frac{1}{W}\frac{\partial}{\partial X_1}[W(E_{12} + E_{13})\frac{\partial G}{\partial X_1}] \tag{49}$$

Yeremenko (1975) shows that, given variable motion in wide channels, the characteristic time for the change of the flow velocity structure across the channel is essentially greater than the characteristic time change of the velocity field in the vertical plane. Consequently, in a number of cases, it is possible to determine the coefficient E_{13} by the relationships determined for the steady-state solution, while the non-steady-state effect can be resolved by the coefficient E_{12}, the value of which is affected by the velocity component V_2. The extent to which the presence of the velocity component V_2 affects the coefficient of longitudinal dispersion is established by calculations for a flow with a parabolic velocity distribution

$$V_h(F-1) = \frac{U*}{\alpha}(\overline{X}_2 - \frac{\overline{X}_2}{2} - \frac{1}{3}),$$

These were used for solving equation 44, presented in the form

$$\frac{\partial^2 C}{\partial X_2^2} - a\frac{\partial C}{\partial X_2} = \frac{h}{\alpha}\left[\frac{1}{\alpha}\left(\ddot{x}_2 - \frac{X_2}{2} - \frac{1}{3}\right)\right]\frac{\partial \bar{C}}{\partial X_1}, \qquad (50)$$

where $a = V_2/\alpha U_*$; $\alpha = 0.067$ is the Boussinesq coefficient; U_* is the dynamic velocity. The velocity component V_2 was assumed to be constant over the depth.

Since the generally accepted form of the dispersion coefficient is the expression $E_{12} = \beta h U_*$, coefficient β had to be determined by expression

$$\beta = \frac{\int_0^1 Vh(F-1)\bar{c}\,dx_2}{hU_*\partial\bar{C}/\partial X_1}.$$

The results of evaluation of β, depending on the ratio V_2/U_*, are given in the following table

V_2/U_*	β
1.00	141,980.0
0.67	2,799.0
0.335	91.4
0.067	11.0
0	7.0
-0.067	5.3
-0.335	1.4
-0.67	0.52
-1.0	0.25

From the table, it follows that coefficient β changes non-symmetrically. Given $V_2/U_*>0$, this coefficient changes by several orders of magnitude, whereas given $V_2/U_*<0$, the coefficient changes by only one order of magnitude. The qualitative verification of the result was carried out experimentally in a hydraulic tray 29 cm wide and 4.0 meters long. The concentration distribution has been measured after a single instantaneous discharge from a point source into the experimental channel. The distributions were obtained in the steady and variable flows with the hydrograph shown in Figure 6a. The results in Figure 6 show the increase in the longitudinal dispersion coefficient in a variable flow.

The integration of equations for determining the BOD and the dissolved oxygen deficit in a steady-state flow is described by Paal (1967).

The original equations are as follows:

$$\frac{\partial L}{\partial t} + V\frac{\partial L}{\partial X} - D\frac{\partial^2 L}{\partial X^2} + K_1 L = 0,$$

$$\frac{\partial \bar{S}}{\partial t} + V\frac{\partial \bar{S}}{\partial X} - D\frac{\partial^2 \bar{S}}{\partial X_2} - K_1 L + K_2\bar{S} = 0 \qquad (51)$$

with the initial conditions $L=0$ and $\bar{S}=0$, given $t=0$.

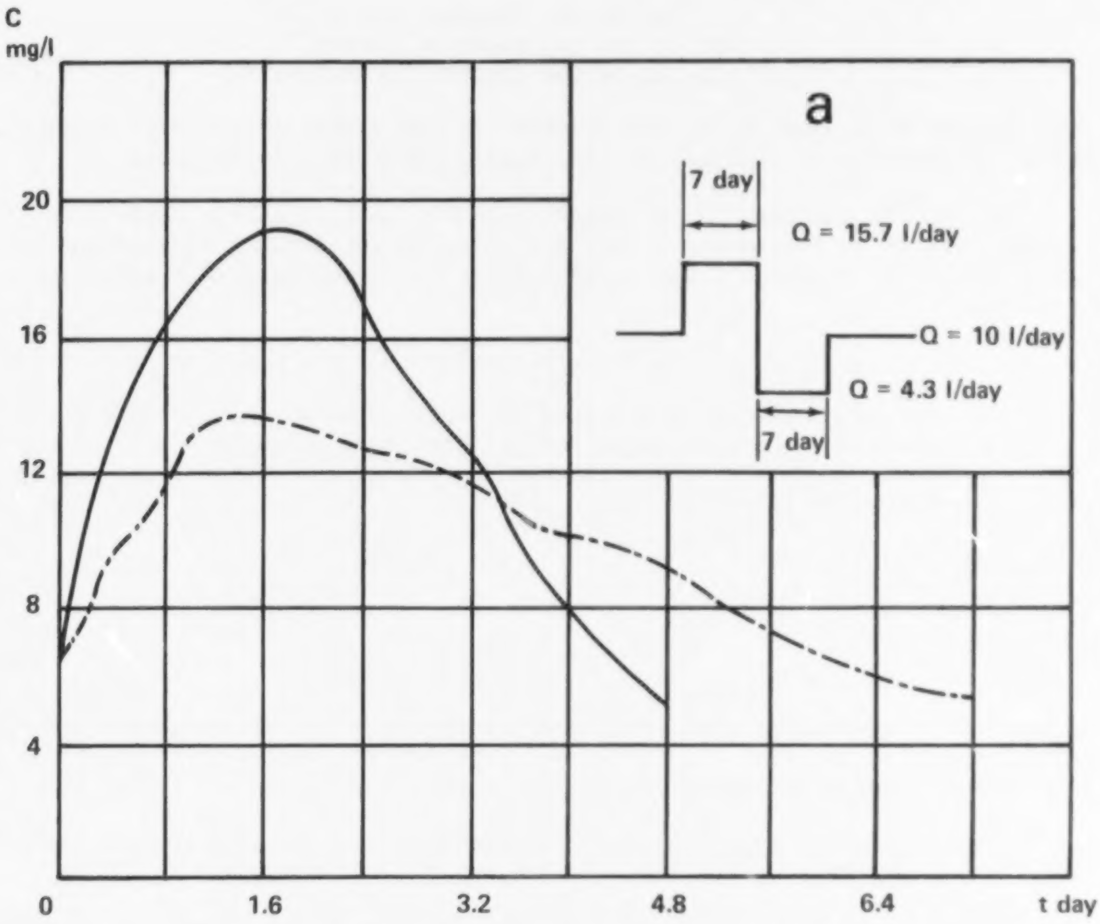


Figure 6. Influence of the unsteadiness of the flow on the dispersion of the pollutant concentration in time: _____ is steady motion; ----- is unsteady motion.

Diagrams of the introduction of effluents are presented in the form of a step function determined for values $t \geq 0$ in the following manner

$$\begin{aligned} &L_0 \text{ in the interval } 0 \le t < \tau \\ &L_1 \text{ in the interval } \tau \le t < 2\tau \\ &L_n \text{ in the interval } n\tau \le t < (n+1)\tau \end{aligned} \tag{52}$$

The average time step, therefore, depends on the required accuracy of calculation; in applied calculations, it is usually taken as $\tau=1...2$ hours.

During the discharge of effluents from the outfall to the water body, the oxygen deficit is considered a constant value in the first approximation $\bar{S}=\bar{S}_{cm}$ given $X=0$. The second boundary condition is $X \to \infty$, the function L should be bounded, i.e.,

$$\frac{\partial L}{\partial X} \to 0, \text{ given } X \to \infty \tag{53}$$

The Laplace transformation method was used to solve equation 51; the boundary conditions in the initial section were determined by the Heaviside unit function. Given the indicated initial boundary conditions, the solution for determining BOD takes the form

$$L = \frac{1}{2}\frac{q}{Q+q}\left\{\exp\frac{X}{2D}\left(V-\sqrt{V^2+4K_1D}\right)\sum_{i=0}^{n} S_{CTi}\cdot\left[\operatorname{erfc}\left(\frac{X-V(t-i\tau)}{2\sqrt{D(t-i\tau)}}\right) - \operatorname{erfc}\left(\frac{X-V(t-(i+1)\tau)}{2\sqrt{D(t-(i+1)\tau)}}\right)\right]\right\} \tag{54}$$

where q is the effluent discharge; L_{CTi} is the effluent BOD concentration within the time interval $t...(i+1)$; $\operatorname{erfc}(Z)=1-\operatorname{erf}(Z)$, while $\operatorname{erf}(Z)=2/\sqrt{\pi}\int_0^Z e^{-\zeta e}\, d\zeta$. For a conservative substance, it should be taken that $K_1=0$.

The dissolved oxygen regime of the channel, with the above-mentioned conditions, is determined from the solution obtained in the form

$$\bar{S} = \frac{q}{Q+q}\left\{D_{CT}\exp\frac{X}{2D}\left(V-\sqrt{V^2+4DK^2}\right) + \left[\exp\frac{X}{2D}\left(V-\sqrt{V^2+4DK_1}\right) - \exp\frac{X}{2D}\left(V-\sqrt{V^2+4DK_2}\right)\right]\right.$$
$$\left.\frac{K_1}{K_2-K_1}\frac{1}{2}\sum_{i=0}^{n} S_{CTi}\cdot\left[\operatorname{erfc}\left(\frac{X-V(t-i\tau)}{2\sqrt{D(t-i\tau)}}\right) - \operatorname{erfc}\left(\frac{X-V(t\ (i+1)\tau)}{2\sqrt{D(t-(i+1)\tau)}}\right)\right]\right\} \tag{55}$$

Equation 55 is a calculating formula for determining the river oxygen regime, if the effluent oxygen deficit remains constant with time, as in real situations.

To evaluate the pollutant concentration in non-steady-state flow, it is first necessary to integrate the Saint-Venant equations. This is described by Vasiliyev and Voyevodin (1975). The solution of the equations, after approximation by difference methods, is carried out by a numerical method using an implicit difference scheme.

A scheme of open channels of canals is naturally represented in the form of a system of intersecting segments and nodes (vertices), each at the end of some segment. On each segment, the required functions satisfy a system of Saint-Venant differential equations with initial conditions, and, in each vertex, they satisfy the boundary conditions or the conjugation conditions. The Saint-Venant equations are written in characteristic form, which in the vector form can be presented as

$$R\frac{\partial U}{\partial t} + \Lambda R\frac{\partial U}{\partial X} = F, \tag{56}$$

where

$$u = \binom{Q}{Z}, F = \begin{pmatrix} V^2\left(B\frac{\partial Zq}{\partial X} + \frac{\partial W}{\partial X_h}\right) - gW\frac{Q|Q|}{K^2} - q(V+C) \\ V^2\left(B\frac{\partial Zq}{\partial X} + \frac{\partial W}{\partial X_h}\right) - gW\frac{Q|Q|}{K^2} - q(V-C) \end{pmatrix}$$

$$R = \begin{pmatrix} 1 & -B(V+C) \\ 1 & -B(V-C) \end{pmatrix}, \quad \Lambda = \begin{pmatrix} V-C & 0 \\ 0 & V+C \end{pmatrix}, \quad K = WC\sqrt{R}, \quad h = Z - Z^0, \quad B = \frac{\partial W}{\partial h},$$

B is the width of watercourse that has a depth h; Z^0 is the bottom of the channel. Equations 56 are approximated by using an implicit difference of the first order accuracy by t and of the second order by X.

$$R^K_n \frac{U^{K+1}_n - U_n}{\tau} + (\Lambda R)^K_n \frac{U^{K+1}_{n+1} - U^{K+1}_{n-1}}{2\Delta} = F^{K+1}_n, \tag{57}$$

Here τ is the interval along the t axis common for all segments; Δ is the interval along the X axis, generally different for various segments.

The second order calculation for the vector F^{K+1}_n follows

$$F^{K+1}_n = F^K_n + \left(\frac{\partial F}{\partial U}\right)^K_n \left(U^{K+1}_n - U^K_n\right)$$

As a result, each segment contains a system of linear algebraic equations of unknowns U^{K+1}_n of the following form

$$A_n U^{K+1}_{n+1} + B_n U^{K+1}_n + C_n U^{K+1}_{n-1} = D_n, \tag{58}$$

where

$$n = 1,2,\ldots,N-1; \quad A_n = -C_n = (\Lambda R)^K_n; \quad B_n = \left(R - \tau\frac{\partial F}{\partial U}\right)^K_n; \quad D_n = B_n U^K_n + \tau F^k_n.$$

For the extreme points of the segment n=0 and n=N, the difference equations are obtained by approximating the first equation for n=0, and the second equation of system 56, n=N. As a result, we obtain

$$B^{11}_0 Q^{K+1}_0 + B^{12}_0 Z^{K+1}_0 + a^{11}_0 Q^{K+1}_1 + a^{12}_0 Z^{K+1}_1 = d^1_0, \tag{59}$$

$$B^{11}_N Q^{K+1}_N + B^{12}_N Z^{K+1}_N + C^{11}_N Q^{K+1}_{N-1} + a^{12}_N Z^{K+1}_{N-1} = d^2_N \tag{60}$$

The system of equations 58-60, together with the conjugation conditions and the boundary conditions, form a closed system of linear algebraic equations in the unknowns Q^{K+1}_n, Z^{K+1}_n ($n=0,1,\ldots,Nm$; $m=1,1,\ldots,m$) within the upper temporal layer t_{K+1}. Given this conjugation, there will be conditions where channels merge in the intermediate points causing an influx of water; for example, the case of a concentrated influx or outflow of water at the boundary of two sections of the channel (Figure 7).

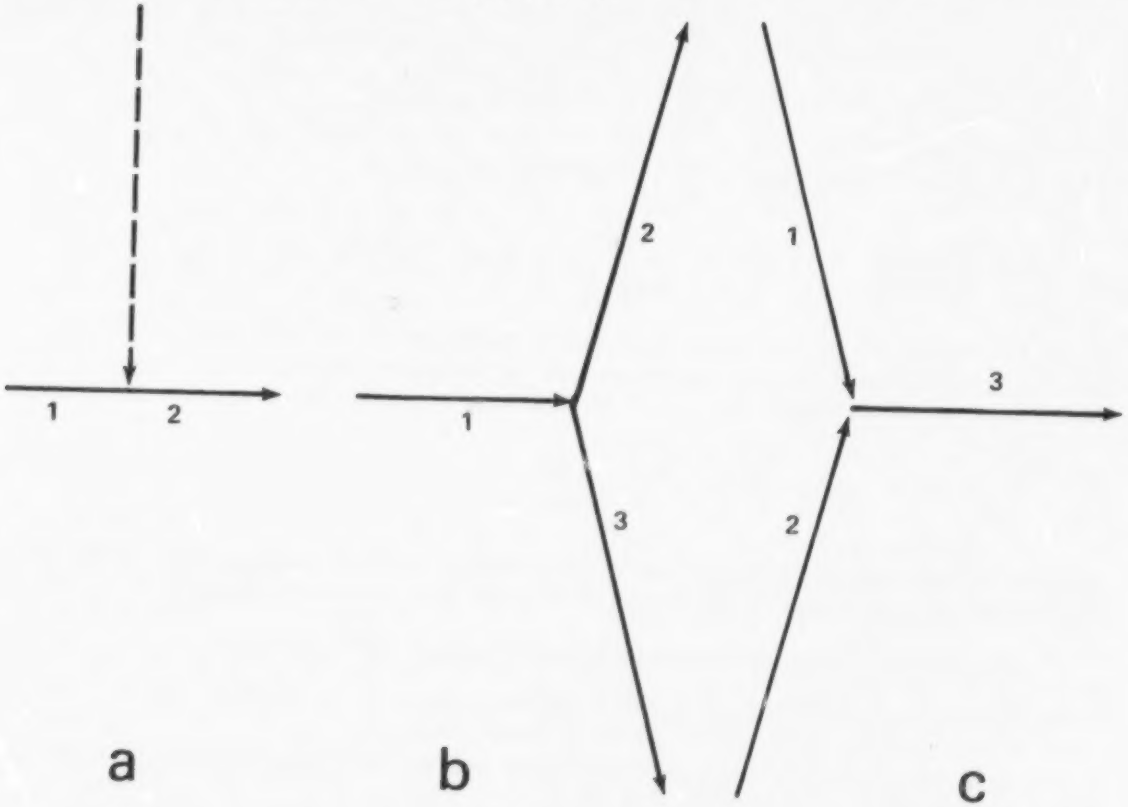


Figure 7. Variants of flow conjugation: a - concentrated inflow (outflow); b - division of flow; c - confluence of channels.

$$Q_2 = Q_1 + Q_k; \; Z_2 = Z_1 \qquad (61)$$

At the separation and juncture of flows in a node (Figure 7), there are conjugation conditions

$$Q_3 = Q_1 + Q_2; \; Z_1 = Z_2 = Z_3 \qquad (62)$$

Without calculating the structure of difference equations and boundary conditions (which in some cases can be even non-linear), the direct solution of the system 58-60, together with the boundary conditions and the conjugation conditions, is practically impossible. Vasiliyev and Voyevodin (1975) suggest two algorithms for the solution of such systems of equations, which consider, first, the type of system of channels, and second, the tridiagonal structure of the matrix of difference equations. The conjugation conditions are presented in the form of the following group of equations:

a) balance relations; b) relations between the parameters on the ends of the segment and the parameters in the vertex, which is the left or right end of this segment; and c) relations between the parameters in the vertex.

For example, during the calculation of hydraulic characteristics, it is convenient to choose as the parameters in the vertex: Q_*^j as additional influx, and Z_*^j as the water level in the vertex. Let j be the number of the vertex, m the number of the segment, and M_j the combination of numbers of segments contiguous to the given vertex; then the boundary conditions or conditions in the vertex can be written in the form

a)
$$\sum_{m \in M_j} \iota_m Q_m = Q_*^j \qquad \iota_m = \begin{cases} 1 \text{ on the left end of the segment} \\ -1 \text{ on the right end of the segment} \end{cases} \tag{63}$$

b)
$$Z_m = Z_*^j, \quad m \in M_j \tag{64}$$

c)
$$Q_*^j = Q_*(t) \tag{65}$$

The following two cases are considered:

1. Ramified system of channels. In this case, it is assumed that the system of vertices and segments constitutes a tree-type graph. The basic property of the graph is that the number of its vertices exceeds by a unit the number of segments. It is not difficult to demonstrate the existence of at least two vertices to which only one segment is contiguous. To one of these vertices is assigned the number 1, to the other (M+1) (where M is the total number of segments in the system). Then the numbered vertex and the segment with the same number are excluded from consideration. As a result, the new scheme of vertices and segments possess the same property as the preceding one; i.e., except for the vertex with number (M+1), at least one vertex continues to exist in the new system to which only one segment is contiguous. The vertex and segment are assigned the following number and the process of numbering is continued until the last segment is numbered. It must be noted that if $m \in M_j$, then $m \le j$. Further, the vertex with the same number as the number of the segment is the left end of the segment, and that axis X is always directed from the left to the right. The sequence of numbering plays an essential role in the process of solving difference equations.

The solution is presented as two stages. In the first stage, the vertices and the segments are considered in a sequence of increasing numbers, beginning from m=1 and i=1. The system of difference equations 58 in each segment is of the form: (Henceforth, index K+1 will be omitted for the sake of brevity.)

$$U_n = X_n U_{n+1} + Y_n, \quad n=1,2,\ldots,N-1, \tag{66}$$

where coefficients X_n and Y_n are determined, with the help of recurrence relations

$$X_n = -(B_n + C_n X_{n-1})^{-1} A_n, \tag{67}$$

$$Y_n = (B_n + C_n X_{n-1})^{-1} (D_n - C_n Y_{n-1}). \qquad (68)$$

Assuming that X_o and Y_o are known, given $n=N-1$, we obtain

$$U_{N-1} = X_{N-1} U_N + Y_N \qquad (69)$$

With the help of relations 69, unknowns Q_{N-1} and Z_{N-1} in equation 60 are eliminated, and, as a result, the following relation for the last point of the m-th segment is obtained

$$(Q_N)_m = \mu_m (Z_N)_m + \zeta_m,$$

and, using condition 64 which is linearized if necessary, we obtain

$$(Q_N)_m = \mu_m^* Z_m^j + \zeta_m^*. \qquad (70)$$

Here, j is the number of the vertex which corresponds to the right end of the segment.

If $m<M$, then relation 70 is used for eliminating unknowns Q_m in equation 63 in the j-th vertex. Thus, condition 63 can be rewritten as

$$(Q_0)_j = Q_*^j + (\sum_{m<j} \mu_m^*)\, Z_*^j + (\sum_{m<j} \zeta_m^*), \qquad (71)$$

and taking into account equation 64 at $m=j$ and 65 on the left end of the j-th section, we can obtain

$$\alpha_j (Q_0)_j + \beta_j (Z_0)_j = \gamma_j \qquad (72)$$

Equation 72 and equation 59 form a closed system relative to the unknowns U_o and serve for computing coefficients X_o and Y_o; and they can be written in a form similar to 66. Here it is necessary to note that conditions 63 on the first segment ($m=1$) now have the form 71; that is, coefficients X_1 and Y_o on the first segment can always be calculated at the beginning of the first stage.

When the segment considered is the last one, equation 78, together with conditions 63-65 in the last vertex, is used for determining the unknown parameter Z_* in the last vertex.

In the second stage, the vertices and the segments are considered in the reverse sequence. First Q_N and Z_N are calculated from known values of Z_*^j (j is the number of the vertex on the right end of the segment) from conditions 64 and 70. Further, the remaining unknowns are calculated by formula 66 for $n=N-1,\dots,0$, and Z_*^m is calculated from condition 64 given known values Q_o and Z_o on the left end of the segment. The unknowns for the rest of the segments are calculated by a similar method.

2. Arbitrary "annular" system. In this case, a graph having an arbitrary number of cycles corresponds to a system of vertices and segments. Here, the numbering is of no importance, and the vertices and segments can be numbered arbitrarily.

The solution algorithm is divided into the following stages (Rozovskiy 1957). At first, only the segments are considered in any sequence. The system of equations 58 on each segment is reduced to the form

$$U_N = X_n U_{N+1} + Y_n + W\bar{U}_o, \quad n = 1,2,\ldots,N-1, \tag{73}$$

Here $U_o = \begin{pmatrix} Z_o^{K+1} \\ 0 \end{pmatrix}$, coefficients X_n, Y_n, W_n for $n=1$ are calculated from the first equation of the system 58, by eliminating the unknown Q_o^{K+1} using equation 59, while for the remaining $n \geq 2$ by the recurrence formulas

$$X_n = -(B_n + C_n X_{n-1})^{-1} A_n,$$

$$Y_n = (B_n + C_n X_{n-1})^{-1} (D_n - C_n Y_{n-1}),$$

$$W_n = -(B_n + C_n X_{n-1})^{-1} C_n W_{n-1}. \tag{74}$$

Only when $n=N-1$ will this relationship be obtained:

$$U_{N-1} = X_{N-1} U_N + Y_{N-1} + W_{N-1} \bar{U}_o, \tag{75}$$

which is used to eliminate the unknowns U_{N-1} in equation 60. As a result, on the right end of each segment, a relation of the following form is obtained

$$(Q_N)_m = (\mu_{11} Z_N)_m + (\mu_{12} Z_o)_m + (\mu_{13})m. \tag{76}$$

In order to obtain a similar relationship on the left end of the segment, equation 59 is rewritten in the form

$$Q_o = -(a_o^{11} Q_1 + a_o^{12} Z_1)/B_o^{11} - (B_o^{12} Z_o)/B_o^{11} + d_o/B^{11}$$

and with the help of relation 75, the unknowns $Q_1, Z_1, Q_2, Z_2, \ldots,$ are eliminated in succession. Calculating equation 76, we finally obtain

$$(Q_o)_m = (\mu_{01} Z_N)_m + (\mu_{02} Z_o)_m + (\mu_{03})_m. \tag{77}$$

Equations 76 and 77, given $m=1,2,\ldots,$ together with the conjugation conditions 61-65 in each vertex, form a closed system of equations relative to the unknown parameters in the Q_*^j Z_*^j vertices and Q, Z on the ends of the segments. The derived system is efficiently solved by the iteration method (Rozovskiy 1957).

After the unknowns on the ends of the segments have been calculated, the solution in all the internal points for all the segments is determined with the help of equation 75.

A computational framework developed on the basis of these algorithms allows calculation of a wide range of hydraulic problems for open channels. The most efficient method of program construction for multipurpose programs is the combination of separate functional elements (modules). The calculation of various and different problems require various combinations of such elements. A series of computational modules which contain the variety of required algorithms is organized and can be applied to specific situations.

Some of the required coefficients (V,W), necessary for solving the transfer equations, are derived from integration of Saint-Venant non-steady-state motion equations. Yeremenko, Kolpak, and Selyuk (1975) integrate the transfer equations by numerical methods using implicit and explicit difference schemes. The conditions differ, depending on the difference scheme. The initial equation is of the equation 36 type and can be closed under special conditions.

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial X} = \frac{1}{W}\frac{\partial}{\partial X}\left(WD\frac{\partial C}{\partial X}\right) - KC + f(X,t) \tag{78}$$

The implicit two-layer difference scheme is approximated by the following differences

$$\frac{C_i^{j+1} - C_i^j}{\tau} + V_i\frac{C_{i+1}^{j+1} - C_{i-1}^{j+1}}{2\Delta X} = \frac{1}{2(\Delta X)^2 W_i}\left[(W_{i+1}D_{i+1} + W_iD_i)(C_{i+1}^{j+1} - C_i^{j+1})\right.$$

$$\left. - (W_iD_i + W_{i+1}D_{i-1})(C_i^{j+1} - C_{i-1}^{j+1})\right] - KC_i^{j+1} + f_i^{j+1} \tag{79}$$

and is solved by the sweeping method

$$C_i^{j+1} = \alpha_{i+1/2}\, C_{i+1}^{j+1} + \beta_{i+1/2} \tag{80}$$

where the sweeping coefficients are determined from recurrence relations

$$\alpha_{i+1/2} = \frac{M_i}{2B_i - A_i\alpha_{i-1/2}}, \quad \beta_{i+1/2} = \frac{A_i\beta_{i-1/2} - G_i}{2B_i - A_i\alpha_{i-1/2}},$$

in which

$$A_i = \frac{\tau}{2\Delta X}\left(V_i + \frac{W_iD_i + W_{i+1}D_{i+1}}{W_i\Delta X}\right), \quad 2B_i = 1 + K\tau + \frac{\tau}{2W_i(\Delta X)^2}(W_{i+1}D_{i+1} +$$

$$+ 2W_iD_i + W_{i-1}D_{i-1}); \quad M_i = \frac{\tau}{2\Delta X}\left(\frac{W_iD_i + W_{i-1}D_{i-1}}{W_i\Delta X} - V_i\right), \quad G_i = C_i^j - f_i\tau$$

With this difference scheme, the left-hand boundary conditions are $C = \eta(t)$. But it is assumed that, in the closing section ($i=N$), the diffusion is negligible compared with the transfer; therefore, $\partial C/\partial t + V(\partial C/\partial X) = 0$ is accepted. Consequently, sweeping coefficients $\alpha(3/2) = 0$; $\beta(3/2) = \eta(t)$, and the concentration

$$C_N^{j+1} = \frac{C_N^j/\tau + \beta_{N-1/2}}{\Delta X/V_N(1/\tau + V_N/\Delta X) - \alpha_{N-1/2}}$$

is determined.

The disadvantage of the above difference scheme is the limitation on the spatial step $\Delta X < 2D/V$ (Maron 1971)

The results of the investigation of various difference schemes by Yeremenko, Kolpak, and Selyuk (1975) suggest that the Dufour-Frankel three-layer explicit difference scheme, which does not require limitations on the spatial step, is the best available method. The initial equation in this case is presented in the difference form as

$$C_i^{j+1} = C_i^{j-1} + 2\tau[-V_i + \frac{1}{W_i} \frac{(DW)_{i+1} - (DW)_{i-1}}{2\Delta X}] \cdot \frac{C_{i+1}^j - C_{i-1}^j}{2\Delta X} + 2\tau D_i$$

$$\frac{C_{i+1}^j - C_i^{j+1} - C_i^{j-1} + C_{i-1}^j}{(\Delta X)^2} + 2\tau[f_i - KC_i^j]. \tag{81}$$

The stability criteria of this difference scheme are

$$\tau \leq \min\{\frac{\Delta X^2}{2D}; \frac{\Delta X}{|V|}\} \tag{82}$$

When the explicit difference scheme is used, the diffusion effects in the closing section are not excluded, and equation 78 is replaced by a finite-difference two-layer pattern

$$C_N^{j+1} = C_N^j - \tau[V_N \frac{C_{N-2}^j - 4C_{N-1}^j + 3C_N^j}{2\Delta X} - D_N \frac{C_{N-2}^j - 2C_{N-1}^j + C_N^j}{(\Delta X)^2}], \tag{83}$$

for which the following stability conditions are derived

$$\frac{D}{V} \leq \Delta X \leq \frac{2D}{V} \qquad \tau \leq \frac{2(\Delta X)^2}{3V\Delta X - 2D}$$

$$\frac{2D}{V} \leq \Delta X \qquad \tau \geq \frac{2(\Delta X)^2}{3V\Delta X - 2D} \tag{84}$$

These stability conditions do not contradict the stability conditions of the Dufour-Frankel scheme. Yeremenko et al. (1975) used the following formula to determine the longitudinal dispersion coefficient

$$D = 1.5BV, \tag{85}$$

where B is the flow width.

Two methods are used for calculating the conjugation conditions when the influx into the river at the junctions E of tributary confluence is considered (Technical Guide, 1963). The following condition is valid if diffusion is not neglected at the point of confluence of flows 1 and 2 into stream 3.

$$C_{E(3)}(Q_{E(1)} + Q_{E(2)}) = Q_{E(1)}C_{E(1)} + Q_{E(2)}C_{E(2)} \tag{86}$$

The concentrations $C_{E(1)}$ and $C_{E(2)}$ are determined by algorithms described above for explicit or implicit schemes.

For cases when the influence of diffusion cannot be neglected, the conjugation condition takes the form

$$(QC - WD\frac{\partial C}{\partial X})_{E(3)} = (QC - WD\frac{\partial C}{\partial X})_{E(2)} + (QC - WD\frac{\partial C}{\partial X})_{E(1)} \tag{87}$$

Various difference approximations of condition 87 are possible. Numerous calculations showed that it is best to use the continuity condition $C_{E(1)} = C_{E(2)} = C_{E(3)}$, in which case the obvious equality $Q_3 = Q_1 + Q_2$, instead of 87 gives

$$WD\frac{\partial C}{\partial X}\Big|_{E(3)} = WD\frac{\partial C}{\partial X}\Big|_{E(2)} + WD\frac{\partial C}{\partial X}\Big|_{E(1)} \tag{88}$$

By using conditions 86 and 87, it can be shown that the influence of diffusion is important only at points adjacent to node E (Figure 8).

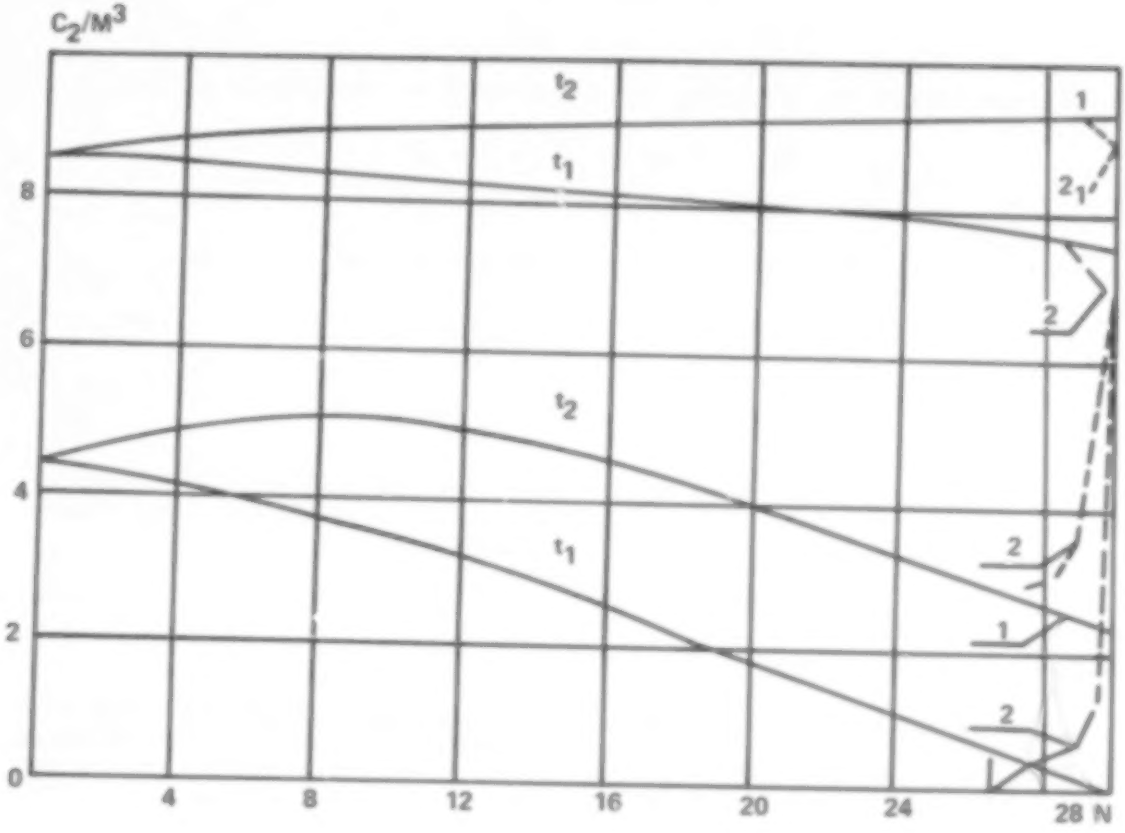


Figure 8. Calculated pollutant concentrations in two confluent flows at times t_1 and t_2. Curves 1 are results obtained without calculating diffusion effects; curves 2 calculate diffusion effects; 0, the value of concentration in conjugation node, with calculation of diffusion.

Similar conjugation conditions are considered by Vasiliyev and Voyevodin (1975). Here, as in the calculation of variable flows, the case of concentrated influx or outflow of water, confluence, or disjunction of flows is considered. For instance, in the case of confluence of flows, the condition $C_{E(1)} = C_{E(2)} = C_{E(3)}$ is equivalent to equation 87. Also, two forms of calculating the flow conjugation, corresponding to cases 86 and 87, are considered. The transfer equation is presented in the form

$$W\frac{\partial C}{\partial t} = \Psi\frac{\partial}{\partial X}\left(\frac{WD}{\Psi}\frac{\partial C}{\partial X}\right) + F(C) \tag{89}$$

where

$$\Psi = \exp\int_0^X (Q/WD)dx$$

and is approximated with the help of the ordinary implicit first order difference scheme in t, and of the second order in X (Maron 1971).

$$W_n^{K+1}\frac{C_n^{K+1} - C_n^K}{\tau} - \Psi_n^{K+1}\left[\left(\frac{WD}{\Psi}\right)_{n+1}^{K+1}\frac{(C_{n+1}^{K+1} - C_n^{K+1})}{\Delta X^2} + \left(\frac{WD}{\Psi}\right)_{n-1/2}^{K+1}\frac{(C_n^{K+1} - C_{n-1}^{K+1})}{\Delta X^2}\right]$$

$$= F_n^{K+1} \tag{90}$$

Here the coefficients and the right hand side of the expression are calculated approximately as follows (Sherenkov 1975):

$$\Psi_n^{K+1}\left(\frac{WD}{\Psi}\right)_{n\pm 1/2}^{K+1} = (WD)_{n\pm 1/2}^{K+1}\left[1 \pm \frac{\Delta}{2}\left(\frac{Q}{WD}\right)_{n\pm 1/2}^{K+1}\right], \quad F_n^{K+1} = F_n^K + \left(\frac{\partial F}{\partial C}\right)_n^K (C_n^{K+1} - C_n^K)$$

As a result, for each segment the following system of implicit equations relative to the unknowns is obtained, C_n^{K+1}, $n=1,2,\ldots,N-1$

$$U_n C_{n+1}^{K+1} + B_n C_n^{K+1} + e_n C_{n-1}^{K+1} = d_n \qquad n = 1,2,\ldots,N-1 \tag{91}$$

where $a_n = \frac{\tau}{\Delta X}\left(Q - \frac{WD}{\Delta X}\right)_{n+1/2}^{K+1}$; $\quad l_n = \frac{\tau}{\Delta X}\left(Q + \frac{WD}{\Delta X}\right)_{n-1/2}^{K+1}$;

$$l_n = W_n^{K+1} - (a_n + e_n) + \tau\left(\frac{\partial F}{\partial C}\right)_n^K; \quad d_n = B_n C_n^K + \tau F_n^K$$

In addition to equations 91, at the points n=0 and n=N, the following equations are developed

$$F_o^{K+1} = Q_o^{K+1}C_o^{K+1} - \left(\frac{WD}{\Delta X}\right)_o^{K+1}(C_1^{K+1} - C_o^{K+1}), \tag{92}$$

$$F_o^{K+1} = Q_o^{K+1}C_o^{K+1} - \left(\frac{WD}{\Delta X}\right)_N^{K+1}(C_N^{K+1} - C_{N-1}^{K+1}) \tag{93}$$

Together with the boundary conditions and the conjugation conditions, equations 91 to 93 form a closed system relative to the unknowns

$$C_n^{K+1}(n=0,1,\ldots,N_m, \quad m=1,2,\ldots,M)$$

in the upper temporal layer. The solution sequence of the equations is similar to the variable motion calculation. This solution algorithm has a disadvantage in the implicit difference scheme which is described by Yeremenko et al. (1975), but in spite of the transformation, the solution of equation 92 remains sensitive to the condition $\Delta X \leq 2D/V$. They also describe the mathematical model which evaluates the effect of surface flow on concentration distribution in a channel, using the turbulent diffusion equation. With an essentially non-uniform concentration distribution in the actual flow section, the transfer equation is

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial X} = \frac{1}{W}\frac{\partial}{\partial X}\left(DW\frac{\partial C}{\partial X}\right) - \frac{1}{W}\frac{\partial T}{\partial X} - q\overline{C} + f, \tag{94}$$

where $1/W(\partial T/\partial X)$ is an additional term which describes the unevenness of the dispersion in the flow section, and

$$T = \int \overset{0}{V}_{1} \overset{0}{C} dW - DW\frac{\partial \overline{C}}{\partial X}, \quad \overline{C} = \frac{1}{W}\int C dW, \tag{95}$$

where $\overset{0}{C} = C(\vec{X},t) - \overline{C}, \quad \overset{0}{\underset{1}{V}} = V_X(\vec{X},t) - V$

The estimation of the term containing T is derived for the case when the concentration of pollutant delivered by the flow does not change with time, while the discharge of influx water is negligible. The influence of the term containing T is investigated by comparing the solution of the three-dimensional problem, in the specified velocity field, with the one-dimensional problem, using equation 95 without calculating the term using the following conditions:

$$\overline{C}(X,0) = 0, \quad \overline{C}(0,t) = 0. \tag{96}$$

The three-dimensional problem

$$V_1\frac{\partial C}{\partial X_1} = D_2\frac{\partial^2 C}{\partial X_2^2} + D_3\frac{\partial^2 C}{\partial X_3^2} + F(X_1,X_2,X_3), \tag{97}$$

with the boundary conditions

$$C(0,X_2,X_3) = 0, \quad \left.\frac{\partial C}{\partial n}\right|_{\gamma} = 0 \tag{98}$$

where γ is the contour of the flow and n is the normal to this contour, was integrated with the help of the known locally unidimensional variable direction method (Samarskiy, 1971). The lateral pollutant influx is specified by the source function $F(X_1,X_2,X_3)$. Calculations were carried out for a trapezoidal channel with a specified cross-sectional distribution of longitudinal velocity. In the calculations, various distributions of sources along the channel were specified as (a) of constant strength, (b) constant for specific periods, and (c,d) sinusoidal. The results of calculating the three-dimensional problem, after the concentration C has been averaged in the cross-section, are shown in Figure 9. The results of the solution to the one-dimensional problem are also displayed in Figure 9; curves 1, corresponding to the solution of the one-dimensional equation, are located lower than curves 2, the plots of the averaged function $\bar{C}$.

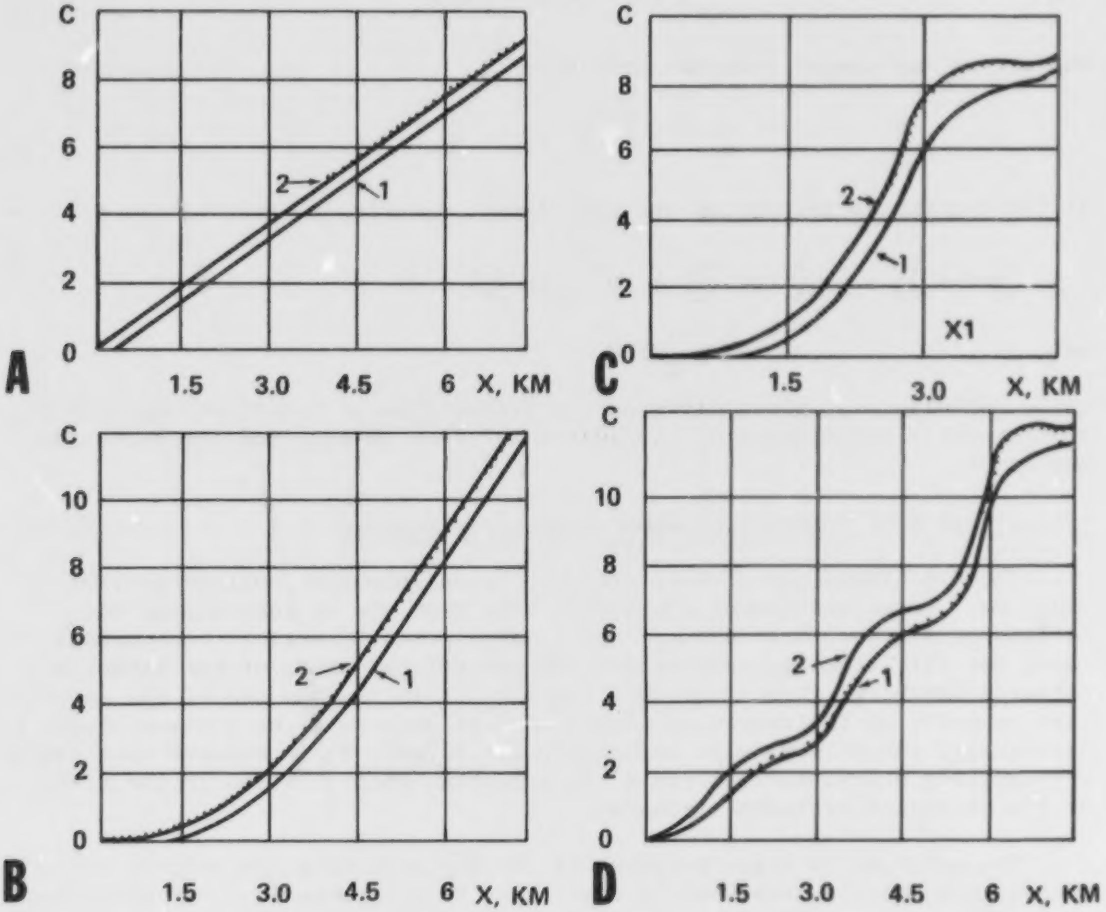


Figure 9. Diagrams of variation in concentrations along the X axis given differed specification of function $F(X_1,X_2,X_3)$ (curves 1 are the solutions of the three-dimensional problem averaged over the cross-section; curves 2 are the solution of the one-dimensional problem without calculating the term; the dots indicate the solution of a one-dimensional problem, with a calculation of the term $F(X_1,X_2,X_3)$.

The error fluctuates within 10% to 25%. In some cases therefore, the additional term in equation 94 should be taken into account and its approximating expression introduced. For this, the pollutant transfer equation is considered without taking into account the diffusion effects

$$V_1\frac{\partial C}{\partial X_1} = F(X_1,X_2,X_3).$$

Integrating this equation by X_1, we have

$$\frac{1}{W}\frac{\partial T}{\partial X} = \overline{F} - \kappa(\overline{F} - \frac{D}{V}\frac{\partial F}{\partial X}), \tag{99}$$

where κ is introduced from the condition

$$\frac{\kappa}{V_1}\overline{F} = \frac{1}{W}\int\frac{F}{V_1}\,dW \tag{100}$$

If the quantities introduced are considered, equation 91 assumes the form

$$\frac{\partial C}{\partial t} + V\frac{\partial C}{\partial X} = \frac{1}{W}\frac{\partial}{\partial X}(DW\frac{\partial C}{\partial X}) + \kappa V^{-1}(f - D\frac{\partial f}{\partial X}) - qC \tag{101}$$

Here $f = \overline{F} = \frac{1}{W}\int F dW$

These calculations, given different specifications of function F and κ=1.05, indicate that calculation of the additional term in equation 101 decreases the error.

Solution of Some Problems of Water Quality Management

The algorithms given above allow us to define water quality control problems. Yeremenko (1975) considered some problems of controlling the quality of effluent discharges. Preparation for discharging from storage lakes for different purposes should include determination of the impact of effluent discharge into a variable flow to obtain maximum use of the assimilative capacity of the receiving water body, to determine the minimum discharge hydrograph, and determine an acceptable discharge. It is assumed that there are existing standards that limit the pollutant concentration in the river at the points of effluent discharge.

The solution to these problems can be attained with the help of the mathematical models described in this paper by using the explicit difference scheme 81.

Determination of the effluent discharge regime required for discharge into a variable flow, can be reduced to the calculation of the source capacity given a limitation on BOD concentration equal to $\overline{L}$; from equation 81 we find

$$f^n_{L_i} = K\overline{L} - [-V_i + \frac{1}{W_i}\frac{(DW)_{i+1} - (DW)_{i-1}}{2\Delta}] \cdot \frac{L_{i+1} - L_{i-1}}{2\Delta} + D_i\frac{L^n_{i+1} - 2\overline{L} + L^n_{i-1}}{\Delta^2} \tag{102}$$

An actual example of this type was solved for a channel having a width of B=150m, initial depth h_o=4.0m and specified $Q_o(X,t)$=227m^3/sec, $Q_o(0,t)=SQ_o$ sin¶t/T (T is the specific given period of time), Q(N)=f(Z); L(X,0)=0; L(0,t)=0; L(4000m,t)=4.2m/m^3=$\overline{L}$=constant; the concentration of dissolved oxygen S(X,0)= 6.0g/m^3, S(0,t)=6.0g/m^3 was given. In the calculations, S_{np}=9.0g/m^3, K_1=0.2 1/day, K_2=0.4 1/day, were accepted. The results of the calculation are shown in Figure 10(a,b), where the time intervals are t_1=3360 sec, t_2=10860 sec,

t_3=18060 sec, t_4=25260 sec.

The feature of pollutant dispersion in a variable velocity flow, as in Figure 10(a), is the relatively small velocity of pollutant dispersion, in comparison with the velocity of propagation of water. Figure 10b indicates a calculated change of assimilative capacity with time.

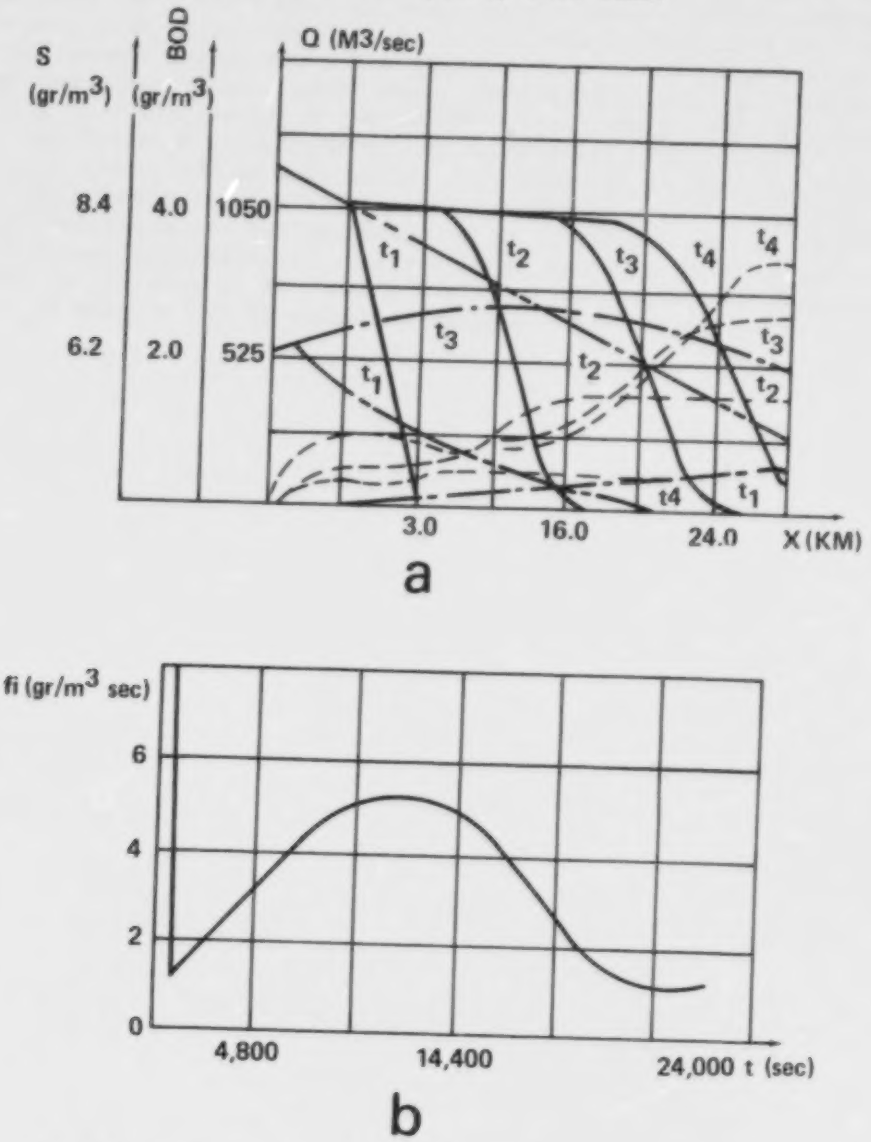


Figure 10. Results of calculating pollutant concentrations and source capacity in an unsteady flow.

The determination of the minimum discharge hydrograph that will permit an acceptable release of effluent into a watercourse was also considered. A typical problem is the release hydrograph during daily control of the load at a hydropower station. At a fixed capacity of pollutant delivery, the minimum discharge should maintain BOD within the required standards. This problem is solved by matching. To speed up the calculations, it is possible to apply standard programs to find the roots of functions on a finite segment. A typical solution for a variable flow in a 500m-wide channel is described in the section X=0 by the hydrograph shown in Figure 11a. The problem is solved when a minimum release discharge is found, which maintains $\max L \leq L$ norm (L norm is the permissible BOD value), along the entire channel at the points of effluent discharge. Within the given 100km long channel, two sources of constant capacity f_{L_1}(X=5.0km) = 3000g/sec, f_{L_2}(X=40.0km) = 700g/sec, were located. The minimum discharge of the hydrograph is Q_1=451m^3/sec. BOD calculations with the given hydrograph showed that at the source f_{L_1}, concentrations may appear which exceed the permissible value L norm=6.0g/m^3, as seen in Figure 11b. In addition, the variation of BOD concentration (L_1) at the source during 24 hr is shown in Figure 11a. Calculations showed that by increasing the discharge Q_1 to Q_2=493 m^3/sec, it is possible to provide the BOD (L_2) values equal to the minimum standards in the section f_{L_1}.

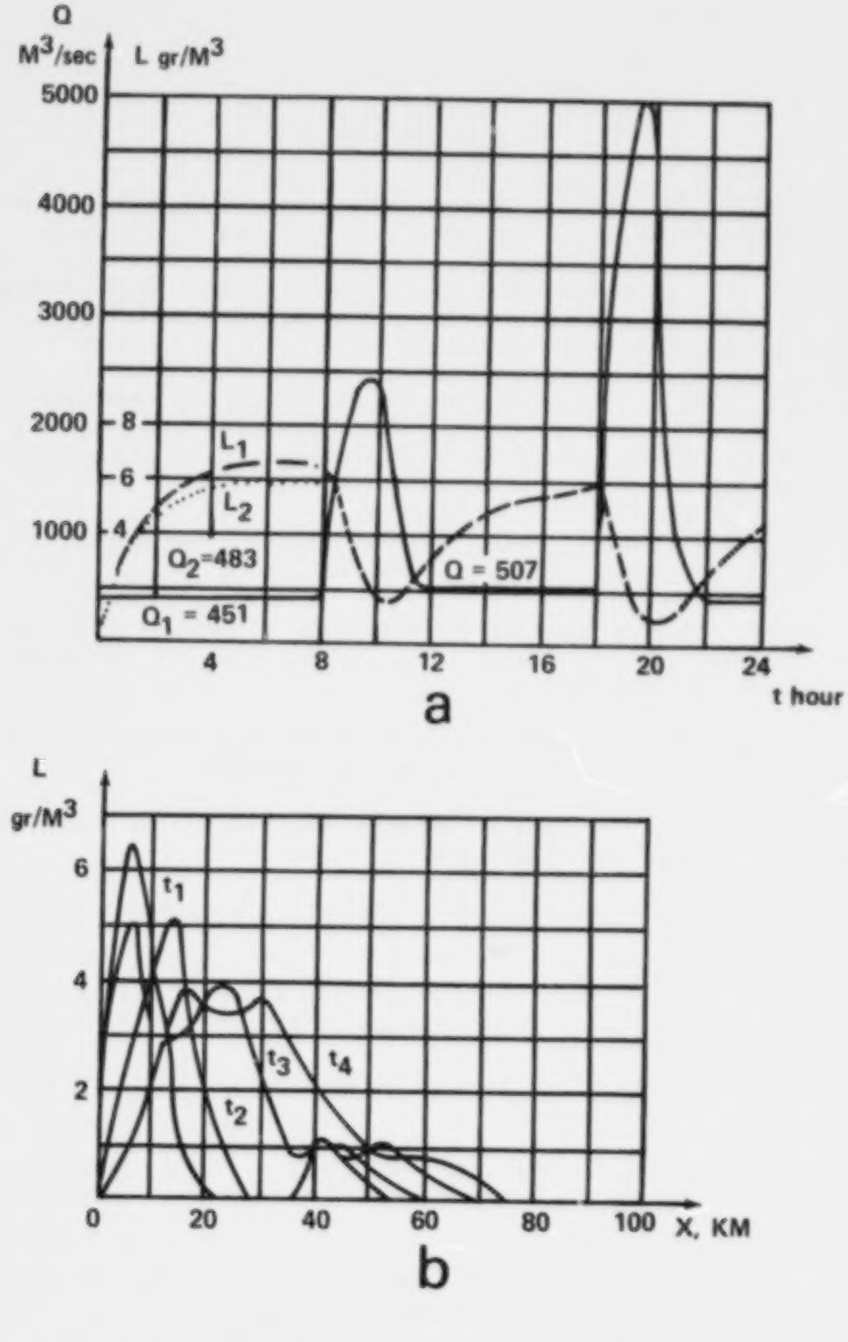


Figure 11. Results of calculating minimum water discharge given water releases for a guarantee of required water quality in the flow.

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DISCUSSION

THOMANN: I am slightly acquainted with the state-of-the-art in modeling water quality in the USSR. The material presented by Dr. Yeremenko illustrates a considerable step forward for Soviet scientists. I congratulate him.

On the basis of practical testing of river water quality modeling in the U.S., it is possible to analytically indicate that results with equations that consider dispersion do not significantly differ from solutions received without this consideration. Can you comment on this situation?

YEREMENKO: The situation mentioned by Dr. Thomann is a known fact. It is correct, for the most part, in stationary flows and steady flows when wastewaters are discharged. In a nonstationary case, the calculation of diffusion processes can play an important role. Therefore, it is necessary to conduct an analysis and to give a basis for the necessity or expediency of calculating diffusion.

YENETSIANOV: I would like to add to this report, namely to the part related to the equation that describes the distribution of the pollutant in a river flow. When describing the physical processes of a river, a linear diffusion equation is used as given, without critical analysis. From our point of view, it is possible to use this equation in the manner indicated only when describing the behavior of nonconservative substances. It reflects rather accurately the physical processes occurring with nonconservative components. Nonconservative pollutants can be divided into two large groups: 1) "heterophased" pollutants that interact physically and chemically with sediments, and 2) essentially nonconservative pollutants for which the law of conservation of mass is not fulfilled.

For the first group we proposed to supplement the equation with a term which describes the interaction with the sediment phase of the pollutant. In both phases, the pollutant must be in a thermodynamic equilibrium. Time is required to establish this equilibrium. If the equilibrium is shifted to the nonliquid phase, then the concentration in solution can essentially differ from the real content of the component on a given stretch of a river. When the equilibrium is shifted to the aqueous phase, a large quantity of the component is extracted from the nonliquid phase.

The second modification concerns the linear diffusion coefficient. The criterion is written in the following manner: D/vL where D is the diffusion coefficient, v is velocity, and L characteristic size. Depending on the value of this criterion, both Dr. Thomann and Dr. Yeremenko are correct.

BEZOBRAZOV: The impression has been created that research on diffusion processes is gradually yielding to wider, more problematic questions of constructing water quality control models. However, you must realize that most of these models at one stage or another must be based on solutions to diffusion problems. Many questions, connected with the diffusion processes, have been solved. Some of these questions received exhaustive analysis in Dr. Yeremenko's report. However, as more questions are solved, more controversial and complex questions arise, in particular, problems of evaluating the relationships of the turbulent diffusion coefficient, or, to be more exact, the relationships of its various components. It is necessary to consider the influence of allowing isotropy or anisotropy of the diffusion coefficient on the dispersion of a passive pollutant.

RODZILLER: I agree with Dr. Thomann's evaluation of Yeremenko's paper. He gave us an exhaustive summary of the material presented for the symposium. However this is not an exhaustive summary of all the work in progress in this country. Mathematical models must influence decision-making which will maintain clean water bodies. I fully agree with Dr. Thomann that it is easy to write a complex equation, but that it is difficult to solve it, and likewise difficult to put it into practice. Therefore, we must heed Dr. DeLucia's words: a simple and accessible solution is better than a mathematically elaborate one.

For practical goals, I consider it necessary to concentrate our attention on solving the water quality question for water bodies depending on the arbitrary quantity of the discharge and the arbitrary composition of the wastewaters. In the models being developed, it is necessary to consider substance transformation processes. I assume that you have such models, perhaps not absolutely exact, but which allow you to solve engineering problems.

SELYUK: When planning water economy activities and regulating quality of water sources, it is necessary to consider self-purification processes. Questions of the detailed study of physical processes in water bodies, and the development and perfection of mathematical models of these processes deserve particular attention.

YEREMENKO: I must agree with Venetsianov and Bezobrazov's remarks on the necessity to continue research on the behavior of nonconservative pollutants in a water body. This work is complex. In the report presented, a determined relationship between the diffusion coefficient's components is actually shown. We understand that this is not a rule for all circumstances. Therefore, we have tried in the report to illustrate the influence of the choice of this value and used its determined meaning to obtain a concrete solution. We thank you for all the valuable and useful comments, which we will try to take into consideration in further work.

THOMANN: By U.S. experience, diffusion processes are not as important as the kinetics of photosynthesis processes, oxygen consumption, transformation of various forms of nitrogen, carbon, etc., for calculations and river basin management. These factors are taken into consideration in the practical models being developed in the U.S.

WATER QUALITY MANAGEMENT MODELS:
SPECIFIC CASES AND SOME BROADER OBSERVATIONS

Russell J. DeLucia and Tze-wen Chi

INTRODUCTION

Some Issues Associated with the Planning Process and Model Development

In the process of planning investment in water pollution control of a river basin, a great deal of analysis is needed to examine many pertinent facets of the problem. There are interacting physical, economic, social, and political subsystems in a basin. Each subsystem can be complex and involve time-consuming study. Models are often used to ensure and facilitate consistent analysis. However, there is no one prescribed approach for model building.

A model can be comprehensive enough to include all subsystems. A model of this type in general will be coarse and simplified for computational convenience and, therefore, will seldom provide sufficient details of the system's response. To remedy the shortcoming, other models are used to generate refined information. Some of these models will be subsystem specific while others will include more than one subsystem. They can be designed to be used conjunctively in that output of some models may be input to others. For example, a highly aggregated optimization model generates plans which can be further analyzed by a detailed simulation model. In addition, models can be designed for feedback iteration. The appropriateness of a single model versus many models and how to structure it (or them) depends upon the analyst's experience, preference, and the particular system under consideration. In practice, one would start with relatively simple and robust model(s) and refine toward more reliable model(s).

In the broadest sense, the development of a water quality model is dependent on the characteristics of the problem being examined. We will not attempt, in this paper, to provide an exhaustive list of the issues that arise in this process, but we do wish to present four areas of concern that influence the development. These are:

1. the relationship of planning activity to the problems of variable specifications;
2. the nature of the institutional or client setting;
3. the type and quality of data available for model development; and
4. the availability of resources for model development and implementation.

In the first area, the issue of the specificity with which the model's endogenous variables must be defined is the primary consideration. For example, although a river basin may have an important nonpoint waste load, this pollution input can be accounted for in the model as an equivalent point source because specific control alternatives for nonpoint loads are not being considered. Another example is a case in which non-treatment alternatives, such as flow augmentation or bypass piping and regional treatment, are under consideration, but they need not be included in the model specification. Rather, such alternatives are dealt with by running the model under an alternative flow regime (to account for flow augmentation) or by developing treatment cost functions which reflect transport costs (to account for piping and regional treatment). There are other issues, of course, having to do with the nature of the pollution problem (e.g., the importance of nutrients) and the nature of the preference function (e.g., the problem of equity versus efficiency).

The second area of concern is associated with the institutional or client setting of the planning problem. Here the issues influence decisions related to the structure of the mathematical model and its level of documentation. Is the model for general use or is it to be employed in the analysis of a specific problem? Is the planning problem strictly a water quality management problem or is the model to be part of a larger planning effort and will it have to be made compatible with other quantitative models?

The more general the model use will be, the stronger the argument for detailed documentation and the inclusion of a variable structure that explicitly deals with many of the issues suggested in the area discussed above. On the other hand, if the model is being developed for a very specific planning problem or is strictly a water quality management model, then both the structure and ultimate documentation of the model can be limited. Strict water quality management problems argue for less complex models. The availability of complementary models suggests that the model's variable structure may be limited and these variables or characteristics should be considered in a second or complementary model.

The last two areas (availability of information and resources) jointly influence both the level of complexity of model variable structure and ultimately its implementing computer code. The more complex the model in general, the more the data, computing, and professional resources necessary. In some cases significant prior information allows the use of a more complex model than would be warranted by the study budget. Consider, for example, the situation in which there is adequate existing field data to allow extensive calibration of parameters. Additional data from the present study could then be developed slowly for the ultimate verification exercise.

The development of the Saint John River model provides an example in which many of the issues in the four areas arise. As the model is presented in the following section, we will highlight the issues we considered most important and discuss the important features of the discussions that led to the development of the model in its present form.

The Saint John Model

The model discussed in this paper was developed for initial use in the Saint John River Basin described briefly below. However, the characteristics of the model reflect not only the particulars of the specific Saint John planning problem for which it is being used. In addition, its characteristics reflect the planning requirements and constraints implicit in the 1972 Amendments to the Federal Water Pollution Control Act (Public Law 92-500). The model was developed and was documented so that it would be useful in examining basin or area-wide planning efforts required or suggested by various sections of the Amendments.

The model was developed for use in a planning study conducted by a team of consultants, under a U.S. Environmental Protection Agency contract with the Northern Maine Regional Planning Commission in a joint venture of E.C. Jordan Co., Inc., and Meta Systems, Inc.

The Northern Maine Regional Planning Commission in cooperation with the state agency is undertaking aspects of water quality planning mandated by P.L. 92-500. The initial model users are members of the consultants' staff who have been involved in the development of this and other models. However, the model development was funded by EPA and is being documented at a level sufficient to allow ready application by other users.

Prior to discussing particular characteristics of the model, it is useful to describe some of the particulars of P.L. 92-500, the nature of the basin for which the planning is being done, and complementary planning efforts that have been done.

The P.L. 92-500 calls for the attainment of "secondary" treatment or better by 1977 for publicly owned treatment works and best-practicable control technology for industries provided that these levels of treatment/control will result in meeting water quality standards. In cases where these levels of technology are insufficient to meet the standards, more stringent technology would be called for. P.L. 92-500 further states that when it is necessary to move beyond secondary and best-practicable, the targeted reductions should reflect economic considerations. More stringent 1983 goals, namely the application of best-practicable by publicly owned treatment works and best-available technology economically achievable by industries, are also required. The explicit two-time period-treatment targets are incorporated into this model that considers both capacity expansion and level of treatment questions.

The Saint John River drains the northern third of the State of Maine in the U.S.A. and portions of the Provinces of Quebec and New Brunswick in Canada. The planning effort which sponsored the development of the model(s) discussed focused on the Maine portions of the basin and the international boundary waters between New Brunswick and Maine. Another planning study was carried out by a Canadian team and there was a series of both formal and informal exchanges of information. In addition, the United States planning study and the development of the Saint John model benefited significantly from the results of an earlier methodological study sponsored by the Department of the Environment of Canada. This earlier study used the Saint John

Basin as an example and included the development of a prescriptive model; a nonsteady-state stochastic simulation model developed in this earlier study of the Saint John Basin was also available for use.

The watershed is sparsely populated, with large areas of timberland and agricultural land both of which contribute significant nonpoint pollution loads. There are, however, a number of large volumes of either untreated or partially treated industrial (pulp and paper, potato processing) and domestic effluents which have created severe water quality problems.

Model(s) Overview

The model(s) developed actually comprise a set of complementary models. There is often no unique mathematical model for a particular problem (Meta Systems Inc, 1971). Rather, a set of linked, sometimes hierarchical models, is most useful. In this study, we have developed both a descriptive and a prescriptive stream/river model and a descriptive lake model. Only the stream/river models are discussed in this paper.

A descriptive model relates pollutant loads with stream quality indicators. This model can be run independently as a steady-state, quality simulation model. It also produces the matrix of descriptive relationships that become part of a prescriptive non-linear (separable) programming model which minimizes present value of costs over a multiple-period, time horizon. Capital costs are distinguished from operating and maintenance costs and appropriate relative multipliers may be introduced when useful. The impact of reimbursement funds (both federal and state) and cost sharing of joint municipal industrial treatment works may be included in this manner. Provision is made in the model for inclusion of various equity and budgetary constraints.

The physical systems relationships are based on stream hydraulics and kinetics with some modification for reservoirs and lakes that may be integral to the system. The dissolved oxygen/biochemical oxygen demand (DO/BOD) regime is modeled by using a modification of the empirical Streeter-Phelps (1925) relationship. The carbonaceous and nitrogenous BOD loads are handled individually (the latter including an appropriate lag effect). The combination of long travel time and pulp and paper industry loads present in the Saint John River, with their resultant significant nitrogenous loads, argued for this inclusion. Point and nonurban nonpoint pollution sources are significant. They are explicitly included with the options of including cost of controls for both point and nonpoint sources since source control options differ. The kinetic characteristics of the major pollutants (pulp and paper waste, potato processing waste, and municipal wastes) can vary considerably. The inclusion of polluter-specific kinetics allows explicit consideration of different control levels for different polluters. The nutrient regime is examined by considering nitrogen and phosphorus as total species and therefore essentially as conservative elements. The analysis of this model is based on the assumption of steady-state regimes (although, for example, variation of the hydrologic regime is easily accomplished).

The structuring of a prescriptive water quality management model which incorporates relationships between pollution loads and stream quality based on "steady-state" solutions of stream and/or estuary equations is not new. The uniqueness of the prescriptive model presented herein lies in: a) its structure reflecting planning problems arising from P.L. 92-500; b) the use of a synthetic pollutant to express the cost removal function; and c) a series of heuristics adopted to overcome possible nonconvex difficulties.

The first of these will be obvious from the structure presented below. The second and third are somewhat interconnected.

The Model--

The basin is divided into activity points or reaches labeled j, j=1, ...,J, and quality points or reaches i, i=1,...,I. For particular i and j, the reach or point may be identical.

The descriptive approach entails the development of a set of transfer functions, T_{ji}, relating the quality at point i with the pollution at point or reach j. T_{ji} is a vector transfer coefficient in that this relationship is developed for each of the ℓ pollutants and quality indicator couples we are examining (DO, BOD, N, P). Letting $T_{ji\ell}$ represent the scalar element for pollutant ℓ in the vector T_{ji}, we note that $T_{ji\ell}$ is a function of geometry, flow, kinetics, etc. $T_{ji\ell}$ is a function of system physics.

The descriptive river model uses stream hydraulics which are overwhelmingly advective with little or no dispersion, and are based on a steady-state analysis. The pollutants (instream) that are discussed below are treated with zero or first-order kinetics dependent on their conservative or nonconservative nature.

The calculation of the transfer coefficient for the nonconservative organic BOD and DO interrelationships is programmed using variations of the Streeter-Phelps (1925) equations as follows: Extensions to the Streeter-Phelps equation given by Camp (1963) and Dobbins (1974) are for a BOD level L,

$$\frac{dL}{dT} = -(k_1 + k_3)L \qquad (1)$$

where the terms k_1 and k_3 represent constants respectively for deoxygenation and sedimentation, and:

$$\frac{dD}{dt} = k_1L - k_2D - A \qquad (2)$$

where k_2 is the oxygen reaeration rate, D is the oxygen deficit, or saturation concentration less actual concentration, and A is the photosynthetic production rate. Integrating (1) for L_t, the concentration at a point corresponding to time t downstream from an initial concentration L_o is:

$$L_t = L_oe^{-(k_1 + k_3)t} \qquad (3)$$

Utilizing equation 3 it is possible to integrate equation 2 for D_t the oxygen deficit at any time t downstream from an initial deficit D_o, thus:

$$D_t = \frac{k_1}{k_2 - (k_1 + k_3)} L_o - \left[e^{-(k_1 + K_3)t} - e^{-k_2 t}\right]$$

$$- \frac{k_1}{k_2} 1 - e^{-k_2 t} + D_o e^{-k_2 t} \qquad (4)$$

Equation 4 thus provides the means for determining transfer coefficients for BOD and dissolved oxygen in the models.

If F_{ji} denotes the flow between upstream activity point j and downstream point i, then stream velocity and depth are functions of flow--$V(F_{ji})$, $H(F_{ji})$. Velocity and depth in turn are determinants of the reaeration.

Using the Thackston and Krenkel (1969) formulation, the reaeration coefficient may be written as:

$$k_2 = \delta \left[1 + \left(\frac{V(F_{ji})}{\sqrt{gH(F_{ji})}}\right)^{1/2}\right] \sqrt{\frac{S}{H(F_{ji})}} \qquad (5)$$

Using these expressions in the river segments and the O'Connor and Dobbins (1956) formulation in impoundments, it is now possible to define a set of transfer functions or coefficients which express the change in oxygen concentration (Q_i) at quality point i in response to a unit load of BOD carbonaceous (L_j=1) introduced into the stream at waste point j (point source)

$$T_{ji}(F_{ji}) = \frac{dQ_i}{dL_j} \qquad (6)$$

The coefficient T_{ji} describes the interrelation between streamflow and waste input for a particular flow. If M_{ji} is the distance between points j and i, then the time of travel is $\frac{M_{ji}}{V(F_{ji})}$, thus:

$$T_{ji}(F_{ji}) = \frac{k_1}{k_2(F_{ji}) - (k_1 + k_3)} \left[e^{-(k_1 + k_3)M_{ji}/V(F_{ji})} - e^{-k_2(F_{ji})M_{ji}/V(F_{ji})}\right] \qquad (7)$$

Since oxygen concentration is also coupled with nitrogenous BOD, a similar transfer coefficient is developed.

Additional transfer coefficients were developed for the other constituents. Nitrogen and phosphorus are treated as total species and therefore are considered as conservative substances. Our ℓ_i, i=1, k, where k denotes the number of pollutants to quality indicator couples, are BOD carbonaceous/DO, BOD nitrogenous/DO, DO/DO, Nitrogen/Nitrogen, and Phosphorus/Phosphorus. DO is not a pollutant, but a transfer coefficient must be developed for this couple since upstream DO influences downstream DO as indicated in equation 4.

The T_{ji} describe the transfer of a pollutant concentration lumped at point j to point i downstream. We have also included in the model provision for pollutant loadings arising by runoff from the area drained by each river section. By assuming a constant increment of flow/length and unit uniform loading along the reach, we can derive a transfer coefficient for the propagation downstream to reach i of a pollutant load linearly distributed along reach j. In contra-distinction to the situation discussed above, the flow in such a reach is not constant, i.e., F_{ji} does not exist; rather, one has an F_j and an F_i such that

$$F_i = F_j + F_j^1 M_{ji} \tag{8}$$

where F is the increment of flow/length in reach j. Then

$$\begin{aligned}\tau_{ji} = {} & \frac{k_1 F_j^1 V(F_i,F_j)}{k_2(k_1 + k_3) F_i} \left[1 - e^{-k_2 M_{ji}/V(F_i,F_j)}\right] \\ & + \frac{k_1 F_j^1 V(F_i,F_j)}{(k_1 + k_3)\left[k_2 - (k_1 + k_3)\right] F_i} \left[e^{-k_2 M_{ji}/V(F_i,F_j)}\right. \\ & \left. -e^{-(k_1 + k_3) M_{ji}/V(F_i,F_j)}\right]\end{aligned} \tag{9}$$

To a good approximation the velocity can be regarded as constant:

$$V(F_i,F_j) \simeq V(F_j) = V(F_{ji}) \tag{10}$$

Clearly, in reaches downstream of j, the pollutant so introduced propagates simply according to the τ_{ji}.

The transfer coefficients then constitute the necessary relationships for descriptive modeling. The downstream quality can be calculated by multiplying the upstream load times the transfer coefficient, and summing over the upstream loads and appropriate couples. For example, if Q_i is the DO at point i, LN_j the nitrogenous BOD load at j, LC_j the carbonaceous BOD load at j, then

$$Q_i = \sum_{j\varepsilon J} LN_j T_{ji\ell_1} + \sum_{j\varepsilon J} LC_j T_{ji\ell_2} + Q_{j*} T_{ji\ell_3} \tag{11}$$

J is the set of pollutant points upstream from i, j* the uppermost point in the system, and ℓ_1 indicates the BOD nitrogenous/DO couple, ℓ_2 the BOD carbonaceous/DO couple, and ℓ_3 the DO/DO couple. The prescriptive model will have constraints of the form $Q_i \leq \overline{Q}_i$ where $\overline{Q}_i$ is the quality target (standards).

In addition to the pollutants and quality indicators mentioned above, the model utilizes an additional parameter, biomass potential that indicates the extent to which substances in the waste stream distort the biological activity of streams beyond natural levels. The biomass potential concept was developed as a part of another study (Meta Systems, Inc., 1973). Certain pollutants (or certain levels and intensities of pollutant loadings) cause irreversible ecosystem disruptions; we do not deal with such regimes. The pollutants in which we are interested are those which have reversible effects at pertinent loadings. These may cause overfeeding and overpopulation, or malnutrition and loss of species diversity; they are responsible for excess productivity which causes only such perturbations to the natural system from which recovery is possible through simple physical and biological activities. For given hydraulic conditions, a measure of the relative extent of excess productivity in a water body is the biomass potential, a parameter that may be estimated from conventional water quality criteria.

$$[\text{Biomass Potential (BP)} = 1.47\ \text{BOD}_5 + 4.57\ \text{TRN} + \gamma P, \text{ where } \gamma P \simeq 30]$$

When this parameter is combined with stream size, dilution ratio, and detention time parameters, a measure of the degree of reversible distortion of an aquatic ecosystem is obtained. Biomass potential is an appropriate water quality criterion to be used in the post-1977 period. At that time, standards will be met and DO criteria will no longer be as relevant. As written above, the biomass potential is expressed in terms of an equivalent estimate of ultimate BOD ($NH_4^+ + 2O_2 + H_2O \rightarrow NO_3^- + 2H_3O^+$; $2 \times MW_{O_2} / 1 \times MW_N = 64/14 = 4.57$) that follows from the oxidation of nitrogen in the reduced stage (as in ammonia NH_3 to nitrate NO_3^-). The basis of the estimate $\gamma \simeq 30$ is discussed in the Meta Systems report (1973). A predecessor to the concept of biomass potential (or equivalent) as a water quality criterion was contained in Standards of the Royal Commission on Sewage Disposal in Great Britain (Fifth and Eighth Reports of the 1898 Commission). The basis of the standards, which were developed from extensive surveys of small streams draining densely populated areas in the United Kingdom, was that the 65°F, 5-day BOD of the receiving water should be less than 4.0 mg/ℓ during the low-flow warm weather season. This was perhaps the first example of a true BOD or biomass standard for streams; similar standards were adopted subsequently in the United States. Most stream standards, however, related to dissolved oxygen concentration; BOD concentration in streams was regarded as significant only with regard to the effect on dissolved oxygen. Dissolved oxygen reserves are directly important only in connection with safety factors pertaining to maintenance of aerobic stream conditions. The dissolved oxygen parameter has little direct correlation with water quality for recreation and aesthetic uses, because stream standards in many regions of the United States today (e.g., New York and the New England states) do not use the oxygen criterion to distinguish among the classes of good quality waters--Classes A, B, and C.

In addition to serving as a useful water quality criterion especially for the post-1977 period, biomass potential proves a useful synthetic pollutant to utilize in costs of pollutant control expressions.

The Prescriptive Model--

Most early water quality management models only utilized the BOD carbonaceous/DO relationship. Many were single period annual cost models (Loucks and

Jacoby, 1972). Cost functions were often expressed as total annual cost as a function of BOD_5 removal efficiency.

Complete treatment plants consist of a series of processes and/or mechanisms acting simultaneously or in series. Although one or more of the processes may often be present primarily for the removal of a particular pollutant (i.e., biological treatment processes for the removal of BOD) treatment plants are in fact "joint product" plants removing BOD, suspended solids, nitrogen, etc., subject to all the difficulties associated with allocating the costs to the joint products. Thus, expressing total costs in terms of a single constituent such as BOD was an artificial mechanism but one that proved useful in models that focused primarily on the BOD_5/DO link.

Programming models which incorporate the nitrogenous BOD in the quality relationships (constraints) while expressing costs in terms of carbonaceous BOD often result in convexity difficulties. The problems stem from the fact that in the range of BOD_C removals for which the marginal costs rise with higher degrees of removal--the same is not true for one of the other "joint products," typically BOD_N.

Planning for capacity expansion problems of models involves nonconvex issues even when dealing only with the BOD_C/DO couple. These follow directly from the existence of economies of scale in treatment plant costs.

In the model, costs are expressed as a function of biomass potential removal and capacity. The specific planning requirements explicit in the P.L. 92-500 allow the use of certain heuristics to examine both degree of treatment and capacity expansion questions in a computationally efficient manner. A convex separation programming algorithm is utilized by allowing the use of readily available computer codes (IBM, 1971).

The model is formulated with two decision periods; one to reflect the decisions necessary to meet the 1977 water quality goals of the P.L. 92-500, the other reflecting the 1983-1985 goals.

The following paragraphs present the cost structure utilized in the model, generalized for the nth point-source polluter. From the discussion above we suggest that cost functions exist in the form shown in Figure 1. For the purposes of discussion, capital cost functions only are shown, and the curves are schematic. Annual operation and maintenance costs are assumed to have the same general shape.

The model requires that, in the neighborhood of a particular design capacity flow, X, $D_1 \leq X \leq D_2$ for a specific waste source the cost function can be approximated in the following form:

$$\text{Capital Costs} = K = \alpha X^{\beta} + f(r) \quad D_1 \leq X \leq D_{i+1} \qquad (12)$$

$$r_1 \leq r$$

where X is the design capacity, and r the design removal efficiency, and where r_1, D_1, and D_{i+1} are the bounds over which equation 12 is valid. Note that in general $f(r)$ is convex, and αX^{β} obviously concave for $X > 0$, $\alpha > 0$, and $0 < \beta < 1$.

The rationale for this approach is to select a particular design load (capacity) X, and approximate the concave function with a linear term. This linearization is schematically depicted in Figure 1.

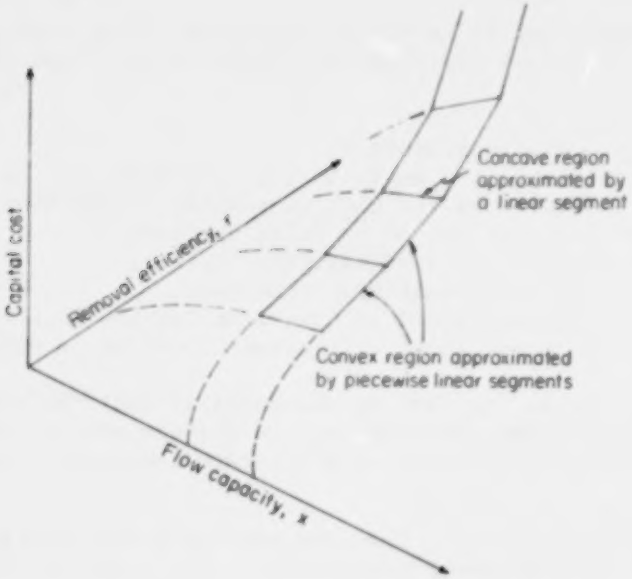

Capital cost
Removal efficiency, r
Flow capacity, x
Concave region approximated by a linear segment
Convex region approximated by piecewise linear segments

Figure 1. Linearization of concave segment of design flow cost function.

As discussed below, the size of the "neighborhood" in which the linearization is valid, and hence the adequacy of the approximation, is a function primarily of the flow growth rates. To consider this method in greater detail, examine a two-period capacity expansion problem. Let D_1 and D_2 represent the design loads for the wastewater flow in periods 1 and 2. They are a function of the flow growth rates for the particular point sources and hence a function of population growth. The smaller the growth rate, the smaller will be the difference between D_1 and D_2, and hence the smaller the neighborhood of approximation.

The decision variables include:
X_1 = total capacity to be built by start of the first time period;
X_2 = incremental capacity to be built by start of the second time period;
r_0 = percent removal in existing facility;
r_1 = percent removal in first period;
r_2 = percent removal in second period.

The parameters include:
D_1 = design flow for first period;

D_2 = design flow for second period;
Δ = $D_2 - D_1$
r_{m1} = minimum percent removal for first period (secondary) (assume r_{m1} is greater than or equal to r_0);
r_{m2} = minimum percent removal for second period (best practicable);
T_1 = length of first period (years);
T_2 = length of second period (years);
($T_1 + T_2$ is the length of the planning horizon).

There are three sets of constraints; namely, those that apply to each treatment site, those that apply to the stream quality, and those that are budgetary.

Treatment Facility Cost Constraints--

The following constraints apply to the definition of the costs of capacity and removal efficiencies for each wastewater treatment plant:

$D_1 \leq X_1 \leq D_2$ (in the cast of no existing plant) (13)
$Q_0 \leq X_1 \leq D_2$ (where Q_0 is an existing capacity) (14)
$0 \leq X_2$ (the non-negativity condition) (15)
$r_1 \geq r_{m1}$ (minimum level of treatment for the first period) (16)
$r_2 \geq r_{m2}$ (minimum level of treatment for the second period) (17)

The explicit assumption made in the model is that the cost functions, for a particular concentration and flow, may be written over a limited range of capacity and removal efficiencies as a separate function of capacity and removal.

A mechanism for expressing the cost functions for each period remains to be developed. If K_1 represents the capital costs prior to the first period, and K_2 the capital costs during the first period, assuming X_0 to be zero, (For purposes of simplicity of presentation, X_0 will be assumed to be zero henceforth. For situations where it is non-zero, simply substitute $X_1 - X_0$ in the applicable cost functions and budget constraints.), then the expenditure prior to the first period must involve three separate terms. The first term is the capital cost associated with a design flow of D_1 and a removal efficiency, r_0. The second term reflects the change in cost resulting from the difference between X_1, at the beginning of period 1, and the design flow D_1, at the beginning of period 1. As explained above, for small differences $X_1 - D_1$, the change in cost can be approximated as a linear function, $a(X_1 - D_1)$. The coefficient "a" is the slope between the capital costs associated with increasing the removal efficiency from r_0 to r_1, assuming that the flow of the treatment is D_1. The removal cost is a convex function of the form depicted in Figure 2. The capital costs to be incurred by the start of the first time period can therefore be written as:

$$k_1(X_1, r_1) = K_1 + a(X_1 - D_1) + f_1(r_1) \qquad (18)$$

where K_1 is defined by equation 12 when $D_i + D_1$ and $X_i = X_0$.

The capital costs incurred during the first period, T_1, depend on whether incremental capacity, X_2, will be constructed during this period, i.e., whether $X_2 = 0$ or $X_2 > 0$. If $X_2 = 0$, then the capital costs of the first period equal only the costs of increasing the removal efficiency from X_1 to X_2,

$$k_2(X_2, r_2) = f_1(r_2) - f_1(r_1), \text{ if } X_2 = 0 \tag{19}$$

as shown in Figure 3. If $X_2 > 0$, the capital cost includes the cost of constructing additional flow capacity, X_2, and increasing the removal efficiency of the total capacity flow, $X_1 + X_2$, to X_2. The cost of additional removal efficiency, i.e., from r_1 at the beginning of period 1 to r_2 during period 1, for a flow capacity of X_1, is defined in equation 19. The unit cost of removing a fraction of the waste, r_2, from an additional flow X_2 can be estimated by a linear approximation, b, of the initial portion of the cost function for capacity X given r_1, $k(X|r_1)$, as illustrated by Figure 4.

$$b = \frac{k(D_2 - D_1|r_1) - k(0\ X_1)}{D_2 - D_1} \tag{20}$$

The removal efficiency costs for X_2 are approximated by the function $f_2(r_2)$ which is based on an X_2 of $D_2 - D_1$. The capital costs may now be written as

$$k_2(X_2, r_2) = bX_2 + f_2(r_2) + f_1(r_2) - f_1(r_1) \text{ if } X_2 \geq 0 \tag{21}$$

This equation is only an approximation of the actual costs associated with X_2 and r_2, based on an assumed D_1 and D_2. If the values obtained for X_2 significantly differ from either 0 or $D_2 - D_1$, then the b and $f_2(r_2)$ should be changed to more accurately estimate the total capital cost before resolving the model.

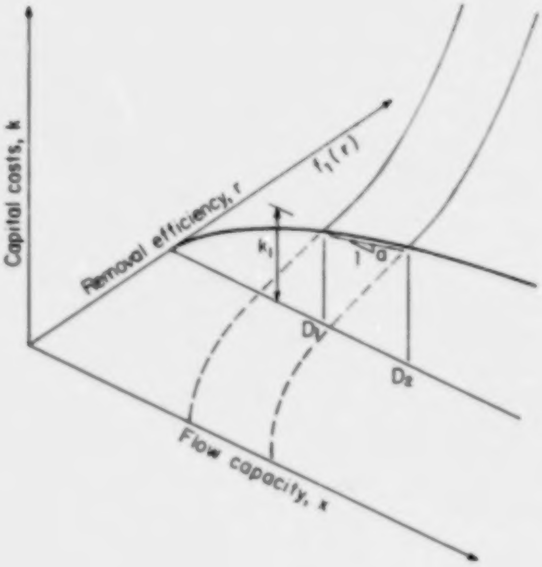


Figure 2. Capital costs incurred by the start of the first time period as a separate function of capacity, X and removal, r.

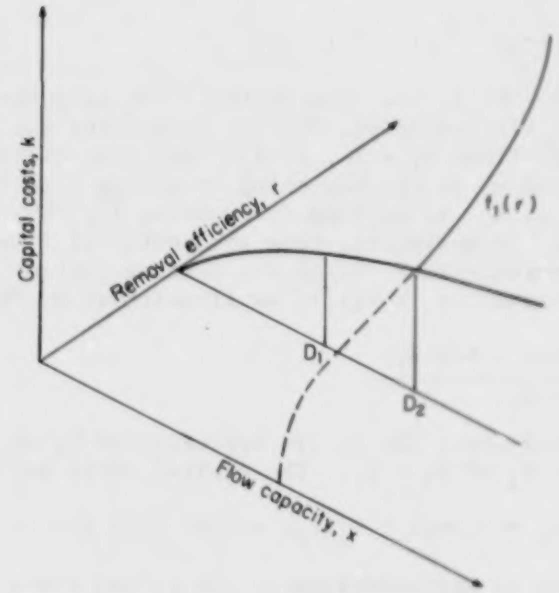

Capital costs, k
Removal efficiency, r
$f_1(r)$
D_1
D_2
Flow capacity, x

Figure 3. Capital costs incurred during the first time period when X = 0.

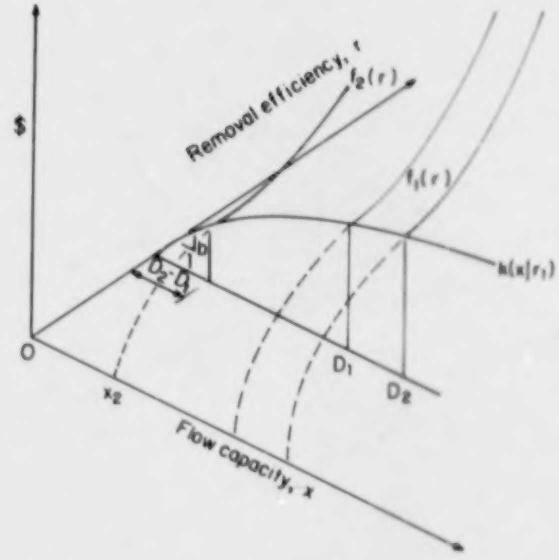

$
Removal efficiency, r
$f_2(r)$
$f_1(r)$
$k(x|r_1)$
b
D_2-D_1
O
x_2
D_1
D_2
Flow capacity, x

Figure 4. Capital costs incurred during the first time period when $X_2 > 0$.

Now there are at least two approaches to handling the expression $k_2(X_2, r_2)$: a) a mixed integer formulation; or b) a heuristic utilizing a "loss function on X_2." Both have been investigated in the Saint John study.

The first approach suggests that I_1 be a dichotomous variable and

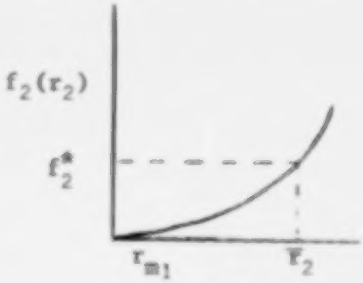
$$0 \leq X_2 \leq \Delta I_1 \qquad (22)$$

then:

$$k_2(X_2, r_2) = bX_2 + I_1 f_2(r_2) + f_1(r_2) - f_1(r_1) \qquad (23)$$

The second approach suggests a linear loss function in X_2 as a replacement for $f_2(r_2)$. It should be noted in general that $f_2(r_2)$ will not be a major term of $k_2(X_2, r_2)$ in the cases under consideration because the capacity expansions during the second time period will be small.

Consider



Let $\overline{r}_2$ be the maximum expected removal for the second period and $f_2^* = f_2(\overline{r}_2) - f(r_{m1})$; then let

$$f_2(r_2) = (X_2 - X_1) \frac{f_2^*}{\Delta} \qquad (24)$$

hence

$$k_2(X_2, r_2) = b(X_2 - X_1) + (X_2 - X_1) \frac{f_2^*}{\Delta} + f_1(r_2) - f_1(r_1 - r_0) \qquad (25)$$

Collecting terms, etc., we can write

$$k_1(X_1, r_1) = a_1 X_1 + f_1(r_1) + \text{constant terms} \qquad (25a)$$

$$k_2(X_2, r_2) = a_2 X_2 - a_3 X_1 + f_1(r_2) - f_1(r_1) + \text{constant terms} \qquad (25b)$$

where a_1, a_2, and a_3 are positive coefficients and f_1 is a convex function. A similar argument leads to the following for operation and maintenance costs:

$$O_1(X_1, r_1) = b_1 X_1 + g_1(r_1) \qquad (26a)$$

$$O_2(X_2, r_2) = b_2 X_2 - b_3 X_3 + g_1(r_2) - g_1(r_1) \qquad (26b)$$

where g_1 is a convex function and b_1, b_2, and b_3 are positive coefficients.

In the objective function both the capital and operating cost functions will be multiplied by discount factors β_1, β_2, and θ_1, θ_2.

The objective function then is

$$\begin{aligned}\min Z = \sum_{j\epsilon N} & \left[\beta_1 \left\{a_{1j} X_{1j} + f_{1j} (r_{1j})\right\}\right. \\ & \left. + \theta_1 \left\{b_{1j} X_{1j} + g_{1j} (r_{1j})\right\}\right] \\ & + \sum_{j\epsilon N} \left[\beta_2 \left\{a_{2j} X_{2j} - a_{3j} X_{3j} + f_{1j} (r_{2j}) - f_{1j} (r_{1j})\right\}\right. \\ & \left. + \theta_2 \left\{b_{2j} X_{2j} - b_{3j} X_{3j} + g_{1j} (r_{2j}) - g_{1j} (r_{2j})\right\}\right] \qquad (27)\end{aligned}$$

where the second subscript j indicates a particular pollution source, of which there are N. The N pollution source points j are a subset of the J activity points defined earlier.

There are four types of constraints:

1. local constraints;
2. removal and load relationships;
3. instream quality constraints;
4. budgetary and equity constraints.

The first of these are equations 13 through 17 associated with each of the cost functions. The second set of constraints relates the removals and loads of the various pollutants to that of biomass potential. To facilitate this, we define the following sets of variables and parameters:

Parameters:

C_{jpt} = the concentration (#/mgd) at waste point j before treatment, of the pth pollution in the tth period. p=1,...Np. p=1 for biomass potential

L_{jpt} = the daily load (#/day)...

= $C_{jpt}D_t$, where D_t is the design flow defined above.

Variables:

r_{jpt} = removal percentage of the pth pollution at the jth waste point in period t

$$r_{jpt} = \lambda_{jpt} r_{j1t} \qquad (28)$$

relates the removal of the other pollutants to that of biomass potential. Within the range of removals considered, the linear relationship is adequate in most cases

$\hat{L}_{jpt}$ = the daily load (#/day) at waste point i after removal of pollutant p after treatment in the tth period

$$\hat{L}_{jpt} = L_{jpt}\lambda_{jlt}r_{jlt} \qquad \lambda_{jlt} = 1 \tag{29}$$

The third class of constraints are instream quality/waste load relationships and the limits (or targets) placed on instream quality. We let Q_{int} be the nth quality measure at the ith stream point in period t. Recalling the structure of equation 11 the instream quality/waste load relationships are of the following form

$$Q_{int} = \sum_{j \in J} \sum_{\ell \in K} T_{ji\ell_k} L_{jp_k t} + Q_{j*} T_{ji\ell} \tag{30}$$

and the quality targets are of the following form

$$Q_{int} \leq \bar{Q}_{int} \qquad t = 1, 2 \qquad i = 1... \tag{31}$$

where the transfer coefficient $T_{ij\ell_k}$ is based on an appropriate low flow regime, usually the 1-year, 7-day low flow.

Here J is the set of pollutant points upstream from i, and j* is the uppermost point in the system. K is the set of pollutant/quality measure couples appropriate and ℓ is the appropriate quality measure/quality measure couple (e.g., BOD nitrogenous/DO couple).

The last set of constraints that may be of interest are budgetary and equity constraints. These constraints may take the form of limitations on capital and/or operating expenditures for a particular polluter or for sets of polluters. Let $K_{tj}(r_{tj})$ and $O_{tj}(r_{tj})$ represent the cost functions of equation 25 and 26 for time period t and pollution source j; then such constraints might be of the form

$$K_{tj}(r_{tj}) \leq \bar{K}_{tj} \tag{32}$$

$$O_{tj}(r_{tj}) \leq \bar{O}_{tj} \qquad \text{for some or all of } j,\ t \tag{33}$$

where the righthand sides of equations 32 and 33 reflect capital and operation and maintenance budget limitations. Possible equity constraints might take the form

$$K_{tj^1}(r_{tj^1}) \leq \alpha_j \sum_{j \neq j^1} K_{tj}(r_{tj}), \ \alpha_j < 1 \tag{34}$$

reflecting the fact that irrespective of marginal costs, one polluter should not bear more than some percentage of the basin costs. There are a number of other equity or political constraints that may be introduced.

In summary, the model selects treatment levels and capacity expansion sizes and timing based on a criterion of minimization of weighted present value of capital and operating costs. The weight can be varied to reflect either federal resource cost criteria or local/regional (after subsidy) costs or other alternative weights.

The selection is constrained by a) minimum treatment and capacity constraints which reflect federal legislation (these are augmented with local constraints to facilitate model solution); b) waste removal and load relationship; c) instream quality constraints which reflect stream standards and/or targets; and d) various budgetary and equity constraints.

Concluding Comments -

The Saint John Basin planning study which sponsored the development of these model(s) is just being completed. Still, some observations can be made regarding the usefulness of the model(s). As has been the case in the past, the models for prescriptive purpose will be of more limited use than the descriptive purpose. There are two reasons for this. First, although knowledge of least-cost solutions of control alternatives of both treatment level and capacity expansion are important in the planning process, the number of polluters in this case is small and an ad hoc analysis with a descriptive model is sufficient to investigate a reasonable range of options. In some measure this was foreseen early in the study and it was the potential for wider application and the availability of EPA sponsorship that led to the prescriptive model development.

Second, there is the need to examine control under a variety of criteria in addition to least cost. For example, these criteria are associated with a concern for such things as: a) cash flow implications for municipalities; b) energy requirements of alternatives; and c) administrative requirements of control alternatives, both during construction phase and continuing operation. All of these are of some concern in the Saint John Basin and the investigation is amenable to use of the descriptive model with some complementary ad hoc prescriptive investigations rather than the prescriptive formulation which is limited in its criteria structure. This issue is briefly discussed in the following pages since the implications for the usefulness of prescriptive models is more general.

The Need to Calibrate/Verify the Descriptive Relationships

This section discusses an indispensable step, namely validation, in water quality modeling. In specifying the functional (or structural) relationship among variables in the previous sections, we have made assumptions, many of which are for convenience, while others are founded on the observational data of similar systems. For example, we have postulated that the Saint John River behaves like the Ohio River and some laboratory systems in that the BOD degradation is proportional to the BOD remaining. We also assume that the rate of degradation in our systems may be different from that of the Ohio River* and is to be determined by the pertinent data. Therefore, it is important to realize that a water quality model is, at best, an approximation of the actual system, and that results obtained are only as reliable as the input information and the accuracy of the operations performed on the data.

Recognizing that a model is only an approximation of the real system, it appears logical to visualize that water quality modeling is merely a curve fitting in a river system. This line of recognition motivates one to apply fruitfully the available curve-fitting techniques in water quality modeling. For instance, statistical theory of estimation and hypothesis testing are all useful in model building. In the following, we present an example to illustrate the application of hypothesis testing. Failure to recognize the quality modeling as curve-fitting has in part contributed in making water quality modeling a field of ambiguity and mystery. Too often the calibration/verification procedure is described as a distinct and creative step in water quality modeling. However, it is merely, in fact, an ad hoc procedure to fill partially the role of estimation and hypothesis testing.

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*The Ohio is the river that the "Streeter-Phelps" formulation first utilized (Camp et al., 1969).

Our example* is a river reach receiving nitrogenous wastes. Travel time between upstream and downstream points is about three days. On the basis of data collected in the summer of 1973, the rate constant of nitrogenous BOD decay is estimated to be 0.04 per day in the reach. In the degradation of nitrogenous BOD, nitrogen is transformed into nitrate. With an intent of proving (or disproving) the value of 0.04 per day, samples were taken during the summer of 1974. Sampling times at up and downstream points were properly spaced to account for the traveling time. Both nitrate concentration and stream flow were measured to compute the nitrate flux at both points. These samples are tabulated as:

Sample Number	Upstream in 1000 lbs.	Downstream in 1000 lbs.	Difference in 1000 lbs.
1	28.3	24.2	-4.10
2	18.0	21.3	3.3
3	13.0	17.7	4.7
4	18.2	20.6	2.4
5	18.1	20.8	2.7
6	17.4	24.5	7.1
7	16.4	24.5	8.1
8	19.6	23.1	3.5
9	23.0	43.9	20.9
10	24.3	56.3	32.0
11	24.3	32.1	7.8
12	44.7	27.2	-17.5
13	55.0	36.9	-18.1
14	26.2	61.0	34.8
15	22.4	59.5	37.1
16	34.0	39.2	5.2
17	33.1	25.0	-8.1
18	23.2	23.9	0.7
19	12.8	19.7	6.9
20	19.2	18.3	-0.9
21	18.8	22.6	3.8
22	20.6	26.2	5.6
23	16.5	29.5	13.0

---

*This example is derived from actual data and analysis undertaken in the provision of technical assistance to EPA Region III under Contract 68-01-2223.

Sample Number	Upstream in 1000 lbs.	Downstream in 1000 lbs.	Difference in 1000 lbs.
24	18.9	23.6	4.7
25	21.1	23.9	2.8
26	21.8	40.8	19.0
27	22.3	57.8	35.5
28	23.6	32.8	9.2
29	46.2	42.0	-4.2
30	57.2	58.2	1.0
31	31.4	68.3	36.9
32	35.5	41.8	6.3
MEAN $\bar{X}$	25.8	34.0	8.2
SD(X)	11.1	14.6	14.0
SD($\bar{X}$)	2.0	2.6	2.5

Simple statistics are also computed in the last lines of the table. Our problem is to prove that the increase in nitrate flux downstream is significant enough to justify a rate constant of 0.04 per day. For this rate constant, an increase of 3.3×10^3 lbs. of nitrate will be observed if an average flux of 25.8×10^3 lbs. is carrying across the upstream boundary. According to the theory of statistical technique, (Miller and Freund, 1965), one can set

$$H_0 \ : \mu_1 - \mu_2 = 0$$

to test against the alternative

$$H_1 \ : \mu_1 - \mu_2 = 3.3$$

Appropriate statistic for tests of this type is

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - \delta}{S\sqrt{N}} = \frac{8.2 - 0}{2.5} = 3.28$$

The stochastic variate, t, is distributed in accordance with a student's frequency with 31 degrees of freedom. The value, 3.28, is within critical value $t_{0.025,31} = 2.02$ of 95% significant level; therefore, H_0 (no significant increase in nitrate downstream) cannot be accepted. While type I error is 5%, type II error of the test can also be calculated as 9.0% (=Pr{1t/> 0.06}). Since 9.0% of type II error is fairly convincing, to accept H_1 (i.e., 0.04 per day, $0.06 - 2.02 - \frac{8.2 - 3.3}{2.5}$) is reasonable.

Admittedly, on many river basins the rigorous estimation and hypothesis testing cannot be carried out due to paucity of data. However, it appears that some less precise and rigorous procedures must be adopted to justify the model before its use. But our point here is that, whatever procedure is used, a clear description of the steps taken, and concurrent adjustment on parameter's values must be provided. In the following we present the procedure used to justify the Saint John Basin model.

Although in the full Saint John study, calibration/verification procedures were undertaken for all of the sub-basins, the discussion below will focus on only a portion of one sub-basin, namely the Grand Falls headpond region. This region is highlighted since it contains interesting hydraulic characteristics and is also the region receiving the largest pollution inputs. A schematic of this segment of the basin is contained in Figure 5.

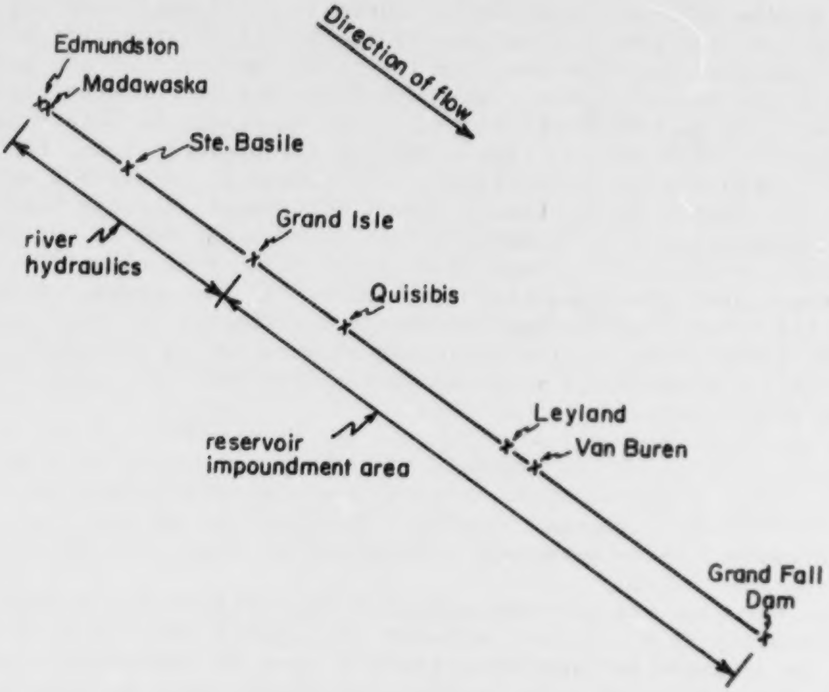

Figure 5. Spatial location of point sources in example application.

Recall from equation 11 that the quality indicator concentration instream at a point i is a complicated summation of products of the upstream loadings and the appropriate transfer coefficients. Within the reliability of sampling theory, the quality indicator level is known from the instream measurements. The problem is then one of examining the system with respect to those unknown values of the loads and transfer coefficients. The large number of unknown parameters (e.g., loadings, instream decay, and sedimentation rates, etc.) necessitate numerous initial assumptions which in turn affect the computed profile. To create a better comparison between measured and computed profiles, the "art" of calibration becomes important in determining which characteristics of the computed profile are incorrectly being models or are in need of incremental adjustment. During the calibration exercise it is essential to maintain an understanding of the physical system being modeled since apparent calibration can be obtained by incorrect adjustment of parameters; however, this would create incorrect forecasts of the water quality levels if the model was subsequently used in a predictive or verification-type mode.

In this example we discuss only the calibration exercises associated with the non-conservative pollutants since their transfer coefficients are more complex.

A series of line schematics identifying the spatial location of sampling stations along the river sections provides a medium of comparison between the measured and computed water quality profiles. On these schematics, the DO-BOD stream sample information collected during a particular sampling is plotted. The ranges of the sample measurements are denoted by the wide vertical bar at each of the sampling stations, the length of the vertical bar indicating the spread of the measurements. The short cross-bar on the vertical bar denotes the mean value of the measurements. It is important to point out that the plotted values for BOD profiles represent the ultimate total BOD values. The need for considerations of ultimate (as opposed to 5-day) BOD values arises because of the extensive time of travel for transit through the river system. The transformation of the instream samples of 5-day BOD is complicated because the ratios of ultimate carbonaceous (BOD_C) and nitrogenous (BOD_N) values to the 5-day values are dependent on the nature of the source. A method for computing the transformation was developed by utilizing the best estimate of the transfer coefficient and the pollution release levels to create a weighted coefficient for the ratio of 5-day to ultimate BOD conditions for the instream-sampled BOD_5 levels.

Manipulations of the sampling results in accordance with the procedures, such as those indicated above, provide the appropriate data to compare with the mathematically computed results. Prior to the mathematical computation phase, however, there were many considerations that received attention, the more important being:

1. Checking the one-dimensionality assumptions--the assumption of one-dimensionality (i.e., cross-sectional uniformity) implicit in the mathematical model is an important approximation that must be critically examined particularly for regions of considerable river width, such as those found in impoundments like the Grand Falls headpond. Experimental evidence of the cross-sectional homogeneity was provided by sampling at intervals across the cross-section. The results identified no strong channeling effects, indicating that one-dimensional assumption is appropriate.
2. Selection of reaeration levels--the reaeration coefficient, k_2, is one of the rate constants incorporated within the mathematical model computation. Since the rate constant cannot be measured accurately in a polluted stream, and is a function of the characteristics of the stream (e.g., velocity, bottom slope, etc.), as a first approximation, the appropriate reaeration levels must be derived from literature equations. Efforts to match sampled and computed profiles were complicated because a considerable instability of the flow regime occurred during the sampling periods. Sensitivity analysis of reaeration coefficient levels was undertaken for both the river regime and the headpond to determine the magnitude of the changes in the velocity and reaeration that probably occurred during the sampling program. This information was useful in suggesting the level of uncertainty and therefore the difference between the experimental and calculated profiles that might be attributed to the selection of the reaeration coefficient level.
3. Since the Grand Falls headpond creates an impoundment for a distance of about 30 miles upstream of the dam, it was important to establish the con-

consequent effects on the pollutant transport processes (e.g., Does the water stratify?). Use of a technique based on a reservoir Froude number (Water Resources Engineers, Inc., 1969) indicated that the reservoir does not stratify. This finding was later supported by experimental evidence. The absence of stratification, however, did not preclude a considerable decrease in velocity in the wide portions of the reservoir and a consequent difficult task of establishing levels for the sedimentation coefficient.

The calibration computations are necessarily an iterative process, each trial refining appropriate parameters and hopefully improving the comparison between measured and calculated water quality profiles (subject to the proviso that no parameter be adjusted without valid reason). A synthesis of the calibration exercises on the Basin is documented in Table I. The information has been tabulated by a) verification run number, b) general comments, c) flow regime, d) point source, e) background contribution, and f) rate coefficients. The second heading describes the general emphasis of the calibration run. The last four columns denote general sub-sectors of the problem and entries in the columns describe the specific refinements made in each run. The successive runs of the calibration generally indicate activity in sectors or columns from left to right (e.g., It is necessary to establish the hydrologic regime prior to refinement of background loading contributions).

Several general comments will clarify elements of the table:

1. The flow regime of the computer program is developed by summing successive contributions along the river system. Therefore, in order to match the hydrologic regime of the sampling program, one must iterate, trying flow additions and then modifying, if necessary, to create a match of hydrology at many points of the system.

2. The literature estimates of background pollutant contributions are necessarily given as a range of values. Successive trials were necessary to establish the general area within the literature range which was appropriate for the Saint John River. (This range factor was particularly important for the conservative pollutants since the principal N and P contributions develop from background sources.)

3. As illustrated in the example results presented in Figure 6, the series of minor adjustments eventually provided good comparisons to the instream measured values upstream of Edmundston. Successive adjustments to parameters for the region downstream of Edmundston, however, failed to create the comparison with the instream measured parameters that was expected. This divergence indicated an important contribution or activity was being omitted. Adjustments to coefficients and loadings produced an improvement in measured and computed profile, but the profiles still demonstrated a wide divergence in behavior. Subsequent bottom sampling indicated a significant benthal activity in the area. This finding represented an important affirmation of the value of the calibration exercise.

The comments included on Figures 6 through 9 indicate, in addition to Table I, the sequence of trials taken prior to a reasonable calibration. Too frequently in discussion of water quality models, it is only the equivalent of Figure 9 and a brief discussion which is presented.

TABLE I. DESCRIPTION OF SEQUENTIAL CALIBRATION EXERCISES

Run	General	Flow regime	Point source	Background contribution	Reaction rates
1	Testing for configuration of data deck, e.g. connectivity of node points	First trial at flow contributions to create hydrologic regime	First trial at loading contributions	First trial at background contributions	First trial at decay rates
	Results--connectivity is okay	Requires adjustment			
2	Evaluating flow contributions	Refinement of contributions			Refinement of travel times in Grand Falls headpond
3	Attention focused on point source contributions	Minor adjustments	Modification of levels to reflect new data, new BOD_5/BOD_u ratios		
	Results--point source contributions much too high to agree with instream measurements	Hydrologic regime in agreement with values from sampling set #1	Reduction in levels needed--checking of assumed conditions		
4	Attention focused on point source contributions		Decrease in point source contributions		
	Results--movement in correct direction but insufficient--background loadings too high		Checking of assumed conditions		

(Continued)

TABLE I. Continued

Run	General	Flow regime	Point source	Background contribution	Reaction rates
5	Attention focused on background levels			Lowering of values in accordance with some literature values	Adjustment of k_1 level for Emundston Fraser's to reflect revised estimates
	Results--concentration levels still too high in computed levels			Contributions too high	
6	Attention focused on background levels throughout river basin			Lowering of background contributions in particular areas	
	Results--large disagreement still prevalent in Grand Falls headpond				
7	Attention focused on Grand Falls headpond	Adjustment of area vs depth relationships to test sensitivity in Grand Falls areas			Adjustment of sedimentation rates in Grand Falls headpond
	Results--agreement better between computed and measured but still significantly different	Computed profiles not very sensitive to adjustments			

(Continued)

TABLE I. Continued

Run	General	Flow regime	Point source	Background contribution	Reaction rates
8	Attention focused on Grand Falls headpond				Reduction in reaeration rate G.F. headpont; upward adjustment of rate
	Results--as in (7)				
9	Attention focused on Grand Falls headpond				Similar to (8) k_2 = .001 k_2 = .8
	Results--same as in (7) and (8)--must be factor causing divergence in addition				
10	Attention focused on Grand Falls headpond			Addition of estimated benthic demand	
	Results--General agreement between computed & measured profiles but further adjustment of benthic & decay rates required				
11	Attention focused on Grand Falls headpond	Incremental adjustments in travel times between points in headpond	Decrease in Edmundston loading		

(Continued)

TABLE I. Continued

Run	General	Flow regime	Point source	Background contribution	Reaction rates
	Results--improvement in agreement but sedimentation and therefore BOD rate of change is too high in headpond				
12	Attention focused on Grand Falls headpond		Decrease in Edmundston loading		Adjustments in k_2 to reflect O'Connor formulation; adjustment of k_3 from .8 to .6 in headpond
	Results--sedimentation rate still too high, magnitude of load must still be too great				
13	Attention focused on Grand Falls headpond		Decrease in Edmundston loading		Adjustments of k_2 to .2 in headpond
	Results--agreement very good between experimental & computed profiles		Edmundston contribution apparently significantly less than originally thought		

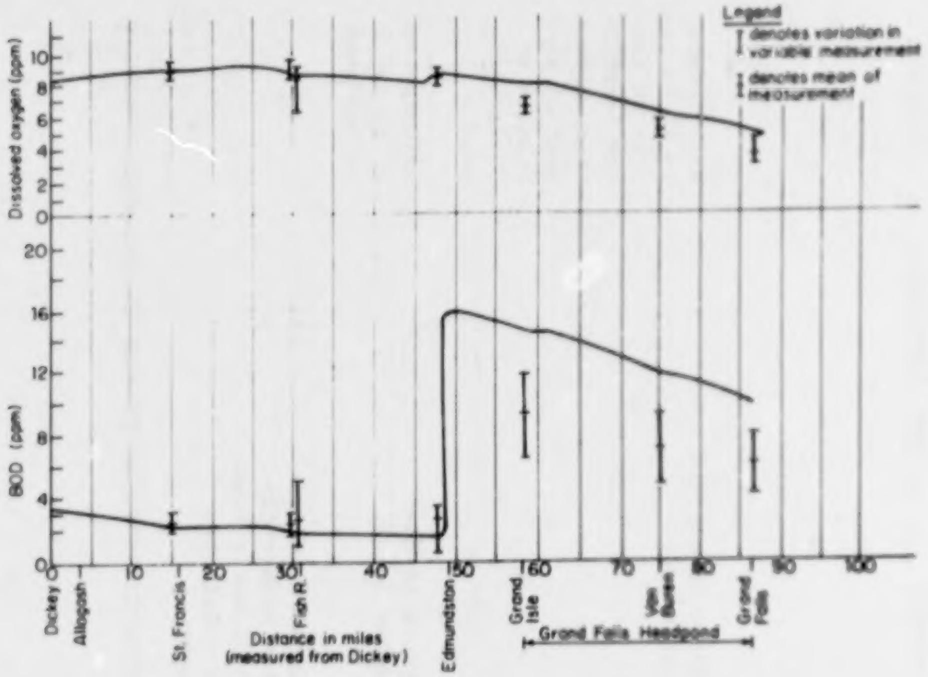

Figure 6. DO and BOD profiles, mainstream St. John River verification runs.

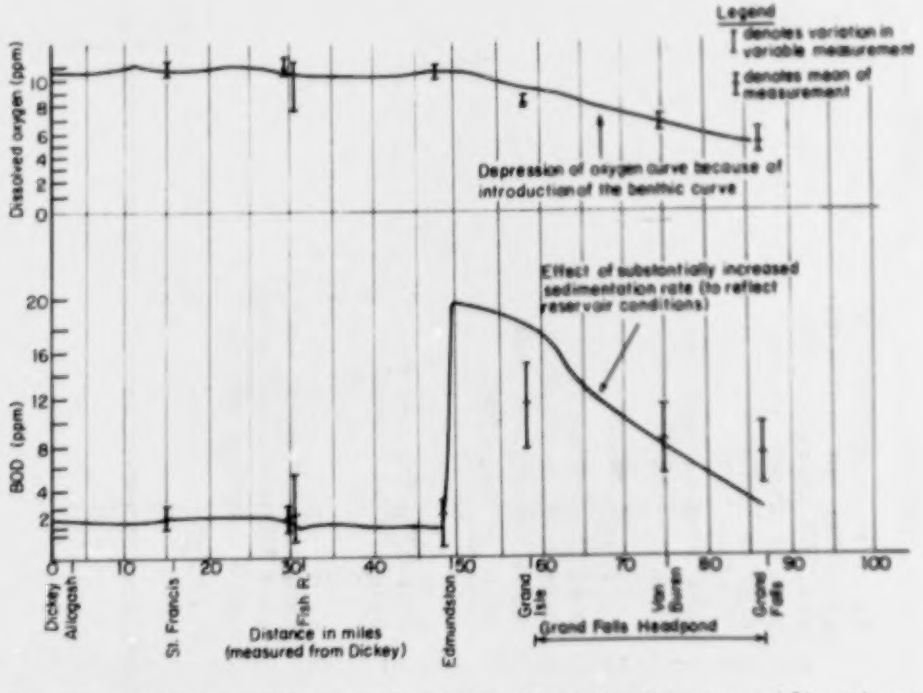

Figure 7. DO and BOD profiles, mainstream St. John River verification runs.

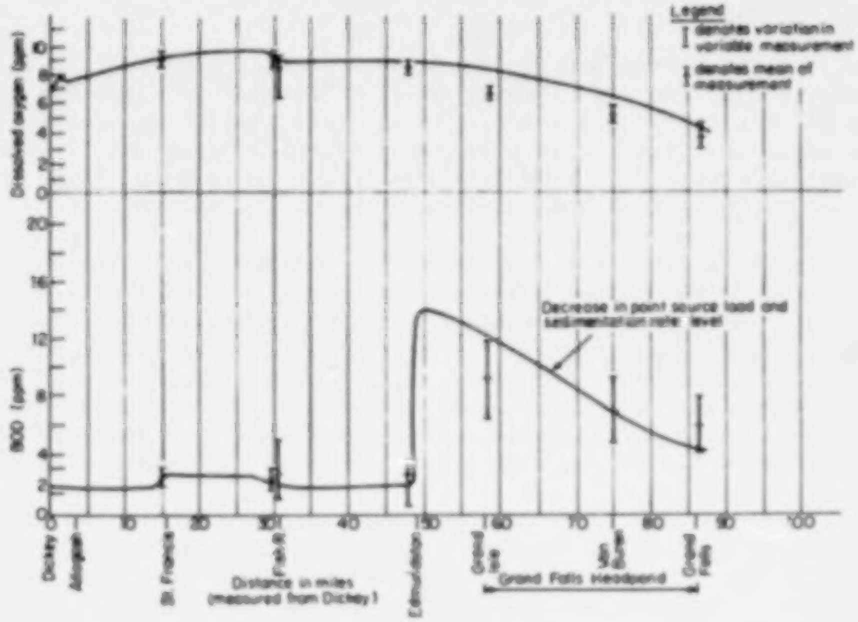

Figure 8. DO and BOD profiles, mainstream St. John River verification runs.

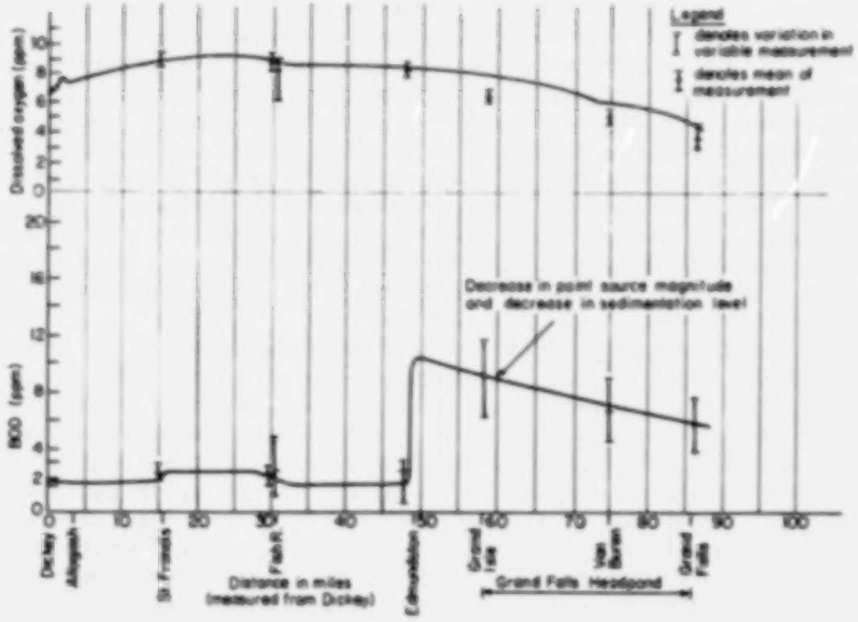

Figure 9. DO and BOD profiles, mainstream St. John River verification runs.

Concluding Comments

We believe that there are two types of application or implementation problems associated with the use of water quality planning models. The first problem is one of acceptance of a descriptive or prescriptive mathematical model for use in an actual water quality planning study. The second is associated with the implementation of a water quality plan that is based at least partially on a study in which a mathematical model has been employed.

Currently there is little impediment to the use of descriptive models in water quality planning in the United States. Model building as a water quality management analysis procedure has been gaining popularity. Advancement in the modeling skill has improved the ability of a model to account for relevant elements of the problem examined. As a consequence, the image of modeling has been enhanced. However, one of the most important stimulants to the application of a model for analysis may be attributed to a shift in the federal legal procedures in dealing with pollution control in the United States.

In a traditional procedure of constitutional governments, an individual awaiting trial in a court is regarded as innocent until he (or she) is proven guilty. The burden of proof that he (or she) is guilty lies with the plaintiff. A similar procedure has been applied in the litigation of pollution control in the U.S. The court (or environmental regulatory agency) is required to assume the burden of proof of damage due to a waste discharge. If the evidence is insufficient to prove damage, then waste producers (private or public) are freed of the charge. Pollution control efforts following this procedure have not experienced a great deal of success in improving environmental quality. Since the burden of proof is on the regulatory agencies, information is valuable to them but not to waste dischargers. However, data collection and analysis efforts assumed by the agencies are inadequate to cope with the problem. Moreover, little cooperation can be expected from waste dischargers, since lack of information acts in their favor. The universal shortage of waste discharge information throughout the U.S. (and an unwillingness to accept results obtained on the basis of a model) may be partially attributable to the reluctance of waste producers to cooperate. The apparent measure to remedy the situation is to reverse the burden of proof; namely, a waste discharger would be required to provide evidence to show his innocence, otherwise any discharge is regarded as harmful to the environment.

The National Environmental Policy Act requires that for any new construction larger than a certain size, an assessment must be carried out to evaluate the environmental impact. The National Pollution Discharge Elimination System (NPDES) specifies more precisely that a waste producer must obtain a permit for waste discharge. Public Law 92-500 also states that any polluter must have a minimum treatment prior to discharge and if the water quality standard is still not met, the level of treatment must be increased to reduce the amount of discharge. In these pieces of legislation, legal procedure appears to shift the burden of proving non-damage to the waste producers. Because of this change, data collection and analysis are essential for polluters. Many large waste producers have built up personnel capable of using some sort of analytical techniques to perform pertinent studies. There is evidence that these staffs are susceptible to a relatively new method--model building--in their continuous efforts to ensure consistent analysis.

The application or use of prescriptive models has not been nearly as widespread. We believe that this results from the inability of the models to produce acceptable control plans to inform those involved in the decision-making process. At fault are the criteria functions employed. The functions are based on cost minimization or benefit maximization. The former criterion is often inadequate because it does not encompass issues associated with distribution or equity. The latter criterion, while suffering from some of these same shortcomings, is of little use in guiding the selection of acceptable plans because water quality benefits are usually not measurable.

On the other hand, there are cases, when the situation is complex and in which it is useful to have the solution alternatives generated by prescriptive models as the basis for comparison, where one finds such models implemented. The Saint John is such a case. In this we sought an implementation of the model but not an implementation of the plan based on the solutions of the prescriptive model. The eventual plan implementation will be based on a number of issues integrated into plan formulation process, only some of which stem from results of the prescriptive model.

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DISCUSSION

UBERMAN: Describe the decision-making process developed from data received from the automatic monitoring system on the Delaware River Basin?

THOMANN: The Delaware River has seven water quality monitoring stations, from the upper river areas to the estuary. The data collected by these stations have been used for at least the past 10 years in making decisions about water quality on that river. Just one example: in 1965, we had a very severe drought and there was a great fear that salt water would penetrate into the estuary and affect the water supply. The mathematical model time variable for salt water in the estuary was used, together with the data from the monitoring stations, to demand releases from upstream reservoirs to control the salt-water intrusion. Each day a two-week forecast of salt-water intrusion was made from monitoring data and the mathematical model.

YEREMENKO: It seems very valuable to me that Dr. DeLucia's report tries to systematize American work on mathematical modeling. This will help us understand the directions in which modeling is headed. In this report, models are divided into descriptive and prescriptive. The descriptive models can be based on two approaches: an enlarged parametric approach and an approach with distributed parameters. The first is characterized by a separation of sections and by an examination of all the processes within the section. This is considered less accurate. However, I would like to hear something about the quantitative comparison of these approaches. I familiarized myself with the existing approaches. An enlarged first approach which was developed by Dr. Thomann, and which is widely used in the U.S., is not evaluated quantitatively in comparison with the second approach. If we use the second approach, then we come up against a whole series of limitations. I am of the opinion that the first approach does not have any limitations. The first model, in the report, was used for the River Tiber, the second for the St. John River. The first model, which is considered a less refined model, is connected with linear dispersion. The second model does not consider linear dispersion. Why is a little-known effect considered in the first less-refined method, and not considered in the second?

KOLOSKOV: Today we heard two very interesting reports by Dr. Yeremenko and Dr. DeLucia. I get the impression that the U.S. and USSR differ in their approach to the development of mathematical models for decision-making. How should further research be carried out? In my view, these directions supplement one another, and we should develop them.

If well-studied rivers are modeled, we possess a sufficient volume of information and it is necessary to use the models which Dr. Yeremenko has talked about. At present, we have rivers for which it is necessary to develop water quality management mathematical models, and the volume of information which we can use and put into the model is very limited. In this case, these approaches which our American colleagues have developed can be used.

We should note that one or another of the models accepted obviously will be determined by strategy and tactics for its use for optimizing water quality management and decision-making and by the time interval which is accepted when solving a given problem. If we have problems with accident situations, and speed and accuracy are required to achieve a decision, then Yeremenko's method allows us to obtain good results. On the other hand, if we create a mathematical model to forecast for a long time period and to develop management systems, then the American system is applicable.

We should note that in our country, methods similar to the American methods are being developed, in particular, in the Moscow River where a model was used to forecast water quality.

DAVIES: The subject of our symposium is "The Use of Mathematical Models to Optimize Water Quality Management," and we are here to discuss the best method for achieving goals. I think that everyone understands that many approaches can be used for a determined set of problems and that often the nature of the problem requires the application of a less complex model. However, there are cases when very complex models are necessary. One of the basic goals of models is to provide the decision-maker with the best available analysis of the problem and the ability to consider alternative strategies. We must be convinced that the models are scientifically credible to have any confidence in their use. We have spoken only very briefly about the verification of a complete model. The easiest and probably the cheapest part of modeling is writing the equations. The usefulness of these equations depends on how closely they are able to replicate real processes. We have heard many papers in which efforts to model river basins are described, but we do not know how well verified these models are. I think that we should turn particular attention to that question.

USACHEV: It seems to me that the problem of water quality management is first of all a monitoring problem. The second problem is joining the scales of the model with the results of the measurements. An actual river is a stratified three-dimensional environment. If we describe the model by means of a one-dimensional equation, we must find a way to average the results of the measurements that will agree with our model. The third problem is accuracy and timing for measurements. I believe that more detailed mathematical research is required on attaching models to specific water bodies.

DeLUCIA: I support Dr. Davies' position on the question of the necessity for strict verification. The evident difference between verification and calibration is absent. In my report, physical phenomena are examined. We should pay attention to analyses of the physical, economic, or demographic aspects of the problem, and likewise to how the decisions will influence people who carry out water management. When discussing the St. John River model, I was not implying that the optimized model can be used only when there is a non-stationary hydraulic system. It can be used with different hydraulic systems.

CONSTRUCTION OF MODELS FOR FORECASTING AND CONTROLLING WATER QUALITY IN A RIVER BASIN ON PRINCIPLES OF SELF-ORGANIZATION

A.G. Ivakhnenko

INTRODUCTION

The purpose of this research is to create mathematical models that adequately describe the dynamics of river basins and are applicable both for forecasting and for deliberately changing and controlling the basins.

Considering the complexity of controlled systems (i.e., river basins), and the incompleteness of information obtainable both with existing monitoring methods and from other factors, self-organization methods are considered the most promising mathematical models for identifying and controlling such systems.

Work in this field from 1971 to 1975 was conducted at the Institute of Cybernetics of the Academy of Sciences, of the Ukraine. Construction of forecasting models for river basins developed for management purposes, was also obtained at the VNIIVO Institute of the USSR Ministry of Land Reclamation and Water Use.

Synthesis of Models of the Principle of Self-Organization

The problem was called direct modeling using experimental data. Previously, similar problems were solved by regression analysis with the method of least squares (r.m.s.). The same type of the regression equation was indicated (its power and number of terms); the result of modeling is subjective. With regression analysis, it is impossible in principle to establish the structure of an equation (model) of optimum complexity, since the rule is: "the more complex the model, the more accurate it is (up to the zero-order error)." This serves as the basis for the theory of model multiplicity (Byr, 1963). In reality, overfitting a model is as bad as oversimplification.

The principle of model self-organization using a computer offers the only model of optimum complexity. Therefore, it is sufficient to indicate, in accordance with Godel's incompleteness theorem (Byr, 1963), a certain external criterion. The self-organization principle is formulated as follows: given gradual complication of the model, the number of criteria (called the selection criteria), with properties of external addition (Byr, 1963), passes through the minimum, determining the model of optimum complexity for each criterion. It is always possible to select such an order of model complexity and sophistication (e.g., increasing the number of terms in the power of the polynomial) at which the minimum will be the only one. The theory of self-organization as a selection criterion mainly uses the criterion of regularity, the value of r.m.s. error as measured at a separate sequence of experimental points:

$$\Delta(1) = \sqrt{\frac{\sum_{K=1}^{Nnp} (\eta_K - \eta_K^*)^2}{\sum_{K=1}^{Nnp} (\eta_K^*)^2}} \rightarrow \min,$$

where K is the point number;
Nnp is the number of points in the test sequence;
η is the actual value of the function in a given point;
$(\eta - \eta^*)$ is the interpolation error.

Recently, several more expedient criteria have been proposed: the criterion of non-displacement, the criterion of the balance variables, the variation criterion, and others (Ivakhnenko, 1973). For the synthesis of a model of optimum complexity and sophistication, a method of sequential testing for models (scanning) with gradual increase in complexity, is used. The value of the selection criterion is calculated for each model, and the model with the smallest value is chosen.

Scanning of models offer a non-parametric method and do not require any data on statistical or other properties of the object (steadiness, non-linearity, non-Markovian, etc.). It is important to note the objective character of the optimum complexity model synthesis. According to the principle of self-organization, the technician needs only to feed the experimental data into the computer and indicate the selection criterion. Then the computer, without human help, can synthetize the model and indicate its authenticity. The depth of the selection criterion minimum (Ivakhnenko, et al., 1967) is a measure of this authenticity. Only when an adequate minimum value is obtained, can it be certain that the problem of model synthesis is solved. If the minimum is not small enough (no more than 0.05), then it becomes necessary to widen the scope of computer scanning, i.e., to charge the composition of arguments, base functions, delay of arguments, etc. All the parameters of group management of data handling (GMDH) calculating algorithms are optimized to obtain the minimum depth of the base criterion selection. In particular, the size and scope of the data base (the size of the table of the initial data) is selected by this rule.

The volume of the tables for gradual complication of the model is determined as follows: The number of table columns is equal to the number of terms of the complete exponential Kolmogorov-Gabor polynomial C_{n+N}^{n} if n=N where n is the power of the polynomial, N is the number of variables. For one, two, three, and four arguments, 2, 5, 21, 70 columns, respectively, are obtained. A number of lines in the last columns are equal to $2^{C_{2n}^{n}}$, i.e., 2^2, 2^5, 2^{21}, 2^{70}, respectively. From this, one may infer that complete model scanning (Table I or II) is possible only if the number of variables is no greater than three or four. The number of variables in real problems reaches one thousand. For problems of such large dimensions, various algorithms to rationalize scanning are used instead of complete model scanning. The goal is to find the optimum complexity model. These algorithms are called the

TABLE I. EXAMPLE OF THE GRADUAL COMPLICATION OF A POLYNOMIAL (FOR SINGLE VARIABLE)

The Terms of the Exponential Kolmogorov-Gabor Power Polynomial				
t	t^2	t^3	t^4	t^5
$Y_1=Q_0+Q_1t$	$Y_2=Q_0+Q_1t^2$	$Y_4=Q_0+Q_1t^3$	$Y_8=Q_0+Q_1t^4$	$Y_{16}=Q_0+Q_1t^3$
	$Y_3=Q_0+Q_1t+Q_2t^2$	$Y_5=Q_0+Q_1t^3+Q_2t$	$Y_9=Q_0+Q_1t^4+Q_2t$	$Y_{17}=Q_0+Q_1t^5+Q_2t$
		$Y_6=Q_0+Q_1t^3+Q_2t^2$	$Y_{10}=Q_0+Q_1t^4+Q_2t^2$	$Y_{18}=Q_0+Q_1t^5+Q_2t^2$
		$Y_7=Q_0+Q_1t^3+Q_2t+Q_3t^2$	$Y_{11}=Q_0+Q_1t^4+Q_2t+Q_3t^2$	$Y_{19}=Q_0+Q_1t^5+Q_2t+Q_3t^2$
			$Y_{12}=Q_0+Q_1t^4+Q_2t^3$	$Y_{20}=Q_0+Q_1t^5+Q_2t^3$
			$Y_{13}=Q_0+Q_1t^4+Q_2t^3+Q_3t$	$Y_{21}=Q_0+Q_1t^5+Q_2t^3+Q_3t$
			$Y_{14}=Q_0+Q_1t^4+Q_2t^3+Q_3t^2$	$Y_{22}=Q_0+Q_1t^5+Q_2t^3+Q_3t^2$
			$Y_{15}=Q_0+Q_1t^4+Q_2t^3+Q_3t+Q_4t^3$	$Y_{23}=Q_0+Q_1t^3+Q_2t^3+Q_4t+Q_3t^2$
				$Y_{24}=Q_0+Q_1t^5+Q_2t^6$
				$Y_{25}=Q_0+Q_1t^5+Q_2t^4+Q_3t$
				$Y_{26}=Q_0+Q_1t^5+Q_2t^4+Q_3t^2$
				$Y_{27}=Q_0+Q_1t^5+Q_2t^4+Q_3t+Q_4t^2$
				$Y_{28}=Q_0+Q_1t^5+Q_2t^4+Q_3t^3$
				$Y_{29}=Q_0+Q_1t^5+Q_2t^4+Q_3t^2+Q_4t$
				$Y_{30}=Q_0+Q_1t^5+Q_2t_4+Q_3t^3+Q_4t^2$
				$Y_{31}=Q_0+Q_1t^5+Q_2t^4+Q_3t^3+Q_4t+Q_5t^2$

TABLE II. EXAMPLE OF THE GRADUAL COMPLICATION OF A POLYNOMIAL (FOR TWO VARIABLES)

The Terms of the Exponential Complete Kolmogorov-Gabor Power Polynomial				
X_1	X_1^2	X_2	X_1X_2	X_2^2
$Y_1=Q_0+Q_1X_1$	$Y_2=Q_0+Q_1X_1+Q_2X_1^2$	$Y_4=Q_0+Q_1X_2$	$Y_8=Q_0+Q_1X_1X_2$	$Y_{16}=Q_0+Q_1X_2^2$
	$Y_3=Q_0+Q_1X_1+Q_2X_1^2$	$Y_5=Q_0+Q_1X_1+Q_2X_2$	$Y_9=Q_0Q_1X_1+Q_2X_1X_2$	$Y_{17}=Q_0+Q_1X_1+Q_2X_2^2$
		$Y_6=Q_0+Q_1X_1^2+Q_2X_2$	$Y_{10}=Q_0+Q_1X_1^2+Q_2X_1X_2^2$	$Y_{18}=Q_0+Q_1X_1^2+Q_2X_2^2$
		$Y_7=Q_0+Q_1X_1+Q_2X_1^2+Q_3X_2$	$Y_{11}=Q_0+Q_1X_1+Q_2X_1^2+Q_3X_1X_2$	$Y_{19}=Q_0+Q_1X_1+Q_2X_1^2+Q_3X_2^2$
			$Y_{12}=Q_0+Q_1X_2+Q_2X_1X_2$	$Y_{20}=Q_0+Q_1X_2+Q_2X_2^2$
			$Y_{13}=Q_0+Q_1X_1+Q_2X_2+Q_3X_1X_2$	$Y_{21}=Q_0+Q_1X_1+Q_2X_2+Q_3X_2^2$
			$Y_{14}=Q_0+Q_1X_1^2+Q_2X_2+Q_3X_1X_2$	$Y_{22}=Q_0+Q_1X_1^2+Q_2X_2+Q_3X_2^2$
			$Y_{15}=Q_0+Q_1X_1+Q_2X_1^2+Q_3X_2+$	$Y_{23}=Q_0+Q_1X_1+Q_2X_1^2+Q_3X_2+Q_4X_2^2$
			$+Q_4X_1X_2$	$Y_{24}=Q_0+Q_1X_1X_2+Q_2X_2^2$
				$Y_{25}=Q_0+Q_1X_1+Q_2X_1X_2+Q_3X_2^2$
				$Y_{26}=Q_0+Q_1X_1^2+Q_2X_1X_2+Q_3X_2^2$
				$Y_{27}=Q_0+Q_1X_1+Q_2X_1^2+Q_3X_1X_2+Q_4X_2^2$
				$Y_{28}=Q_0+Q_1X_2+Q_2X_1X_2+Q_3X_2^2$
				$Y_{29}=Q_0+Q_1X_1+Q_2X_2+Q_3X_1X_2+Q_4X_2^2$
				$Y_{30}=Q_0+Q_1X_1^2+Q_2X_2+Q_3X_1X_2+Q_4X_2^2$
				$Y_{31}=Q_0+Q_1X_1+Q_2X_1^2+Q_3X_2+Q_4X_1X_2$
				$+Q_3X_2^2$

group method of data handling (GMDH). The method of scanning rationalization is of no importance. It depends on the character of the problem and the talent of the researcher. The number of newly proposed GMDH algorithms grows continuously. By now, about a hundred different versions of the GMDH algorithms have been published (see "Automatika" journals). The purpose of all the algorithms is to obtain a result by complete scanning of all models in sequence. Regarding the selection of a set of the model arguments, the GMDH algorithms provide for two methods: algorithms with protection of variables (all arguments introduced by the technician), and GMDH algorithms without protection of the variables. (An excess of possible arguments are given.) The computer selects some arguments to ensure the minimum of the main selection criterion. Examples of successful computer self-organization of optimum complexity models for forecasting and management by the GMDH algorithm without protection of the variables follow.

Mathematical Models for Forecasting River Flow

The self-organization principle of model scanning with gradual increasing complexity, allows determination of river flow forecast models that contain diverse sets of arguments and types of base functions.

Synthetized models can be presented as the sum of the so-called "trend" and "remainder."

$$Q = f_1(t) + f_2(t, X_1, X_2, \ldots, X_m),$$

where $f_1(t)$ is a trend, the sum of the optimum number of harmonics with non-multiple periods;

$f_2(t, X_1, X_2, .. X_m)$ is the remainder, the optimum complexity exponential polynomial, which considers the influence of different environmental factors on the river flow.

In the results of modeling the flows of the Dnieper and the Severski Donets rivers, the value of the remainder is small. The inference is that the river runoff is basically determined by the trend ordinates in the time functions. The GMDH algorithm intended for separation of the harmonics trend at non-multiple frequencies is described.

Insufficient attention has been given in applied mathematics to the harmonics analysis of fluctuating processes with non-multiple harmonic frequencies. The Fourier series expansion pertains to the case of multiple frequencies (Ivakhnenko, 1973). A special criterion, called harmonics ordinates balance criterion, is used as the selection criterion. It consists of a set of linear equations that express a specific determined relation among the neighboring process ordinates. Figure 1 gives an example of frequency determination, when separating one or two harmonics. If frequencies are known, it is easy to determine their amplitudes by the r.m.s. method, and then determine the imbalance of the regression equation (Ivakhnenko et al., 1975). In accordance with the GMDH algorithm, the sum of imbalance in points of the verification test succession becomes more accurate as the number of separated harmonics in the trend separation program increases.

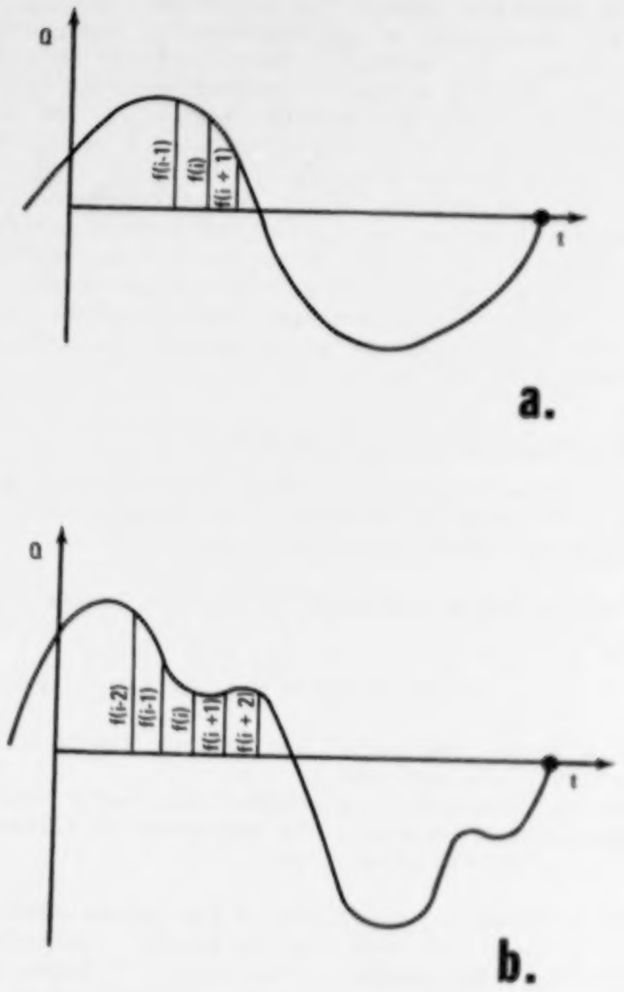
a
f(i-1)
f(i)
f(i + 1)
t
a.
a
f(i-2)
f(i-1)
f(i)
f(i +1)
f(i + 2)
t
b.

Figure 1. Determination of the W and W_1, W_2 harmonics frequencies by the harmonics ordinate balance criteria.

a. Harmonics W

$$\frac{2\cos W = \dfrac{f(i+1) + f(i-1)}{f(i)}}{W=?} \; ;$$

b. Harmonics W_1 and W_2

$$a_1 + a_2\cos W_1 = \cos 2W_1;$$

$$a_1 + a_2\cos W_2 = \cos 2W_2;$$

$$\frac{f(i+2) = a_1[2f(i)]}{W_1=?} + \frac{a_2[f(i-1) + f(i+1) - f(i-2)]}{W_2=?}$$

The simplest algorithm is one with sequential separation of one result in each set (F=1). However, the version of the algorithm in which several (F>1) harmonics are separated for each set is often more accurate. In defining the first set of all given points, possible trends are selected with one, two, three, etc., harmonics. The maximum number of harmonics is $M_{max} \leq N/3$. From these, no single one is selected, but rather a F number of trends of each type that most completely satisfy the ordinate balance criterion requirements. Afterwards, F remainders are calculated. (The remainder is the difference between the process ordinates and each trend of the first set.) In selecting the second set of remaining given points, the Mmax of trends is separated with a number of frequencies from 1 to Mmax, respectively. From the whole set F· Mmax of trends obtained for the second set, F of the best trends of this set is selected by the same criterion. The trends are separated for all points, but selection of the best F trends are carried out by the points of a separate verification test succession. The described procedure is repeated in the following sets. Complexity of the model, depending on the number of sets to be selected, increases until the "imbalance" value of the ordinate balance criterion, measured by means of the test sequence of points, decreases. In the last set, a single solution is selected corresponding to the minimum deviation from the ordinate balance criterion. Thus, selection has optimized both the number of harmonics and the number of separated remainders. The method of problem solving is rational sequential scanning by the ordinate balance criterion with a gradual complication of the model (the number of harmonics to be summed up).

Long-Term Forecast of the Average Annual Flow of the Dnieper River

The harmonic components frequencies may be selected by using the new GMDH algorithm described above, and also the sequential scanning of a discrete set of frequencies by the GMDH algorithm with the regularity criterion (Ivakhnenko et al., 1973). The frequencies are analytically determined by the new GMDH algorithm with the ordinate balance criterion. This criterion requires that a determined ratio between the advanced and delayed ordinates of the process be expressed through the sum of the harmonics with non-multiple frequencies (Figure 1). The forecast calculation time on the BESM-6 computer is reduced from three hours to five minutes; the accuracy of forecasting for the Dnieper River flow is generally not impaired.

The initial data (Ivakhnenko et al., 1973) were used to forecast average annual flow of the Dnieper at the extrapolation interval of T_y=10 years (from 1971 to 1980 inclusive) by the ordinate balance criterion. Data on Dnieper flow for 100 years (from 1871 to 1970 inclusive) were used for creation of the training (95 points) and verifying test successions. In the second verifying sequence, a part of the forecasting interval, data for 1971, 1972, 1973, were used.

The results of the forecast and the best forecast obtained by Ivakhnenko et al. (1973) are given in Table III. The error in the examination sequence for forecasting using the proposed GMDH algorithm with F=1: $\delta(2)$=20.2%, with F=5: $\delta(2)$=11.0%. The error in the sequential scanning algorithm of the discrete values of frequencies, proposed by Ivakhnenko et al. (1973), is equal to $\delta(2)$=17.2%. The new algorithm is actually almost as accurate as the previous one, but calculation time is much shorter. In Table IV, the parameters of the harmonic trend of the optimum complexity, which produced the smallest value of $\Delta(2)$ in the examination sequence, are included.

TABLE III. FORECASTS OF THE DNIEPER RIVER FLOW AT KIEV BASED ON DATA FOR 1871 to 1970

(Cubic meters per second)

Year	Forecast of Harmonic Trend			Actual flow	Spread of forecasts by GMDH
	Regularity criterion	Criterion of the balance of variables freedom of selection F=1	F=5		
1971	1635	1560	1600	1830	1460-1900
1972	1061	1266	1095	1030	1050-1400
1973	909	795	1128	1060	800-1050
1974	925	1018	1156	1300	8003-1300
1975	1218	1326	1344	--	1050-1350
1976	1328	1284	1480	--	1150-1550
1977	1145	1219	1132	--	1000-1300
1978	1197	1480	1000	--	1000-1300
1979	1413	1787	1586	--	1300-1600
1980	1500	1641	1435	--	1300-1700

Forecasting the Average Monthly Values of River Flows--

Studies of the river flow data showed that the flow of water can be determined for each month with sufficient accuracy by regression equations:

$$Q_i = f_i(Q_{year}, Q_{i-1}) \quad i=1,2,3,\ldots,12$$

where Q_i is the average monthly water flow for the given month;
Q_{year} is the value (or forecast) of the annual flow; and
Q_{i-1} is the mean flow for the previous month.

Properties of the river flows are such that the regression equation of $Q_i = f_i(Q_{year})$, gives a certain accuracy only for April during the annual flooding period. For other months, sufficient accuracy of regression is not obtained unless the annual rainfall is indicated (low, medium, high water) or if the regression contains both Q_{year} and Q_{i-1}. This is clearly shown in Figures 2a, 2b, and 2c.

The regression equation using only $Q_i=f(Q_{year})$ for the Dnieper River shows too large a dispersion. The scatter of points, given as an example in Figure 2a, cannot be expressed with sufficient accuracy by a line. Only April provides an exception; here the points have comparatively small dispersion around the line of regression (Figure 2b).

Introduction of one more axis, Q_{i-1}, is sufficient to show that all the points are sufficiently close to a certain surface (Figure 2c), corresponding to the equation with two indices: $Q_i = f(Q_{year}, Q_{i-1})$.

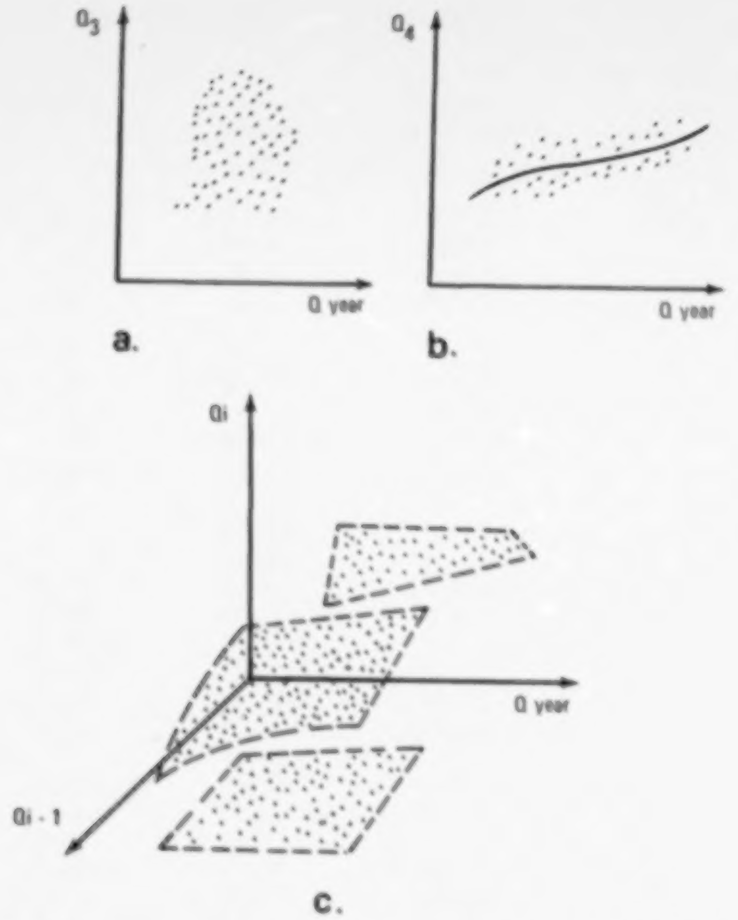

Figure 2. Reduction in the dispersion of points at the transition from the regression equation Q_i=f(year) to the equation Q_i=f(Qyear Q_{i-1}):
a. Scatter of points for all months, except April;
b. Regression curve for April;
c. Regression surface for all months including April.

Figure 2a may be considered a projection of the points of Figure 2c on to the Q_{i-1}-Qyear plane. These figures define the accuracy of the equations: the regression surface in Figure 2c is at a small angle to the axis of Qyear, so that even a considerable error in the forecast of average annual flow shows little influence on the results of the average monthly flow forecast. A close approximation to the average annual flow may be obtained. For example, it is sufficient to point out the type of rainfall projected for the year, high, average, or low, to considerably reduce the dispersion of the points from the regression surface.

TABLE IV. OPTIMAL HARMONIC TREND STRUCTURE

Set #	Quality of harmonic component	Frequency in Radians	Periods (years)	Coefficient	
				A	B
1	2	3	4	5	6
1	4	0.434	14.48	26.9	2.8
		1.084	5.80	34.3	2.7
		1.886	3.33	-72.6	-11.7
		2.716	2.31	6.6	48.5
2	5	0.351	17.90	5.8	-79.3
		0.877	7.16	0.8	45.4
		0.480	4.25	-48.2	-46.9
		2.124	2.96	6.3	46.1
		2.752	2.28	-93.7	6.9
3	8	0.219	28.69	95.7	-10.4
		0.621	10.12	-72.9	-8.4
		0.907	6.93	35.9	5.5
		1.293	4.86	-90.1	45.6
		1.747	3.60	-100.1	3.0
		2.149	2.92	26.9	1.2
		2.596	2.43	50.3	-69.5
		2.922	2.15	-7.8	-19.0
4	7	0.263	23.45	-51.4	-9.6
		0.574	9.32	-132.4	45.1
		1.147	5.48	92.8	-40.4
		1.507	4.17	53.3	31.7
		2.011	3.13	-3.0	22.9
		2.477	2.54	-1.7	-22.6
		2.880	2.18	-13.2	46.0

It is possible to select the f_i function of optimum complexity by complete scanning of all possible regression equations since the number of arguments is small (two). Complete scanning is also used in the so-called combination GMDH algorithm (Ivakhnenko et al., 1973). One equation of the optimum complexity is selected for each month.

Forecasting Average Monthly Flow in the Dnieper River--

Forecasts of the average annual flow in the Dnieper have been taken from the work of Ivakhnenko et al. (1973). Average monthly flow values were based on data from 1878 through 1967 inclusive. Ninety experimental points were divided into the training and test succession. The points were arranged by the value of dispersion. The points with lesser dispersion (40 points) were included into the test succession, and the points with larger dispersion (50 points) in the training sequence (the third method of regularization). The values of the regression equations coefficients were determined initially only in the training sequence. The complexity of the equations increased until the error measures in the test sequence (regularity criterion) were reduced. After determining the equations of optimum structure (separately for each month), their coefficients were recalculated for all the points ("coefficient adaptation stage").

By scanning all the possible polynomial models with two arguments up to the third power inclusively, we were able, by the regularity criterion, to select twelve polynomials of optimum complexity of the following type:

$$Q_i = Q_{0,i} + a_{1,i}Q_{year} + a_{2,i}Q_{i-1} + a_{3,i}Q^2_{year} + a_{4,i}Q_{year}\cdot Q_{i-1} + a_{5,i}Q^2_{i-1},$$

$i=1,2,3,\ldots,12.$

The values of coefficients obtained are given in Table V. Results for forecasting are given in Table VI and in Figure 3. The error of the forecast in the examination sequence (Table VII) is equal to $\Delta(2)=25\%$. This points to the considerable accuracy. The error does not increase with the increase of prediction time; the 1977 forecast is as accurate as that for 1974. A separate study (carried out by Stepashko) showed the stability of the forecast from calculation errors; the 1974 error occurs only in the results closest to 1975 and has almost no influence upon the accuracy of further forecasts.

TABLE V. VALUE OF COEFFICIENTS FOR AN EQUATION FORECASTING AVERAGE MONTHLY FLOW IN THE DNIEPER RIVER

i	$a_{0,i}$	$a_{1,i}$	$a_{2,i}$	$a_{3,i}$	$a_{4,i}$	$a_{5,i}$
1	43.03	0.186	0.966	0	0.000288	0
2	633.10	-0.298	0	0	0.000526	0
3	-16.80	0	1.312	0	0.000284	0
4	-1236.00	4.253	0	-0.000348	0	0
5	-799.80	3.945	-0.212	0	0	0
6	692.90	0	0.0668	0	0.000103	0
7	275.80	0	0.308	0.0000897	0	-0.00000262
8	21.49	0	0.943	0	0	-0.000112
9	492.90	-0.599	0.717	0.000227	0	0
10	525.60	-0.260	0	0	0.00054	0
11	137.20	0	1.017	0	0	0
12	935.50	-0.570	0	0	0.000719	-0.000339

Mathematical Models for Forecasting the Quality of River Water

It is not necessary to discuss the importance of forecasting reliable river water quality indices. Predictions of the content of chlorides and dissolved oxygen in the water of the Severski Donets River were compared by methods based on regressive analysis, on Lagrange interpolation polynomials, and group method of data handling (GMDH).

The self-organization principle is most fully utilized by the GMDH. One algorithm was described by Chukin (1973). It is designed to forecast nonstationary cycle processes. The initial information is provided by the time series, from which the following matrix is formed:

$$Y_{i,j}(i=\overline{1,l};\ j=\overline{1,n}),$$

where l is the length of a separated cycle; and
n is the number of cycles in the training sequence.

The following polynomial is accepted as the approximating function:

$$Y_i = a_0 + a_1 Y_{i-1} + a_2 Y_F + a_3 Y_{i-1} Y_F + a_4 Y^2{}_{i-1} + a_5 Y^2{}_F + \ldots$$

The algorithm allows the power of the polynomial to be changed by accepting the assignment for N.

N=10-complete cubic polynomial;
N=8-polynomial without "particular" cubes;
N=6-full square polynomial;
N=3-linear polynomial.

The polynomial coefficient are determined by the r.m.s. method. The selection of coefficients is determined by the GMDH.

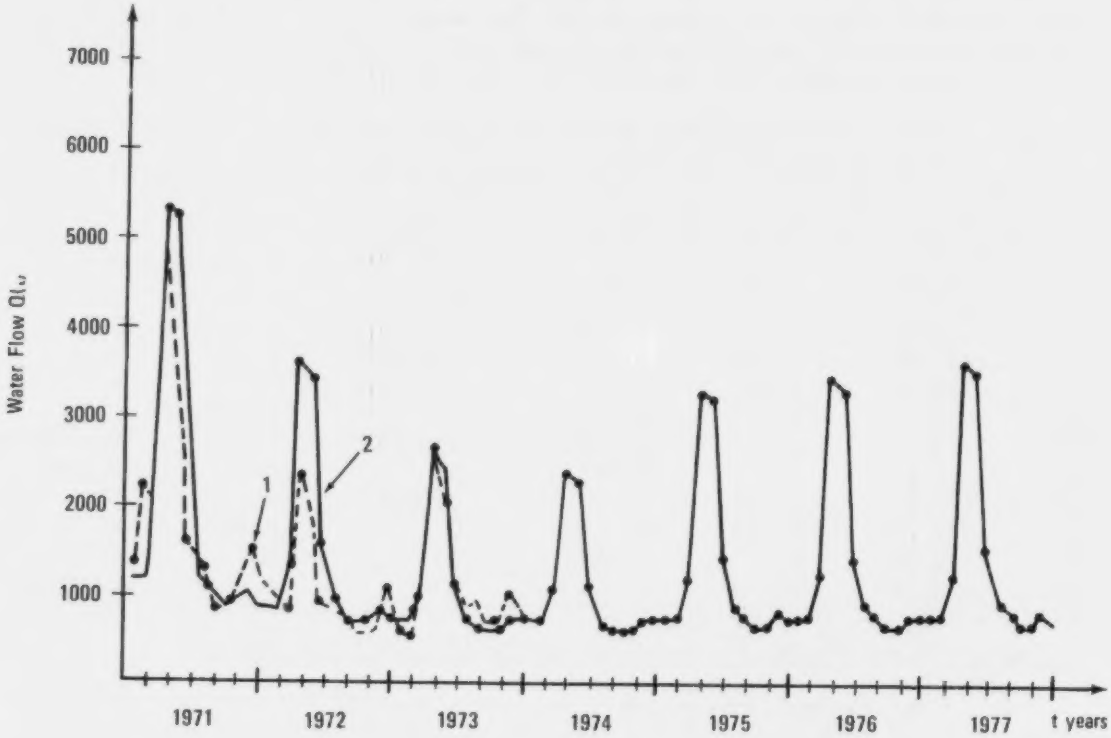

Figure 3. Average monthly Dnieper River flow forecast graph.
1. is the actual flow; 2. is the forecast.

An ALGOL-60 language program was compiled for this calculation and the values of $l=6$, $N=3$ were selected. The experimental data (analyzed every 4 hours) served as the basis for selecting the length of the cycle to be separated. Accuracy was maintained by the use of linear regression, and selection (by the computer) of an optimum Lagrangian polynomial of the first power that served as the basis for selecting the linear polynomial.

TABLE VI. FORECAST OF DNIEPER FLOW, M^3/Second

Year	Average Annual Flow	Average Monthly Flow											
		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
1971	1814	1129	1168	2119	5332	5229	2016	1182	979	856	892	1045	693
1972	1398	806	809	1368	4035	3870	1507	911	787	664	663	812	730
1973	930	726	711	1104	2418	2358	1075	682	612	571	571	718	710
1974	909	713	703	1087	2342	2291	1060	674	606	570	569	716	711
1975	1180	708	721	1171	3227	3158	1286	793	698	603	603	751	708
1976	1217	706	722	1180	3424	3278	1321	812	713	612	611	759	710
1977	1283	705	727	1202	3647	3491	1386	846	739	628	627	775	715

TABLE VII. RESULTS OF THE FORECAST OF AVERAGE MONTHLY FLOW IN THE DNIEPER IN THE EXAMINATION SERIES (a); GIVEN ACTUAL VALUES OF ANNUAL FLOW (b); AND VERIFICATION OF FORECAST STABILITY (c), M ^{3}/Second.

Year	Average annual flow	Average monthly flow												r.m.s. for the Year
		I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1968a	1200	850	616	830	407	2870	1070	628	610	561	814	797	706	--
b	1200	705	720	1174	3366	3223	1305	803	706	608	607	755	709	0.173
c	2000	691	764	1419	5876	5848	2284	1325	1074	974	1057	1213	1039	0.705
1969a	1340	772	674	666	2910	3760	1660	1320	882	767	694	828	1220	--
b	1340	704	729	1219	3837	3675	1444	877	762	645	643	792	721	0.237
c	3000	707	855	1835	8387	9263	4164	2322	1606	1893	2812	2997	2641	1.590
1970a	2490	1290	1290	1360	10000	5970	2310	1160	908	784	943	1670	1630	--
b	2490	686	789	1578	7193	7503	3111	1766	1337	1369	1719	1885	1684	0.317
c	4000	295	610	132	10200	12820	6814	8691	1973	3149	6286	6532	2969	0.909
1971a	1830	1380	2230	1950	4900	3150	1530	1340	951	801	931	1290	1470	--
b	1830	1123	1168	2124	5380	5282	2038	1194	987	866	905	1058	903	0.348
c	1830	1687	1711	3119	5380	5282	2038	1194	987	866	005	1058	903	0.361
1972a	1030	1200	806	772	2310	1950	900	798	684	556	568	647	1110	--
b	1030	840	781	1237	2775	2677	1155	724	645	579	580	727	707	0.288
c	1030	840	781	1237	2775	2677	1155	724	645	579	580	727	707	0.288
1973a	1060	600	522	1031	2624	1979	1159	852	895	668	667	968	859	--
b	1060	708	712	1131	2880	2773	1180	731	655	583	584	731	707	0.226
c	1060	708	712	1131	2880	2773	1180	737	655	583	584	731	707	0.226

Comparative analysis of the results (Table VIII) indicates that the methods of the regression analysis, the modified method of the Lagrangian interpolation polynomials, and the GMDH can be used for water quality forecasting. The GMDH method gives the most accurate results and should be considered as the most practicable method for water quality forecasting.

TABLE VIII. FORECAST OF THE CONCENTRATION OF CHLORIDES AND DISSOLVED OXYGEN (mg/ℓ)

Experimental data		Method									
		Linear regression		Quadratic regression		Lagrangian polynomials^{-1}		Lagrangian polynomials^{-2}		GMDH	
Cl^-	O_2	Cl^-	O_2	Cl^-	O_2	Cl^-	O_2	Cl^-	O_2	Cl^-	O_2
482	7.20	453	7.40	459	7.40	454	6.50	469	6.80	474	7.40
489	7.90	453	7.40	458	7.40	454	6.20	462	7.50	473	8.00
482	8.00	452	7.40	457	7.30	433	5.60	474	7.80	471	7.80
468	7.30	452	7.30	456	7.30	425	5.70	418	6.30	470	7.00
468	6.90	451	7.30	455	7.30	418	6.50	420	7.60	480	7.10
440	6.80	450	7.30	454	7.30	411	7.10	411	7.10	465	6.80
Average quadratic error of forecast		25	0.42	24	0.45	38	1.40	35	0.49	14	0.19
Relative error, %		5.3	5.60	5.1	6.00	8.1	18.60	7.4	6.70	3.0	2.50

In "Possibilities of Applying Perceptions to the Problems of Evaluating Conditions of Water Objects" (VNIIVO, Kharkov), Belogurov and Uberman proposed that the Kolmogorov formula be used in a continuous network of non-linear transformers. The Kolmogorov formula presents the continuous function of several variables by super-position of the continuous functions of one variable and the equation:

$$Q = f(\alpha_1\alpha_2\ldots\alpha_n) = \sum_{i=1}^{2n+1} q_i[\sum_{j=1}^{n} X_{i,j}(X_j)]$$

This provides a mechanism for accepting individual water quality measurements from a monitor. Output from 30-35 water quality sensors in a monitor can be fed into this non-linear transformer to determine Q. Belogurov and Uberman associated this idea with the theory of perceptrons and called the proposed non-linear functional transformer an analog continuous perceptron (ACP). Since accuracy of the ACP has not been studied, and no experimental verifications have been made, the concept is still preliminary.

Methods of Managing the Discharge of Power Plant Reservoirs

A plan to regulate power plant discharge must consider several objectives:

1. Power to provide the planned electrical energy;
2. Water transport to maintain the navigation channel depth;
3. Irrigation to maintain water for irrigation; and
4. Fish farming to maintain conditions for aqua-culture.

This task is a multiobjective management project. For brevity, an example is cited: the Dnieper reservoir cascade managed only by two criteria: a) power and b) fish farming. The problem is solved with the equations for water body dynamics and the equations of the loss functions. Application of the principle of self-organization is most effective for the synthesis of these equations on the basis of a small set of observations, especially when synthesizing the loss functions.

Synthesis of Equations for the Discharge in Dnieper Reservoirs--

The dynamics of the Dnieper discharge of water reservoirs may be described by the following system of difference equations. (In this system, the water flow lag time is not taken into consideration, since the time intervals are considerably larger than water flow time in the reservoirs. Therefore, the Kanev reservoir dynamics are not considered.)

$$\begin{cases} Z_t^{(1)} = f(Z_{t-1}^{(1)}, V_t^{(1)}, V_{\Sigma t}^{(1)}); \\ Z_t^{(2)} = f(Z_{t-1}^{(2)}, V_t^{(1)}, V_t^{(2)}, V_{\Sigma t}^{(2)}); \\ Z_t^{(3)} = f(Z_{t-1}^{(3)}, V_t^{(2)}, V_t^{(3)}, V_{\Sigma t}^{(3)}); \\ Z_t^{(4)} = f(Z_{t-1}^{(4)}, V_t^{(3)}, V_t^{(4)}, V_{\Sigma t}^{(4)}); \\ Z_t^{(5)} = f(Z_{t-1}^{(5)}, V_t^{(4)}, V_t^{(5)}, V_{\Sigma t}^{(5)}), \end{cases}$$

where Z_t^i, $i=\overline{1,5}$ is the water level at the dams at the end of the t-time interval respectively for the Kiev, Kremenchug, Dnieprodzerzhinsk, Dnieper, and Kakhovka water reservoirs (state variables);

V_t^i, $i=\overline{1,5}$ is the volume of water passed through the hydraulic units of the Dnieper power stations at the t-time interval (variables of control);

$V_{\Sigma t}^i$, $i=\overline{1,5}$ is the total volume of water flowing into the Dnieper water reservoirs at the t-time interval (variables of perturbations).

Water balance equation:

$$V_{\Sigma t}^i = V_{1t}^i + V_{2t}^i + V_{3t}^i + V_{4t}^i + V_{5t}^i + V_{6t}^i + V_{7t}^i + V_{8t}^i, \quad i=\overline{1,5},$$

where V_{1t}^i, $i=\overline{1,5}$ is the basic inflow into the i-th reservoir during the t-time interval;

V_{2t}^i, $i=\overline{1,5}$ is the base flow into the i-th reservoir during the t-time interval;

V_{3t}^i, $i=\overline{1,5}$ is the volume of effluents discharged into the i-th reservoir during the t-time interval;

V_{4t}^i, $i=\overline{1,5}$ is the volume of atmospheric precipitation into the i-th reservoir during the t-time interval;

V_{5t}^i, $i=\overline{1,5}$ is the volume of water evaporated from the i-th reservoir during the t-time interval;

V_{6t}^i, $i=\overline{1,5}$ is the volume of water for municipal needs taken from the i-th reservoir during the t-time interval;

V_{7t}^i, $i=\overline{1,5}$ is the volume of water discharge in the i-th reservoir during the t-time interval;

V^i_{8t}, $i=\overline{1,5}$ is the percolation losses in the i-th reservoir during the t-time interval.

Water volumes $V^i_{1t},\ldots,V^i_{8t}$, $i=\overline{1,5}$ are determined from an additional forecast for the given number of control intervals $t=\overline{1,N}$.

The problem of identifying the equation lies in determining the f function. These functions can be found either by analyzing the cause and effect links (determinate method) or by the GMDH algorithms (self-organization method) for a given series of observation points. In the latter, the complexity of f functions generally increases, lowering the regularity criterion.

Identification of the Equations of Dynamics of the Dnieper Reservoirs--

Equations of the Dnieper reservoirs dynamics were identified on the BESM-6 computer on the basis of statistical data (Table IX) from the hydrologic annual report, according to the program given by Ivakhnenko (1971), and are as follows:

A. Kiev reservoir

$$Z^1_t = 2.71 \cdot (0.76 \cdot Y_1 + Y_2 - 3.1 \cdot Y_1 \cdot Y_2) + 2.71;$$

$$\begin{aligned} Y_1 = {} & 0.09 - 0.24 \cdot (V^1_t - 2.54)/2.54 + 0.42 \cdot (V^1_{\Sigma t} - 2.68)/2.68 - 0.12 \cdot \\ & \cdot [(V^1_t - 2.54)/2.54] \cdot (V^1_{\Sigma t} - 2.68)/2.68; \end{aligned}$$

$$\begin{aligned} Y_2 = {} & 0.03 + 0.96 \cdot (Z^1_{t-1} - 2.7)/2.7 + 0.15 \cdot (V^1_t - 2.54)/2.54 + 0.19 \cdot \\ & \cdot [(Z^1_{t-1} - 2.7)/2.7] \cdot (V^1_t - 2.54)/2.54 \end{aligned}$$

B. Kremenchug reservoir

$$Z^2_t = 4.22(1.34 \cdot Y_1 - 0.35 \cdot Y_2 - 0.02 \cdot Y_1 \cdot Y_2) + 4.22;$$

$$Y_1 = -0.0173 + 0.7544 \cdot X_2 + 0.2902 \cdot X_3 + 0.1168 \cdot X_2 \cdot X_3;$$

$$Y_2 = -0.0231 - 0.0302 \cdot X_1 + 1.0682 \cdot X_2 + 0.2486 \cdot X_1 \cdot X_2;$$

$$\begin{aligned} X_1 = {} & -0.37 - 1.05(V^2_t - 3.46)/3.46 - 0.08(V^2_{\Sigma t} - 0.98)/0.98 + 1.37 \cdot \\ & \cdot [(V^2_t - 3.46)/3.46] \cdot (V^2_{\Sigma t} - 0.98)/0.98; \end{aligned}$$

$$\begin{aligned} X_2 = {} & 1.15(Z^2_{t-1} - 4.22)/4.22 + 0.35 \cdot (V^1_t - 2.5)/2.5 - 0.12 \cdot (Z^2_{t-1} - 4.22)/ \\ & 4.22 \cdot (V^1_t - 2.5)/2.5; \end{aligned}$$

$$\begin{aligned} X_3 = {} & -0.09 + 1.32 \cdot (Z^2_{t-1} - 4.22)/4.22 + 0.26 \cdot [(V^2_{\Sigma t} - 0.98/0.98] - 0.3 \cdot \\ & \cdot [(Z^2_{t-1} - 4.22)/4.22] \cdot (V^2_{\Sigma t} - 0.98)/0.98 \end{aligned}$$

C. Dnieprodzerzhinsk reservoir

$$Z^3_t = 3.77 \cdot (0.02 + 0.74 \cdot Y_1 - 1.29 \cdot Y_2 + 20.22 \cdot Y_1 \cdot Y_2) + 3.77;$$

$$\begin{aligned} Y_1 = {} & 0.06(V^2_t - 3.45)/3.45 - 0.02(V^3_{\Sigma t} - 0.17)/0.17 + 0.06 \cdot [(V^2_t - 3.45)/3.45] \\ & \cdot (V^3_{\Sigma t} - 0.17)/0.17; \end{aligned}$$

$$Y_2 = 0.01 + 0.25 \cdot (Z^3_{t-1} - 3.78)/3.78 + 0.04 \cdot (V^3_t - 3.57)/3.57 - 0.8 \cdot \cdot [(Z^3_{t-1} - 3.78)/3.78] \cdot (V^3_t - 3.57)/3.57$$

D. Dnieper reservoir

$$Z^4_t = 7.42 \cdot (-0.003 + Y_1 + 0.53 \cdot Y_2 + 25.66 \cdot Y_1 \cdot Y_2) + 7.42;$$

$$Y_1 = 0.01 \cdot (V^4_t - 3.7)/3.7 - 0.002 \cdot (V^4_t - 0.03)/0.03 + 0.001 \cdot [(V^4_t - 3.7)/3.7] \cdot (V^4_t - 0.03)/0.03;$$

$$Y_2 = -0.007 + 0.387 \cdot (Z^4_{t-1} - 7.41)/7.41 - 0.01 \cdot (V^3_t - 3.59)/3.59 - 1.84 \cdot \cdot [(Z^4_{t-1} - 7.41)/7.41] \cdot (V^3_t - 3.59)/3.59$$

E. Kakhovka reservoir

$$Z^5_t = 3.63 \cdot (0.017 + 0.39 \cdot Y_1 + 0.55 \cdot Y_2 - 2.17 \cdot Y_1 \cdot Y_2) + 3.63;$$

$$Y_1 = -0.3 \cdot (V^5_t - 3.23)/3.23 + 0.01 \cdot (V^5_t + 0.42)/(-0.42) + 0.31 \cdot [(V^5_t - 3.23)/3.23] \cdot (V^5_t + 0.42)/(-0.42);$$

$$Y_2 = -0.05 + 1.62 \cdot (Z^5_{t-1} - 3.62)/3.62 + 0.16 \cdot (V^4_t - 3.71)/3.71 - 2.22 \cdot [(Z^5_{t-1} - 3.62)/3.62] \cdot (V^4_t - 3.71)/3.71$$

TABLE IX. INITIAL DATA FOR IDENTIFYING THE DYNAMICS EQUATIONS FOR THE DNIEPER RESERVOIR*

Variable	Month											
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
$Z^{(5)}_t$,m	3.17	2.76	2.26	3.47	4.00	3.90	3.70	3.76	3.84	4.09	3.88	3.95
$Z^{(5)}_{t-1}$,m	3.53	3.17	2.76	2.26	3.47	4.00	3.90	3.70	3.76	3.84	4.09	3.88
$V^{(4)}_t$,km^3	3.44	3.97	4.67	5.87	5.90	2.99	1.91	1.83	2.34	3.09	3.16	3.58
$V^{(5)}_t$,km^3	4.08	4.57	5.11	2.88	4.09	2.18	1.51	1.29	1.72	2.32	3.39	3.48
$V^{(5)}_{\Sigma t}$,km^3	3.21	4.49	2.80	2.52	1.82	1.60	3.90	5.33	-0.11	-0.13	-0.36	-0.29

*For the sake of brevity, only data on the Kakhovka reservoir for 1968 are presented. Analogous data were used for all other reservoirs. Levels are presented with respect to the elevation of 12 m.b.c.

Synthesis of Loss Functions--

Reservoir dynamics equations may also be obtained by determinate reasoning. To obtain the loss functions, however, self-organization appears to be the only method possible. The synthesis of two loss functions,(for power generation and for fish farming), is given here as an example of a two-criteria problem of power station discharge management.

The power generation by the Dnieper discharge is considered the first control criterion. Electric power generation depends on water levels and volumes of discharged water as follows:

$$E = \sum_{t=1}^{N} \sum_{i=1}^{5} E_t^i (Z_{t-1}^i, Z_t^i, V_t^i),$$

where E_t^i is the consumption of electric energy of the i-th hydroelectric power station during the t-time interval; and

N is the number of control intervals.

The forecast of fish catch in the T+1 year at the Kremenchug and Kakhovka water reservoirs is based on the mathematical dependence of fish catch for the Kremenchug and Kakhovka reservoirs on the water level regimes of these reservoirs (Ovchinnikov, 1972; Pirozhnikov, 1969):

$$Y_{T+1} = \sum_j V_{T+1}^j (Z_{IIIT}^j, Z_{VIIIT}^j, Z_{VIIIT-1}^j, Z_{VIIIT-2}^j, Y_{T-1}^j),$$

where Y_{T+1}^j, j=2,5 is the forecast fish catch in tons in the year T+1 in the j-th reservoir;

Y_{T-1}^j, j=2,5 is the fish catch in the T-1 year in the j-th reservoir;

Z_{IIIT}^j, j=2,5 is the level of j-th reservoir decrease in March of the current year T, characterizing the winter decrease in the reservoir;

Z_{VIIIT}^j, $Z_{VIIIT-1}^j$, $Z_{VIIIT-2}^j$, j=2,5 is the level of j-th reservoir decrease in August of the present and previous year, two years, characterizing the summer decrease in the reservoir.

The self-organization GMDH algorithms allow us to determine the function type and evaluate the coefficients in the criteria equations.

Synthesis of the Energy Loss Function for the Dnieper Hydropower Stations--

Synthesis of the control criteria for each separate Dnieper hydroelectric power station included in the total power criterion was carried out on the BESM-6 computer on the basis of the statistical data presented by the United Monitoring Office of the South, according to Todua et al. (1973). The following criterion dependencies were obtained. The variables E_t^i $(i=\overline{1,5})$ are measured in relative units of electric energy generation, Z_t^i $(i=\overline{1,5})$ in meters, V_t^i $(i=\overline{1,5})$ in kilometers.)

$$E_t^1 = (60.7 + 18 \cdot Z_{t-1}^1 \cdot Z_t^1 - 20Z_{t-1}^1 Z_t^1 V_t^1 - 88V_t^1 + 25.3Z_{t-1}^1 V_t^1 + 28.4V_t^1) \cdot 10^{-4};$$

$$E_t^2 = (30 - 2Z_{t-1}^{(2)} + 38Z_t^{(2)} - 5.1Z_{t-1}^2 Z_t^2 + 20V_t^2) \cdot 10^{-4};$$

$$E_t^3 = (129.3 - 38Z_t^3 - 15V_t^3 - 1.3(V_t^{(3)})^2 + 1.8Z_{t-1}^{(3)} V_t^{(3)} + 12.2Z_t^3 V_t^3) \cdot 10^{-4};$$

$$E_t^4 = [-44.6 + 60Z_{t-1}^4 - 8.35Z_t^4 + 100.5V_t^{(4)} - 1.5(V_t^{(4)})^2] \cdot 10^{-4};$$

$$E_t^5 = [66.5 + 1.9Z_{t-1}^{(5)} - 13.4Z_t^{(5)} + 11.4V_t^{(5)} + 4.4Z_t^5 V_t^{(5)}] \cdot 10^{-4}$$

Synthesis of Fish Farming Loss Functions for the Dnieper Water Reservoirs--

Synthesis of the forecasting criterion dependencies of fish catch for the Kremenchug and Kakhovka water reservoirs were carried out on the BESM-6 computer on the basis of the statistical data (Table X) presented by the Piscicultural Scientific Research Institute of the Ministry of Fish Husbandry, according to Todua et al. (1973).

$$Y^{(2)}_{T+1} = 22402 - 152.7Z^{(2)}_{VIIIT-1} - 2369Z^{(2)}_{VIIIT} - 436.8Z^{(2)}_{IIIT} + 920Z^{(2)}_{VIIIT-1}Z^{(2)}_{IIIT} - 1.5Y_{T-1};$$

$$Y^{(5)}_{T+1} = 15816 - 1325Z^{(5)}_{VIIIT-1} + 378.6Z^{(5)}_{VIIIT} - 114.7Z^{(5)}_{VIIIT-1}Z^{(5)}_{VIIIT} + 2816Z^{(5)}_{IIIT} - 1.25Y^{(5)}_{T-1}$$

Although only a small number of observations are presented in Table X, the synthesis could be performed by using the GMDH loss function.

TABLE X. INITIAL DATA FOR THE SYNTHESIS OF LOSS FUNCTIONS IN FISH FARMING

Variables	Year						
	1967	1968	1969	1970	1971	1972	1973
			Kakhovka Reservoir				
$V^{(5)}_{T}$, ton	8440	9395	8814	7501	7367	7002	9332
$Z^{(5)}_{III,T}$, m	3.83	3.76	2.76	3.41	3.46	3.02	3.62
$Z^{(5)}_{VIII,T}$, m	3.62	3.63	3.72	3.76	3.95	3.84	4.40
$Z^{(5)}_{VIII,T-1}$, m	2.68	3.62	3.63	3.72	3.76	3.95	3.84
$Z^{(5)}_{VIII,T-2}$, m	3.42	2.68	3.62	3.63	3.72	3.76	3.95
			Kremenchug Reservoir				
$Y^{(2)}_{T}$, ton	7000	6454	7619	7591	8671	7778	9399
$Z^{(2)}_{III,T}$, m	5.79	1.47	0.87	0.87	0.96	3.06	2.31
$Z^{(2)}_{VIII,T}$, m	4.98	5.38	5.54	5.82	5.84	5.64	--
$Z^{(2)}_{VIII,T-1}$, m	6.12	4.98	5.38	5.54	5.82	5.84	5.64
$Z^{(2)}_{VIII,T-2}$, m	4.84	6.12	4.98	5.38	5.54	5.82	5.84

Synthesis of the Multicriteria Optimum Management of Water Reservoir Discharge--

The principle of self-organization allowed us to find the power generation and fish farming loss functions on the basis of a small series of observations, which may be written in the following generalized form:

$$\sum_{i}\sum_{t} E^{i}_{t} = E(Z^{i}_{t-1}\ Z^{i}_{t}\ V^{i}_{t}),\ i=\overline{1,5},\ t=\overline{1,12},$$

$$\sum_{j} Y^{j} = Y(Z^{j}_{VIIIT-1}\ Z^{j}_{VIIIT}\ Y^{j}_{T-1}),\ j=\overline{2,5}.$$

In each point of sixty-dimensional space (five parameters and twenty time intervals), it is possible to calculate electric energy generation and fish catch. In the indicated hyperspace, there are points where, within the limits, electrical power generation is maximum (point $Q_E$max, Figure 4), as well as a point which corresponds to a maximum fish catch (point $Q_y$max, Figure 4). In accordance with the existing theory of multicriteria management, this problem of two-criteria management should be solved on the $Q_E$max - $Q_y$max line of tangency for isoclines of two extreme hills, i.e., the so-called Pareto line (space).

The idea of vector optimization consists of an option by the expert of any point on this line (from the maximum energy to the maximum fish catch).

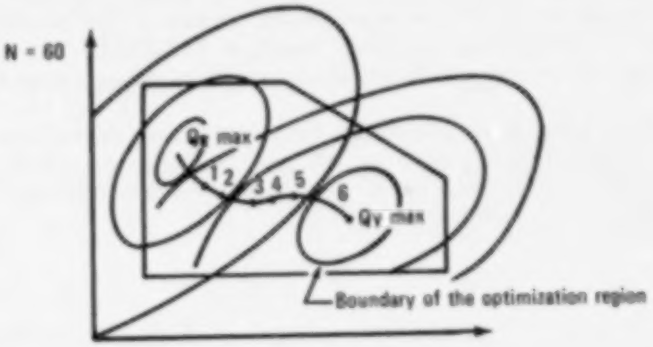

Figure 4. Conditional description of 60-dimensional hyperspace for two-criteria optimization:
$Q_E$max is the point of maximum generation of electric power;
$Q_y$max is the point of maximum fish catch;
$Q_E$max - $Q_y$max - Pareto line, on which the compromise optimum solutions should be searched;
1,2,3,4,5, and 6 - points of vector optimization (see Table IV);

Synthesis of Two-Criteria Optimum Control of the Dnieper Water Reservoirs Discharge--

The dynamics equations of the Dnieper water reservoirs, the energy loss and the fish farming loss functions, were synthesized. The values of the total inflows are indicated in Table XI. At certain limitations in the optimization space (Ivakhnenko and Ovchinnikov, 1975), six of its points on the Pareto line were obtained:

	1	2	3	4	5	6
E_T, Relative Units Electro Energy Generated	0.77	0.76	0.75	0.73	0.72	0.70
$Y_{T+1,\overline{m}}$	15431	15792	16354	17812	18340	18942

The above mentioned Pareto points were determined by the Ovchinnikov (1974) method. The vector optimization consists of selecting any one of these points.

Synthesis of Discharge Management with Forecast Optimization--

The above example illustrates how certain optimum management for a current year is obtained. It will be more reasonable to direct optimum management toward obtaining the long-term optimum, rather than the short-term optimum situation. The goal of optimization is continuously displaced forward for the given preceding interval. The difference between the synthesis problem of managing the discharge from hydroelectrical power reservoirs by two criteria given a projected management period, as opposed to a solution given a fixed period, is that the table of effective solutions (in Pareto sense) must be recalculated at each new management cycle. The management period, i.e., the number of preceding cycles, remains constant, but the object, the first value of an effective problem solution management sequence, is realized.

The investigations have shown an extraordinary stability of management with forecast optimization: at a sufficiently long projecting interval, management becomes stable even when given an unstable object [the regime of "artificial stability" (Ivakhnenko, 1971)].

CONCLUSION

Complete scanning of models with their gradual complication, or rationalized scanning according to the GMDH algorithm based on the principle of self-organization, allow us to objectively solve important problems of forecasting, identifying, and managing water ecosystems.

Examples have been given of the methods application, including forecasting of river flow (with accuracy around ± 20%). This figure indicates the relationship of determinative (80%) and stochastic (20%) components of river flow. Synthesis of the dynamics equations of the hydroelectrical power station discharge water reservoirs and the loss functions (loss for energy generation and fish farming according to short tables of initial observation data) have been shown.

The equations obtained (mathematical models) were used for synthesis of two-criteria management of the hydroelectric power station discharge in accordance with the ideas of multicriteria optimization.

TABLE XI. VALUES OF TOTAL INFLOWS (PERTURBATION VARIABLES)

Total Inflow km^3	Month											
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
$V_{\Sigma t}^{(1)}$	1.96	1.41	1.05	1.08	1.88	1.75	1.40	1.17	0.81	0.95	8.15	6.80
$V_{\Sigma t}^{(2)}$	0.48	0.41	0.25	0.85	0.58	0.70	0.55	0.43	0.50	0.82	2.32	3.44
$V_{\Sigma t}^{(3)}$	0.05	0.02	0.01	0.01	0.20	0.17	0.12	0.21	0.07	0.12	0.70	0.28
$V_{\Sigma t}^{(4)}$	-0.06	-0.06	-0.07	-0.07	-0.01	-0.01	0.01	0.02	-0.03	0.01	0.01	0.01
$V_{\Sigma t}^{(5)}$	-0.80	-0.87	-0.63	-0.40	-0.16	-0.11	-0.02	-0.12	-0.44	-0.49	-0.32	-0.58

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DISCUSSION

OSTROMOGILSKIY: Have you compared harmonic analysis with spectral analysis?

IVAKHNENKO: Analytical determination of frequencies was conducted by a colleague from the Institute of Cybernetics of the Ukrainian Academy of Sciencies, Vysotskiy. Interestingly, it establishes the physical frequencies in the water body not revealed by rapid furie transformation. Thus, the furie transformation, by not revealing frequencies, gives only an approximation.

GLUTSYUK: How accurate are your predictions of yearly flow with the GMDH method?

IVAKHNENKO: We believe that we can project 2 to 3 times the time span of available data. Using data for a period of 100 years, we can obtain data for 200 to 300 years ahead.

GLUTSYUK: Is it possible to unify the process of forecasting yearly flow for various rivers?

IVAKHNENKO: The algorithm is universal. We are trying to heighten the accuracy of forecasting, taking into consideration the discharge and flow rate of a series of rivers, for example, the discharge of the Nieman for forecasting the Dnieper, and vice versa. This reflects, to a certain degree, Professor Kalinin's ideas. He affirms that one river repeats the discharge of another river with a displacement of 2 to 3 years.

GLUTSYUK: Have you compared the data from your analysis and then produced a proposition using your equation? Have you compared the forecast with your data? Have you compared the results of the forecast with individual points?

IVAKHNENKO: For river flow, data for a large number of past years, e.g., for the Dnieper since 1912, is available for several rivers. We can verify the accuracy of forecasting on the data for that past time period. Still one more verification method exists "to change the direction of the time flow" to calculate the path of the process in a reverse direction. This method is used when a small number of points are present.

OSTROMOGILSKIY: Is it possible with the help of the given method to forecast anomalous climatic phenomena and their consequences, for example their influence on river flow?

IVAKHNENKO: The algorithm of monthly river flow was used by one of the Kiev organizations for forecasting soil humidity. This algorithm was verified.

Droughts for example are random occurrences, and do not enter into the forecast. Average forecast values for a decade are performed with a 10 to 15% accuracy.

OSTROMOGILSKIY: In order to construct your system for forecasting, you need information on a river. How do you take into consideration morphometric river changes such as thresholds disappearing on the Dnieper and construction of dams and water reservoir cascades? What, in this case, is the value of long-term forecasts for 100 to 200 years?

IVAKHNENKO: Several scientists confirm that the Dnieper flow is the same now as it was 1,000 years ago. We forecast average yearly flow. Each year, in the month of February, the water reservoir is emptied in order to collect the fresh spring water. The Ukraine gets in an average spring 50 km^3 of water from the atmosphere. The basic function of water reservoirs is to hold this water. It is necessary to increase the volume of the reservoirs. The yearly cycle takes place exactly on time. The yearly flow does not depend on the presence of water reservoirs. An average monthly discharge of flow is made for the upper Dnieper where there are no reservoirs.

GEOGRAPHIC-ECONOMIC ASPECTS OF POLLUTION CONTROL SYSTEMS

Craig ZumBrunnen

INTRODUCTION

The purpose of this paper is to examine four questions involved with water pollution. The first question deals with the geographic-economic distribution or equity problems arising from either water pollution generating activities or water pollution control measures. Essentially this short initial section attempts to apply benefit-cost concepts within a spatial context. The second section examines the issue as to whether the mere territorial size or jurisdictional scale of a pollution control system enhances or impedes progress in achieving its abatement objectives. Regardless of the specifics, any pollution control system must have a geographical component. In other words, what are the inherent advantages and disadvantages associated with pollution control regions at various geographical scales? This paper will investigate the utility and disutility of the various scales of two types of water quality control regions, namely, political-administrative jurisdictions and natural regions such as river and lake basins. The third part briefly evaluates four broad strategies for water resource management: a) moral suasion, b) bargaining, c) government regulation, and d) economic inducements. Both of the last two water pollution control strategies are predicated upon governmental legislative action, but the latter is presumed to involve more decentralized decision making. Finally, current U.S. pollution control policies are reviewed.

Rather than focusing upon technical approaches to water pollution control, this paper deliberately addresses itself to some of the legal, territorial, and economic strategies of water pollution control systems because the author believes that the specific institutional arrangements play a primary role in determining the successfulness of any pollution control measures regardless of the technical components of the scheme. For instance, even a technical process or apparatus which cheaply and effectively intercepts a given pollutant or renders it innocuous will prove to be an ineffective safeguard of water quality if the legal, organizational, and/or economic components of the scheme do not provide sufficient incentives or inducements to ensure that the technical apparatus is installed, operated, and maintained efficiently and properly.

Although the paper refers primarily to the institutional context of American, Canadian, or West European water quality management, most of its arguments are applicable as well to Soviet water quality management despite certain differences in the institutional environments.

Finally, two assumptions are implicit in all of the following discussions. First, the true price or value of pure water, while certainly not infinite, may be greater than both the U.S. and the USSR have assumed thus far. Second, and following from the above assumption, the objective of any pollution control system should be to minimize the total costs of residual disposal (i.e., the sum of total damage and treatment costs) which is achieved by equating the marginal costs (marginal costs are equivalent to the first derivative of a total cost curve) of abatement and the marginal costs of pollution damages.

While the costs of treatment can be ascertained because of the costs of resources utilized in such abatement measures, a price tag on pollution damages is exceedingly difficult to determine. In both the U.S. and the USSR, technical as well as political approaches are used to arrive at surrogate damage costs. The former refers to the use of knowledge about the biological and physical effects of pollutants on the environment to arrive at acceptable or minimum water quality standards (Freeman et al., 1973). The latter refers to the political process whereby the costs of achieving a certain minimum water quality are judged to be either too high or acceptable (Freeman, et al., 1973).

In terms of economic efficiency in a systems context, the key constraints for ensuring a least-cost solution require: a) the equating of marginal abatement costs among all pollution control options and b) among all effluent dischargers. The economic and legal institutions of the United States essentially preclude such an optional solution as undoubtedly such measures would necessitate changes in the competitive advantage of privately-owned, competing enterprises. Although technical change and innovations do alter the competitive advantages of such firms continually, voluntary actions favoring such systems optimization seem remote and such solutions promoted by local, state, or the federal government seem very difficult to implement as is suggested later in the paper. In this respect, the Severskiy Donets project undertaken by VNIIVO appears to possess a solid theoretical foundation.

Spatial Impact of Water Pollution

The question immediately posed is whether it is possible for generalizations to be made concerning the spatial distribution of positive and negative externalities that ensue from activities generating water pollution. A second question arises regarding the spatial distribution of benefits and costs accruing from pollution control efforts.

Positive externalities refer generally to direct or indirect pecuniary benefits that individuals or firms derive from any given pollution-creating activity, that is to say, direct returns in the sense of profits and wages and indirect in the sense of lower real resource costs or production costs. Negative externalities refer to such items as increased expenditures for municipal water treatment, reduced amenity values, and bioecological disruption. Although these first two types of losses can be measured or estimated in monetary terms, it is almost impossible to assess monetarily the deleterious bioecological impacts of water pollution

The benefits of pollution control are manifested chiefly as improved recreational opportunities and amenities (Freeman, 1973), as well as reduced outlays for purification of urban water supplies. The costs of pollution abatement can be grouped into two broad categories, namely real resource costs and factor income costs. The former pertain to labor, land, and capital that must be devoted by various means to curtailing the quantity (and harmful quality) of pollutants otherwise discharged, whereas the latter represent alterations in labor costs and in capital incomes resulting from pollution control measures (Freeman, et al., 1973). Certainly there are inverse relationships between positive pollution externalities and control costs, and between negative pollution externalities and control benefits. The point at issue is thus to determine whether or not these aspects of pollution and pollution control generate a coherent geographical pattern.

Dales (1972) has argued that American literature on pollution exhibits a very strong "uni-directional" bias. He has claimed that, because most Americans live on or near river systems (as opposed to lakes), it appears to be fairly easy to identify geographically who pollutes whom. (The bathroom graffito in Pittsburgh admonishes us to "flush the toilet because Cincinnati needs the water," whereas Cincinnati claims Memphis needs it, and in Memphis, that New Orleans does.)

Lake pollution, on the other hand, often displays no strong directional bias. Water exchange and flushing may be slow, even in lakes with large outflows. Hence, some lakes function in a manner similar to the oceans in acting as pollution sinks. The geographical problem of culprit--victim identification thus appears to be much more complex in the case of lake pollution than in river pollution.

However, even with river pollution, can the situation be accurately depicted as one of upstream culprits and downstream victims? Maybe with respect to negative bioecological externalities, the answer is yes (assuming, of course, that biological self-purification is less than 100 percent effective). However, the situation as regards the geographical pattern of pecuniary effects is much less clear. For the positive pecuniary externalities to be localized upstream and the negative ones downstream, the assumption of no inter-community movement of goods and residents must be made (Dales, 1972).

The foregoing rather simplified assumptions, at least, would have to be satisfied before a distinct "polluter--pollutee" spatial pattern could be drawn. Obviously, the assumption of immobile goods and people is untenable. Hence, no easy generalization is possible concerning the spatial distribution of positive and negative externalities of water pollution for either river or lake.

With regard to the second question, that of the spatial impact of the benefits and costs of pollution control measures, no simple generalizations can be made, except that amenity values downstream are improved as river pollution is abated. The issue is further complicated by the fact that there is no difference in economic efficiency whether the polluter or the pollutee bears the financial burden of pollution control costs and/or damages (Coase, 1972; Turvey, 1972). Hence, the spatial benefit-cost distribution is at least partially dependent upon the specific, legal delimitation of rights.

Such questions really deal with equity considerations, or the impact of control measures upon the distribution of income and welfare. Although improved air quality in the U.S. would probably benefit the inner city poor and black populations more than it would the more affluent suburbanites, improved water quality would probably accrue disproportionately to the more affluent, as they alone have the financial resources to avail themselves of many water-based recreational activities and riverfront lots. If the costs of control measures are passed on to the consumer, they will, in all likelihood, constitute regressive solutions. Freeman and coworkers (1973) have discussed two broad categories of cost-shifting policies, namely cost subsidy and adjustment assistance. These policies could do a great deal to ameliorate the inequitable features of pollution control measures.

Pursuing this direction of thought, it is difficult to make any simple generalizations concerning the spatial impact of water pollution on the one hand and pollution control measures on the other. An investigation of the relative merits of various types and scales of territorial organization with respect to effective abatement of water pollution may, however, contribute to more efficient resolution of the different interests involved.

Geographic Scale of Control Systems

Criteria for Optimal Control Scale--

In discussing the optimal scale for control measures, the first criterion should be that net social welfare including ecological considerations is maximized. This can be accomplished by selecting that pollution control level where the marginal costs of pollution abatement are equal to the marginal benefits, as depicted schematically in Figure 1 (Koleda, 1971). There are two criteria of optimal coordination scale: jointness efficiency and distribution efficiency. Cox (1974) has argued that jointness efficiency depends upon two factors: the degree of internalization of externalities and the exploitation of scale economies. Tullock (1969) has linked these two factors to the jurisdictional area by assuming a) a negative exponential function relating the costs of uninternalized externalities to the size of jurisdiction (see Figure 2) and b) a U-shaped function relating the marginal (pure water) production cost to the size of jurisdiction (see Figure 3). By itself the first assumption means that jointness efficiency increases with jurisdictional area, whereas the second clearly specifies the optimal size as being the one at the bottom of the U-shaped, marginal-cost curve.

Distribution efficiency is achieved if individuals can consume their perceived optimal amounts of the public good (in the form of pure water) at the given price such that their marginal private benefits are equal to their marginal private costs. Distribution inefficiency can occur under two dichotomous situations. On the one hand, non-uniform provision of the good may exist within a context of homogeneous demand curves. On the other hand, heterogeneous demand curves may exist with uniform provision of the good (Cox, 1974). However, in the case of water pollution control, the demand curves for the pure water would probably be heterogeneous and, because of

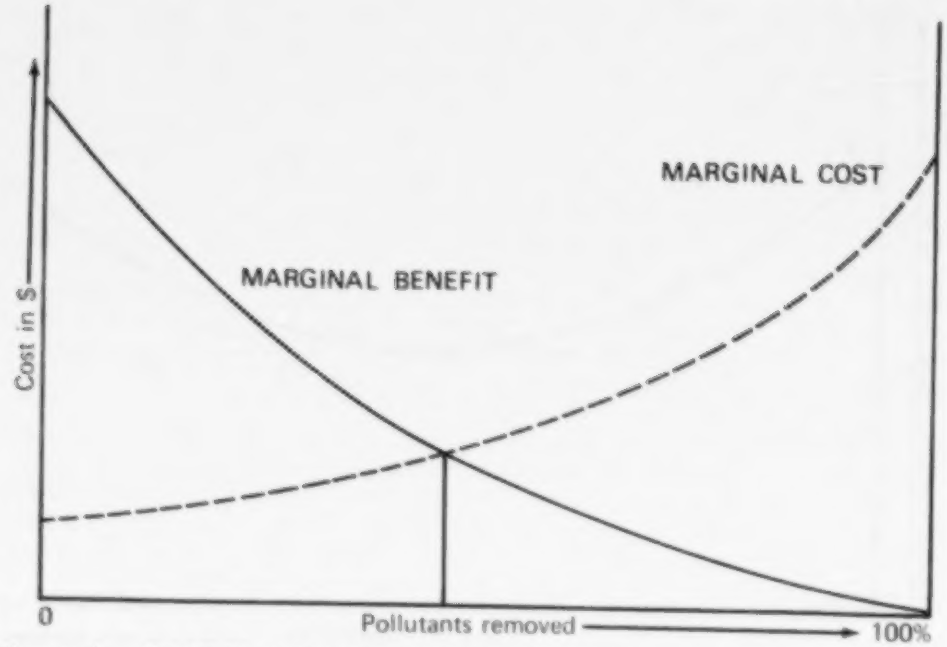


Figure 1. Optimal Pollution Control (Q = optimal abatement level).

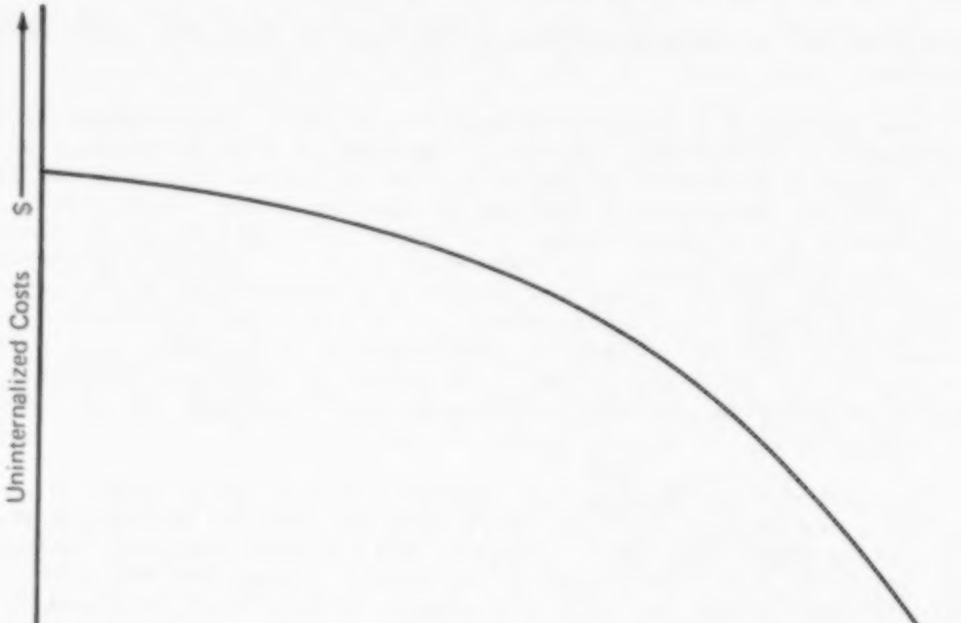


Figure 2. Uninternalized costs and jurisdictional size.

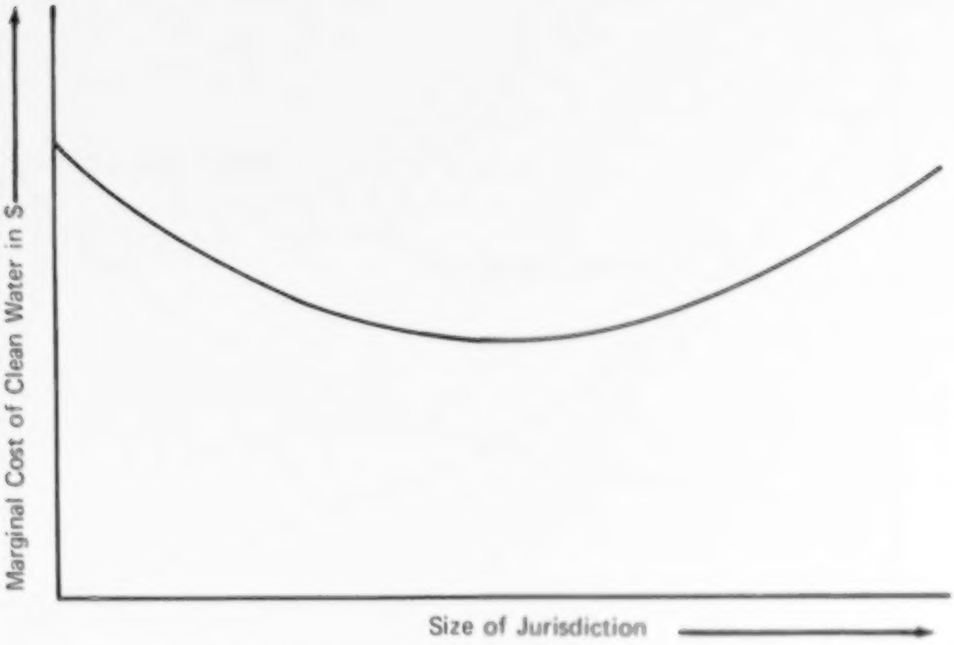


Figure 3. Marginal production of clean water.

geographical and accessibility factors, the provision of pure water would be non-uniform.

There are tradeoff problems between the criteria for jointness efficiency and distribution efficiency. As the jurisdictional size is enlarged to internalize a greater proportion of the externalities and to reap the benefits of scale economies, the economic preferences, and hence the demand curves, of the residents in a given region become more disparate. This situation is likely to lead to a decline in the distribution efficiency. However, in the case of water pollution control, where non-uniform provision of the public good is the rule, the tradeoff is not so simple. Cox (1974) has argued plausibly that the theoretical problem of optimal territorial scale becomes one of selecting the scale," ...at which the ratio of the prices of jointness efficiency and distribution efficiency is proportionate to their marginal rate of substitution" (Cox, 1974).

This theoretical discussion of optimal criteria for territorial scale has one glaring drawback and that is the lack of any satisfactory way of measuring and utilizing these criteria. There are probably many empirical factors that affect jointness and distribution efficiencies. Hence, for some conflicts in territorial partition, Cox (1974) has suggested the use of empirical movement networks to delimit de jure territories. One of the major caveats that he attaches to his procedure is that it reflects only the significance of human movements, and hence would be of little utility in trying to obtain the optimal jurisdictional scale for flood control or pollution control districts. (This supposed limitation will be mentioned again later in the discussion.)

Despite the limitations mentioned in the previous paragraph, it still seems possible to develop some useful rules of thumb concerning the relationships between the size of control districts and the effectiveness of any abatement system. Thus, this analysis will first focus upon the advantages and disadvantages of various political jurisdictions.

Political Jurisdictions--

As we know, in the United States, most of the readily used political tools for environmental control are firmly attached to the lowest levels of government because they involve the ownership and use of the land and associated waterways.

Nevertheless, the disadvantages of local jurisdictions (such as a municipality or county) seem to outnumber their advantages. Projects developed at the local level tend to be single-purpose ones (such as an urban water supply or a sewage treatment plant). Accordingly, unless forced to do so, project planners rarely consider the results of their actions upon other water users (Davis, 1968). In fact, intramural jealousies among neighboring communities often act to promote inefficiencies in the system and stifle possible economies of scale in the construction of regional projects. If the pollution control strategy were to involve effluent charges, a local political unit could not readily extract payment from polluters located outside its territorial authority, even if their effluent contaminates its waterways. Therefore, the application of charges to polluters located within the control district might not result in any appreciable improvement of local water quality unless pertinent agreements between regions can be made. A similar jointness inefficiency can occur if water quality or effluent standards are the primary tools for control. This is true especially when the volume and quality of effluent discharged locally is insignificant in comparison to the total waste load entering the local waterbodies.

Local revenue from any charges might not be sufficient to improve water conditions. Even though large-scale, multi-stage, waste treatment facilities may be the most efficient and effective, a local governmental unit probably would not be able to afford them. The dynamics of the situation call for political integration or, at least, financial assistance from higher level political units.

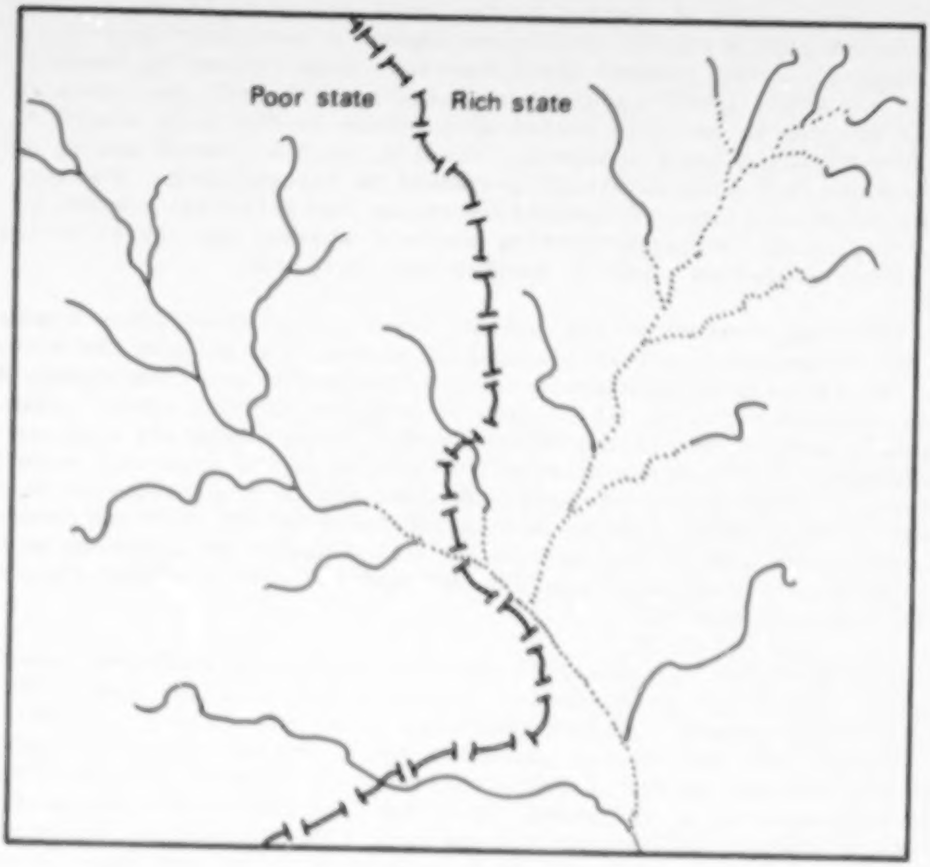
On the other hand, a strategy predicated upon effluent standards might be even less satisfactory because it would have at best a weak revenue generating capacity. The only logical mechanism for producing revenue would be the levying of fines upon enterprises that violate the established standards. However, local jurisdictions are very susceptible to pressure applied by large industrial enterprises, especially those that employ a large percentage of the local labor force. Bowen's (1970) account of the 1948 air pollution disaster in Donora, Pennsylvania is an all too poignant example of the ineffectiveness of local political units in situations of monopsony. If the fines are low, offending firms may simply pay them and pollute as usual. Large fines levied by local governmental bodies are more likely to produce protracted and losing battles in court than augmented revenues or improved water quality. This has been demonstrated with Lake Erie problems and also the residuals discharged into Lake Superior by the Reserve Mining Company of

Silver Bay, Minnesota. Furthermore, local governmental bodies may even be pressured by surrounding local jurisdictions into ignoring unpopular pollution controls in order to attract industry rather than to chase it away.

On the positive side, local political units possess, in principle, a better knowledge of local conditions, both hydrologic and economic, than do the larger scale political units. They may be in the best position to estimate the importance of different levels of water quality to the local population. In addition, they may be able to devise a plan for land use that would minimize the effects of pollution or maximize the efficiency of an abatement system. Unfortunately, these advantages would probably be offset, at least partially, by difficulties in attracting and retaining a competent professional staff. Similarly, local agencies are likely to be the least competent in the latest and most appropriate field, laboratory, and planning techniques.

Some potential improvement in the effectiveness of water pollution control may occur at the state level. In the first place, states are usually willing to attempt the formation of coordinated agencies dealing with outdoor recreation, water quality control, fish and wildlife management, irrigation, and water supply. They can probably get a reasonable gauge on what demands exist, and exercise considerable control over these activities. Secondly, states are in a better position to control water quality of major parts of river basins (by internalizing externalities) through comprehensive investigation of water conditions, and the institution of differential methods such as zoning, charges, or standards. Thirdly, state revenues might be sufficient to undertake large-scale projects, if needed.

On the other hand, there usually is strong competition for available state tax funds, especially from highway and education agencies. If federal funds are accepted, they often have strings attached. Richer states that are not dependent on dirty industries may institute tighter controls. Poorer states might then be tempted to establish lenient control systems to attract industry and thus increase their income base. Such a situation may even be indirectly beneficial in reducing per capita income disparities between states. However, any economic benefits to the residents of the poorer states would be at least partially diminished by losses in amenities and by increased hazards to health. Furthermore, as state boundaries are not coterminous with river basins, this differential control might conceivably occur in a non-optimal manner within particular basins. For instance, the dirtier part of a river might be in a rich state that institutes stringent controls, even though the water is less susceptible to further pollution damage than is clean water in a poor state. In other words, the marginal loss from additional contamination is less in the rich state than in the poor one (see Figure 4). An alternative scenario of interstate "standards cutting warfare" suggests that suboptimal pollution control levels (namely too little) may be reached as firms either move, or threaten to move, to states having more lenient controls. Efficiency of a state control system for a particular river or lake basin probably varies directly with the fraction of the basin within the state boundaries. Obviously, the inclusion of the headwaters territory is crucial. For example, all else being equal, pollution control over the Mississippi River would be more effective within the state of Minnesota than within the state of Louisiana.



LEGEND

⊢⊣⊢⊣	State boundary
〰	Clean stream
⋯⋯	Polluted stream
MC_{pp}	Marginal cost of pollution in a poor state
MC_{pr}	Marginal cost of pollution in a rich state

Figure 4. Marginal pollution costs and political boundaries $(MC_{pp} > MC_{pr})$.

These limitations of control at the local and state levels may lead to the conclusion that a federal pollution control system would be preferable. Indeed, control at the federal level does have some factors to commend it. First, it can avail itself of greater monetary, technical, and human resources. Second, it has more political leverage to deal with monopsony problems at lower jurisdictional levels. Finally, at the federal scale, a larger share of water pollution externalities would be internalized. However, the emergence of multinational corporations raises the potential specter of international pollution "standards cutting warfare" whereby the threat to the entire global ecosystem could be dangerously increased.

In addition, control at the federal level has at least three drawbacks. First, the larger the scale of the control system, the greater the bureaucratic maze and tendency to fracture responsibilities for water management into agencies concerned with specific types of programs (Davis, 1968). Also, in the specific case of the United States, lobbying interests are very powerful, and therefore, certain types of solutions may be looked upon with more favor than others. Large-scale, capital-intensive, public projects, such as sewage treatment plants, may be favored because industry is not affected directly, and manufacturers involved in selling pollution abatement equipment will stand to gain, and such projects will be "visible" benefits from a political viewpoint (Davis, 1968).

A program at the federal level may simply promulgate minimum standards designed to be uniformly applicable. However, such a policy might merely translate itself into spatially homogeneous levels of mediocre pollution control. Furthermore, information gathered from a federal viewpoint might not possess a sufficient degree of differentiation in terms of the characteristics of, and influences upon, different river and lake basins, nor appropriate awareness of the various demands and needs of the surrounding populations. For example, waters already polluted to a certain extent are less susceptible to additional degradation than clear, clean water. A more rational division of uses in such a situation might be to zone the former as a repository for industrial waste and the latter for recreation. An optimal solution within a basin, or between nearby basins, may be to superimpose a strategy for management of venous-arterial water quality, utilizing currently differentiated conditions of water quality as one criterion for territorial partition. Unfortunately, with the exception of interior drainage basins (such as the Caspian Sea) the world's oceans seem to serve as the ultimate sinks for industrial, agricultural, and human wastes. Hence, the ultimate control over water quality necessitates an international scale of control (that is to say, the complete internalization of externalities). Because collective decision making, even at the international scale, is plagued by free-rider problems, there is great need for altruism.

Two tenative generalizations about the optimal scale of political jurisdictions for effective pollution control may be enunciated. First, the theoretical effectiveness of the control system seems to increase as externalities are internalized. Second, with the possible exception of the global scale, all political jurisdictions seem not only sub-optimal but more importantly inappropriate units for the task envisioned. An interesting alternative which may help resolve some of the problems of political jurisdictions, is the use of river and lake basins as the most suitable divisions for water pollution control systems.

Natural Jurisdictions--

The fact that both air and water pollution ignore political boundaries, and the existence of a plethora of inter-jurisdictional conflicts over water management authority in the United States, is a convincing argument for the judicious conversion to the basin concept of water quality control. With slight modification, Cox's (1974) theoretical arguments, linking movement networks to territorial organization, can be used to support the adoption of river and lake basins as the most appropriate territorial units for any system of water pollution control. Instead of focusing upon networks of human movement, it is simply necessary to concentrate upon networks of water movement as the basic organizing principle for selecting the optimal geographical scale of pollution control districts. Obviously, control districts conterminous with drainage basin boundaries are ideal for maximizing jointness efficiency.

In terms of the optimal scale for a natural jurisdiction, the dynamics seem to call for nothing less than integral river or lake basins, regardless of whether they are interior drainage systems or part of an oceanic drainage system. Specific basin control measures could be added to enhance distribution efficiency by seeking to collect and utilize both differential hydraulic and socio-economic data. Where a particular basin encompasses two or more nations, the optimal solution calls for international, basin-wide control. Again, the ideal solution seems to be some form of effective, global, collective bargaining agreement to control ocean pollution.

The appropriateness of river or lake basins as the territorial units used for pollution control systems seems intuitively obvious. Yet in the United States such natural planning and management regions are few in number, have evolved sluggishly, and in all instances known to this author, have very circumscribed authority over multipurpose or multi-use water resource management. Accordingly, the universal adoption of the basin management concept seems to be a logical requisite for substantive improvement in the efficient and ecologically prudent management of America's water resources. Unfortunately, the multitude of entrenched political interests and governmental agencies involved with water resource management do not augur well for the widespread adoption of the comprehensive basin approach to water resource management in the U.S.

The People's Voice

Fisher (1970) has pointed out that a network of knowledgeable voluntary associations and private technical associations can be a useful link to relate public and private agencies with jurisditional decision making. In the usual cost-benefit analyses conducted on behalf of agencies, benefits are generally overstated, and costs fail to take into account the more remote damages and, more particularly, the social contexts of water quality control. The Brandywine study (1968) attempted to alleviate this problem of single purpose planning by specifically incorporating the people's view of the role that the natural environment plays in their lives.

Unless some means is established to combine water basin with political region, quality will be pursued in a piecemeal fashion. Political jurisdictions will continue to be involved in tradeoffs, and appointed bodies will continue to appease those they represent. More than ever, we must focus on the political democratic process as well as on economic theory. In doing so, we accept the responsibility for maintaining a dynamic dimension, and determining its receptiveness to infusions of new information and mechanisms of feedback.

Strategies of Control

In a summary fashion four pollution control strategies will be evaluated in terms of four criteria: a) economic efficiency, b) incentive structure, c) transactional or information costs, and d) enforcement burden.

Moral Suasion--

A policy or strategy of water pollution control based upon the use of moral persuasion to convince polluters to voluntarily refrain from polluting or reduce their water-borne residual discharges as an obligation to society can be summarily dismissed as an ineffective policy doomed to failure within the context of a socialist as well as market economy. First of all, without accompanying subsidies or payments, no strong economic incentive exists to implement pollution abatement measures because of price competition in market economies and free-rider problems in both forms of economic organizations. The private firm that altruistically and unilaterally takes positive actions to either reduce or treat its wasteload incurs increased production costs (in many instances) which can either manifest themselves as lower profit margins (and even outright losses) or higher-priced, less competitive products. Thus, firms have an economic incentive to continue to pollute.

To make matters worse, there is no guarantee that the altruistic firm's abatement measures will be economically as efficient as the potential ones a more "business-like" enterprise which decides against control measures could have adopted.

While transactional costs may be rather modest, moral persuasion has little effective means of enforcing compliance. One could argue that environmental groups in America, through the skillful use of the mass media, could tarnish the public image of polluting enterprises which choose to ignore the moral urgings of the citizenry for cleaner water with the hope of creating a consumer boycott of the offending firm's products. Such boycotts are also susceptible to free-rider problems, especially if the polluting firm's goods are priced lower (as could be expected) than competing businesses who yield to moral force. Then, too, a firm may discover that it is cheaper to hire a public relations firm to resurrect its public image than to reduce the environmental damages which its activities generate.

In summary, a policy based on moral persuasion leads not to the control of pollution, but rather to the continuation of pollution thus reducing both net GNP (Gross National Product) and the welfare of society (Dorfman, et al., 1972).

Bargaining Solutions or the Tort System--

Bargaining solutions represent an improvement over moral exhortations. Such a strategy is predicated upon certain prescribed legal rights and obligations with regard to water use. In certain limited situations, negotiations leading to either bribes or mergers may produce economically efficient solutions if bargaining is perfect. For example, if A and B are (profit-maximizing) firms with the former representing the polluting actor and the latter the polluted or harmed actor, then negotiations between them can yield three economically optimal outcomes: a) Pollution costs could be internalized by a merger, b) B might agree to pay A to modify the nature or scale of its polluting activities, or c) If the law allows B to be compensated for losses, then A might pay B to accept the optimal amount of loss imposed by A. In the last two scenarios, the optimum is defined as A's type and scale of activities and B's reaction to them which maximize the algebraic sum of A's gain and B's loss as compared to the situation where A imposes no diseconomy on B. As was noted previously, the fact that compensation liability is reversed in the last two bargaining outcomes has no impact whatsoever on economic efficiency (Turvey, 1972). Thus, when negotiations are possible, governmental intervention in the United States is not for the sake of efficiency, but for a societal sense of justice. However, as Turvey (1972) illustrates graphically, for optimality to exist it is necessary that any levy assessed against A be paid to B, otherwise sub-optimal conditions will obtain.

In the example above, both protagonists have effective incentive to bargain with each other. However, real world situations are rarely that simple. Water is a flow resource as opposed to a stock resource and as such has many of the attributes of a free good or a public good. It is exceedingly difficult from a physical point of view to assign property rights to a fluid resource. Accordingly, except for certain limited situations, water resources in the U.S., as in the USSR, are publicly owned. Furthermore, in both societies water has a multitude of different users (and uses), many of which have conflicting if not incompatible interests in water resource management.

The key limiting factor to the applicability of a bargaining or tort system of water pollution control is the sheer number of potential participants. In brief, transactional costs of bargaining solutions increase exponentially as the number of actors increases. Finally, bargaining solutions are plagued with free-rider problems which aggravate enforcement problems.

Government Regulation--

For the purposes here government regulations can be divided into three general categories: a) prohibiting certain uses, b) limiting certain uses, and c) prescribing protective measures.

The most stringent policy, prohibition, is needed in certain cases such as bans on the use of persistent biocides and polychlorinated biphenol (PCB). The cumulative effect or biological magnification of certain deleterious residuals may yet prove that the policy of prohibition has not been invoked frequently enough. In general, however, optimality does not require the complete elimination of pollutants. Provided the penalties established for noncompliance are high, the enforcement burden is likely to be low as are transactional costs. The chief problem with a prohibition policy is that it represents an extreme position with limited applicability. However, because of ecological response time lags, prohibition has probably not been invoked often nor expeditiously enough in the U.S. and elsewhere.

A policy of limiting certain uses is of more general applicability and may take two common forms: a) instream water quality standards (in which case it is often exceedingly difficult to assign responsibility for violated standards in situations where there are several dischargers), and b) discharge standards based upon some technical-political-economic estimate of the total allowable discharge (the assimilative capacity) for a given region. The latter system would probably include a licensing of polluters. Both forms are difficult to enforce (especially with regard to occasional infractions), require costly monitoring, are likely to be inefficient, and have few self-regulating properties or incentives to conform to governmental directions. While creating lower transactional costs, and being politically expedient, discharge standards which are geographically and sectorally uniform are sub-optimal.

The third regulating device involves prescribing protective measures, such as uniform secondary treatment processes or specific types of pollution control apparatus. This solution by action approach has limited applicability as polluters usually have better knowledge as to the technical and cost effectiveness of various technical options. Thus, specific means of pollution control ordered by government are likely to be both technically and economically inefficient. On the other hand, such an approach has low transactional costs and perhaps an acceptable enforcement burden.

In a more systematic fashion, Dorfman and Dorfman (1922) enumerate four general difficulties that beset governmental attempts to regulate water quality: a) inadequacy (and cost of information, b) crudeness of regulatory instruments, c) practical difficulties of enforcement, and d) haphazardness of burdens. The section discussing current U.S. water quality management supports their general dismal thesis.

Economic Inducements--

Economic inducement policies may be divided into two broad categories: a) subsidies or payments and b) taxes or effluent charges.

Subsidies, tax incentives and the like may effectively discourage or encourage different (socially and ecologically more desirable) ways of water resource use. They appear appropriate where capital costs are the only stumbling block to the improvement of water quality. Transactional costs and enforcement burdens would likely be modest and certainly lower than the just cited examples of more direct forms of government control and regulation. An effective positive incentive structure can be created as part of either a subsidy or a tax incentive program of water pollution control. The chief liability of these "carrot" approaches is that there is little likehood of the policy being even remotely geographically, ecologically, or economically efficient. Then, too, there are some blackmail or coercion dangers in some payment policies. For example, Shkatov (1968) suggests a scheme in which a Soviet enterprise's incentive fund would receive a two-kopeck bonus from the state for every cubic meter of completely purified sewage which it discharged.

In the U.S. such a proposal, if adopted, would create a host of new and profitable firms engaged in "the business of not creating pollution!" These and other problems with subsidies are discussed in the section dealing with current U.S. water quality.

Tax and effluent charge policies seem far more encouraging for a number of reasons. A tax policy here refers to a tax on pollution generating products while an effluent charge policy refers to a charge (or tax) on the effluent discharged (differentiated according to the quantity, quality, time, and place of the discharge). Both approaches would internalize pollution externalities and significantly reduce enforcement burdens, compared with other possible policies.

A tax on pollution generating commodities would likely have both low information costs and enforcement burdens provided the tax was uniform according to commodity. However, such a strategy would be sub-optimal because it is improbable that damage functions for the manufacture of a given commodity are homogeneous for all production sites. The tradeoff for increased efficiency would be increased information costs as to commodity-specific and location-specific estimates of damage functions.

A charge scheme could be established similar to the federal income tax system in which firms would have to monitor (record) their emissions, report their composition and quantity (file tax forms) to a public agency, and make the required payments (taxes) to the agency. Audits and unannounced inspections would, of course, be necessary. While involving considerable expense (information costs), an effluent standards policy would require similar expenditures. On the other hand, for optimality to obtain the governmental agency only needs to know the quantity and quality of the effluent discharges and not marginal cost curves of individual emitters--the latter being information essentially unobtainable as costs are often regarded as enterprise secrets.

Not only would a charge system create a permanent incentive system to reduce residuals, but furthermore, individual firms have the best knowledge as to the most efficient option to employ to equate marginal pollution control costs with marginal damage costs (effluent charges). Thus, a residuals charge system would tend toward optimality.

The "correct" charge, unfortunately, could not be ascertained easily by trial and error without violating parato-optimality. For instance, while charges could be changed readily, fixed capital in pollution control facilities are far less flexible (Dorfman et al., 1972). Thus, considerable benefit-cost analysis should precede the adoption of a charge policy. It seems that some of the linked econometric-biologic-hydrologic simulation models discussed at this symposium could greatly facilitate such types of analysis and guide decision makers involved in the elaboration of the specifics of a residuals charges policy. However, in most cases there might well be far greater risks of initial underinvestment in "effluent charge-avoiding" treatment facilities and altered production processes than the converse simply as a consequence of national economic expansion.

It appears that a carefully conceived residuals charge policy has more to recommend it on both theoretical and practical grounds than any of the other major strategies thus far proposed or implemented.

Current U.S. Pollution Control Policies--

In the United States, the failure of moral suasion, the limited applicability of bargaining solutions, and the public good aspects of water resources in the face of deteriorating national water quality conditions culminated in a public demand for government regulation of water quality.

The potentially effective federal Refuse Act of 1899 which prohibits the discharge into any waters of "any refuse matter of any kind or description whatsoever" without the polluter having first obtained a permit from the U.S. Army Corps of Engineers has been, in effect, repealed by the U.S. Justice Department's policy of selective non-enforcement (Freeman et al., 1973).

The Water Pollution Control Act of 1956 established a federal subsidy program for the construction of municipal waste treatment facilities. However, the subsidy program pertains only to the initial capital construction costs and not to maintenance and operating expenditures. Therefore, the initial incentive to build treatment plants is not coupled with an incentive to operate them efficiently. Such an incentive, however, could be created fairly easily.

Since many industrial and commercial waste generators are connected to municipal sewage lines, the federal grants program results in reduced sewer charges and hence, a substantial subsidy to business. Furthermore, it undercuts the incentives for all effluent dischargers to seek alternative pollution control measures. Recent federal water pollution control legislation stipulates that all municipalities must use secondary waste treatment processes (85% organic removal) and that all discharges from all sources of pollution are to cease by a target date of 1985. The first regulation leads to a misallocation of resources as some regions will undoubtedly experience insufficient treatment costs and others excessive ones. On the other hand, such a uniform treatment regulation greatly conserves on transactional or information costs. The second stipulation, albeit laudable, will probably be counterproductive as it sets unattainable expectations (and implicitly assumes that the price of pure water is infinite). The enforcement burden of such a water quality goal (zero discharge) would indubitably precipitate a massive breakdown in the system of water quality management.

The second major U.S. policy instrument intended to encourage investment in waste treatment facilities is contained in the Tax Reform Act of 1969 which allows accelerated depreciation of such investments for tax purposes. (Really a tax subsidy program should be categorized as an economic inducement strategy.) This tax subsidy reduces the after-tax costs of pollution control investments (Freeman et al., 1973). However, both the federal grants program and the tax subsidy, in effect, allow dischargers to create and dispose of large quantities of residuals without incurring the full costs of their actions and at the same time use taxpayers' revenue to cleanse the waters they have dispoiled.

This last statement is predicated upon the author's support--with certain critical qualifications--of the polluter pay principle. Subscribing to such a principle appeals both to a sense of justice and economic principles. With regard to the latter, whenever someone does not bear the full costs of one's actions, there is a natural tendency to squander underpriced resources. The crucial qualifications entail the development of a pollution pay strategy which is not regressive in its functioning. For example, in any polluter pay system, the costs of compliance will unavoidably be passed on to the consumer. Such an arrangement is economically desirable, but may be socially inequitable for consumer necessities having low price elasticities of demand. Thus, without an accompanying income adjustment component (such as a guaranteed income), a polluter pay principle will lead to socially regressive resolutions of environmental quality problems.

Although U.S. water pollution control statutes provide for various legal sanctions for non-compliance such as fines and the legal authority to halt the operations of polluters, in practice compliance has been more voluntary than compulsory. In some cases various governmental enforcement agencies have resorted to fines, court action and the like, but they have not been very effective. Overall the effectiveness of existing U.S. pollution control policies have been characterized more by failure than success. Accordingly new policies and strategies need to be tried, especially ones using a combination of residual charges and income adjustment programs.

Conclusions

A number of tentative conclusions can be enumerated at this time. First, it is difficult to state any simple generalizations about the human-geographic manifestations of water pollution on the one hand, and pollution abatement measures on the other. Second, political jurisdictions are far less appropriate than river or lake basins as the territorial organizing principle behind any pollution control system. In this regard, the USSR may be ahead of the U.S. Third, current U.S. water pollution control policy leaves much to be desired. Fourth, and most importantly, a plea is made for the acceptance of the polluter pay principle and the establishment of charges or fees for the use of nature's freshwater assimilative capacity.

Effluent charges currently exist in a number of both East and West European countries. It seems more than a little ironic to this author that many of the socialist countries of Eastern Europe have accepted the concept of residuals pricing while the so-called price-oriented market economies of England and the United States have stubbornly resisted such a strategy. Any number of Soviet academicians and economists have suggested an effluent charge which is: a) proportional to the quantity and quality of unpurified effluent discharged and b) differentiated by industry and product. Brown and coworkers (in press) add the additional economically rational requirements that: a) charges be differentiated by production techniques within industries, b) charges be differentiated geographically to reflect local conditions, c) all polluters regardless of size should pay the same unit charge provided the opportunity costs of their residual emissions are the same, and d) charges ought to vary through time to reflect changes in the opportunity costs of pollution.

As Freeman and coworkers (1973) similarly conclude, a residuals charge strategy has four strong factors to recommend it. First, a charge strategy will produce a given level of pollution abatement at a lower total economic cost than alternative strategies because of the economic incentives it creates and the decentralized decision making as to the scale of emission reduction and the specific measures. Second, not only is the incentive to control residuals discharge direct and powerful, it is durable. Third, governmental policy makers utilizing a charge system need less detailed information about individual polluters to achieve optimality than if they depend on an alternative system such as direct regulation. Fourth, a residuals charges strategy will create less of an enforcement burden on the existing judicial system because collecting the charge is similar to the paying of taxes. To be economically, geographically, and socially desirable, the residuals charges strategy should be superimposed on a basin-wide water resources management system and combined with an income adjustment program to ameliorate the possible regressive aspects of the policy.

Finally, the first part of this paper pointed to the need for global cooperation with regard to environmental pollution control. This joint Soviet-American symposium on water pollution control is not only in the spirit of this need, but also is a tangible example of such needed international cooperation.

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DISCUSSION

DeLUCIA: I would like to make a few comments that in some cases will amplify and reinforce, and in some cases disagree with Dr. Zumbrunnen's remarks. First, with respect to his comments on regulatory approaches, one very important thing to remember is that frequent discussions of regulatory approaches implicitly assume zero or little transaction time, but the empirical evidence in the United States is that this is not so. Hence, the resulting transaction costs due to delays and to the resulting regulatory process are significant. Second, with respect to market mechanisms and effluent charges, I would suggest that there has been recent work that explicitly estimates mechanisms which, when used with a combination of regulatory and effluent charges, minimize the risks of underinvestment, deal with the issues of uncertainty of response, and estimate the implications of single charges versus commodity or specific charges. In fact, this work is an example of a Zumbrunnen proposal--the use of some of the models, such as those we have been discussing, to examine effluent charges and other mechanisms. This work is referenced in my paper. Lastly, I would like to take very strong exception to Dr. Zumbrunnen's strong positive comment on the "polluter pays principle." In my opinion, speaking solely of the situation in the United States, the 1972 amendments to the Water Pollution Control Act are markedly different from earlier water resource legislation in the United States in that, in both the preamble and in the text, the Congress makes it clear that it views environmental quality as a national goal and objective. It is quite clear from reading the legislative history that the Congress's intent is that a significant portion of the financial burden should be borne nationally. Moreover, and perhaps most importantly, the legislation mandates a behavior, but gives no option or flexibility to the local or regional institution. In contrast, earlier water resources legislation in the United States gave options to states and municipalities. Earlier legislation was the basis for the economic literature of cost-sharing in proportion to benefits received.

LANDYZHENSKIY: When I was familiarizing myself with Dr. Zumbrunnen's paper, I was struck by the fact that so many questions presented in this report relate to conditions in our country. Our countries are very large and have a high concentration of industrial enterprises in specific regions--frequently in regions with limited water resources. In the U.S. and USSR, there are so-called "hot spots," where the controlling and the executive agencies are late in passing preventative measures. As a result, difficult situations arise. In order to improve these situations, it is urgently necessary to take cardinal measures, often with considerable expense or material loss to stop the activity of the polluter. All this affects the interests of many agencies and many people. These interests are often contradictory, and the search for the best decision is exceptionally difficult.

The report shows that when solving a question of water protection, even the most effective measures will not produce the desired result, if the jurisdicial, organizational, and economic acts will not create the necessary conditions for this. The problem of so-called "jurisdicial territory" in the U.S., on which the jurisdiction of the controlling agencies is propagated, also exists in our country. In the Soviet Union, government control for protecting of water bodies is vested in local agencies. Determining the best sizes for jurisdicial territory remains an unsolved problem. Research and practical experience show the expedience of creating an administration for river basins. Using the Kuzin terminology, such an administration should be called "global." In our country, only one such administration has been created for protecting the Caspian Sea. The whole water body enters into its jurisdiction. Other similar organizations--of which there are slightly more than 100--have a limited territorial sphere of influence which affects the coordination of decisions for complex water protection programs for the whole river basin. The Volga River is controlled by four basin inspectorates. We should agree with the conclusion that the most suitable administrative division for systems for controlling water pollution is at the river basin level. It is evident that areas of control that coincide with the boundaries of river basins are ideal for maximizing the effectiveness of consolidation. When a given water body includes the territory of two or more governments, the best decisions require international control within the boundaries of the whole basin. The scheme for complex use and protection of the Tisa River requires the participation of Czechoslovakia, Hungary, Rumania, the Soviet Union, and Yugoslavia.

YAVOROVSKIY: I will only touch upon one point--the application of the "polluter pays principle." Solving technical, mathematical, and technological problems is possible under identical conditions, on an identical basis under socialist economic conditions, as well as under capitalist conditions. Solving organizational problems is a different matter; and still more differences arise when examining economic questions. Significant differences exist in the approach to the economics of water protection on the strength of differences in the original conditions. As Dr. Zumbrunnen noted in his report, the problem of water protection in the U.S. is very specific. It is solved under conditions by which the subjective economic interests of the enterprises take precedence over the objective economic interests of society. The fact that a single water body under U.S. conditions is administratively separate is the second essential difference. This arrangement leads to the expediency of applying the "polluter pays principle"; i.e., the system takes into consideration the subjective harmony of production and the interests of individual establishments, not the objective differences of the functioning of the water bodies. In this case, the clear advantage of this method is its simplicity. However, we feel that its basic flaw lies in its inability to fully consider the natural differences in conditions where various water bodies and separate parts of a single water body are used.

Water protection measures under conditions in a socialist system have their own economic specifics. Our problems are solved under conditions in which differentiated economic requirements to protect a water body from pollution are basic to water protection. The fact that expenditures for treatment should be reduced not to a common level but to objective, socially necessary costs is another characteristic. At present, we are faced with a task which our colleagues are solving. For various enterprises, equal economic conditions should be created for cost accounting that does not depend on the location of the enterprise and the water protection factor.

We have not solved all economic questions of water protection, nor have you in the U.S. However, the "polluter pays principle" obviously cannot be used under our conditions, since it does not produce the best solutions within regions or basins. Dr. Zumbrunnen said that this principle guarantees the introduction of the necessary means. However, it must be said that excess financing is possible in this case, but when there is an accurate economic solution to a problem, this is not the best variant. Therefore, at present, we feel that an economic basis to water protection must be found on the following premises:

1. Costs for water protection measures should be differentiated. This type of cost should be taken into consideration in the formation of production net cost and should influence the economic results of the industrial enterprise.

2. Cost accounting should provide for differentiated revenue payments based on the location of the industrial production's water bodies. This type of cost should equalize the original conditions of the enterprise's production activity.

Such an economic position in the approach to water protection questions is formulated in the work of the Corresponding Member of the USSR Academy of Sciences Bunich and Academician Fedorenko, in particular the latter's work, "The Mechanism for Economic Stimulation under Socialism."

A method that requires payment for water resources and considers the cost of water in projecting the net cost of industrial production would be most effective for solving economic problems related to water protection.

In conclusion, I must say that the problems confronting us and our colleagues are the same. However, the paths for solving these problems, in particular the economics, are different.

LOZANSKIY: I agree with Dr. Yavorovskiy and must add that water legislation in our country forbids water pollution. Therefore, it is not a case of payment for pollution, but of assessing guilt or criminal responsibility. It is prohibited to pay money for breaking laws.

The question of payment for discharging waste in our country is based on this premise: the mass of pollutants accepted by a water body should be determined within the framework of its ability to assimilate so that actual water quality standards are observed.

THOMANN: I have three questions and one preparatory remark. It is fashionable to say in our country that the present system is not working, yet there is much evidence to the contrary; evidence that the present system of government control has produced improvements in water quality. On the other hand, I personally am very sympathetic toward the concept of effluent charges. I believe they can and should be experimented with under our present legislation.

My questions, however, relate to my belief that effluent charge problems have not been sufficiently analyzed. First, what should be done with municipal waste discharges? All discussion of effluent charges has been directed toward industrial firms. Second, water quality problems are associated with the discharge of many types of wastes, not just BOD. Previous work not withstanding, I have not yet seen a convincing analysis that relates effluent charges to the many variables in water quality of interest (for example phosphorus, nitrogen, BOD); how do effluent charges account for different water quality variables? Third, the proponents of effluent charges talk about a trial and error method. But how do you measure the success of this program? For example, if I established a fee system for a river during years of high flow, I may observe good water quality for that system; then the drought comes, bringing low flows and water quality deterioration. I find out my charge system was not adequate; but by then it is too late--we may have money, but the fish have died.

DeLUCIA: I would like to clarify my last comments. When I said I disagree with the "polluter pays principle," I disagree only with respect to public institutions. What do you do with the municipal polluter? You build into your fee system both a charge and a subsidy so that you can design a charge that "keeps the system intact" by forcing a municipality to pay at a particular level based on the marginal unit of waste discharged, but you allow a significant subsidy based on what is removed. This subsidy is not given to industry.

As for other pollutants, we have argued strongly that the appropriate regulatory mechanism is a combination of regulatory and charge systems. In the latter case, problems associated with a trial and error mechanism are largely eliminated, since there are direct upper bounds on the discharge, and lower bounds on the treatment or removal. The effluent charge is utilized to efficiently allocate resources in the range of the cost removal curve where the marginal costs are highest. I would further suggest that flow-dependent, administrative requirements, although feasible, may outweigh the benefits. Additional alternative market mechanisms have been suggested for eliminating the trial and error method.

BARTSCH: I want to inject a different and perhaps simplistic point of view. I personally disagree with the "polluter pays principle." I believe that this approach gives industry the opportunity to do what it has always done, and that is to attempt to get by with the very least. From my own observations on industrial performance, I doubt very much that matching pollution control to assimulation capacity is a potentially realistic objective. I call attention to the fact that in the U.S. (and I suspect that in this regard we are no different from the USSR), we have been especially efficient in exploiting the total environment. I call attention to the manner in which we have cut trees while speaking about a sustained yield. To exploit the land, we have destroyed the buffalo. There are many examples of ways in which we have degraded the air. We have also been most efficient in insulting the total environment. I believe we need only look at the history of litigation between industry and official bodies in connection with water pollution control to raise serious questions about the efficacy of this approach. The issue and the goal is not how much can we get by with; the issue ought to be how clean can we make our environment.

ZUMBRUNNEN: I see no problem for instituting effluent charges that are differentiated both according to the quantity and the quality characteristics of the effluent. One possible use of mathematical models would be to make some connection between the charges and simulated capacity of the water resource, thus resulting in a type of collected social decision-making. Accordingly, we should come up with an absolute limit of allowable abuse or misuse for a particular body of water, and accordingly, correlate our charges, perhaps through an auctioning system. It seems that effluent charges would be difficult to apply, except to point sources of pollution. I am not suggesting that effluent charges should be the only water pollution strategy; but rather one tool which has not been used in the U.S. but might be used. With regard to Dr. Bartsch's comments, I am in complete agreement with him. However, it seems to me that by using an effluent charge, we can build in an economic incentive for industrial firms to reduce their discharges of wastes. Indeed, it seems to me that the Soviet Union does have an advantage over the U.S., at least in theory, because of central planning of industrial locations, industrial expansion, and so forth. At this point I would like to ask a question. What in fact are the inputs in terms of the location of industry, the production of industry, and the estimated effluent? I am curious as to whether some of the outputs of VNIIVO's efforts here would flow back to say, GOSPLUN, so that some of the modeling work here might suggest that industrial expansion should not take place on this river basin but elsewhere. I understand that your law states that any pollution is essentially illegal; we have the same law in the United States--the 1899 Refuse Act. I understand the legal implications, but I see no difference in introducing the charge for water use and introducing a charge for effluent.

MODELING AND DECISION-MAKING ASPECTS OF WATER QUALITY MANAGEMENT

Mark A. Pisano

INTRODUCTION

Several of the papers presented at this symposium discuss the manner in which mathematical modeling components and approaches may be utilized in the solution of complex water quality problems. The papers also discuss several key economic and political concepts that are used in the water quality management planning process of the United States. Finally, the papers discuss some specific case study applications of these modeling approaches, as well as how some of the key planning concepts were employed in a particular area.

This paper will present an overview of how the above approaches and concepts have been employed in the United States in implementing the Federal Water Pollution Control Program. Modeling, water quality analysis, and comprehensive planning approaches have played a very fundamental role in the implementation of our national program. This paper will describe that role and provide an assessment of modeling as it has contributed to our water quality management decisions.

Historical Background

To properly understand the key role of modeling in the decision-making process established under the Federal Water Pollution Control Act of 1972, it is necessary to understand the basic approach embodied in this statute.

The central thrust of the Water Program is that control measures themselves, that is, the application of technology, will establish the basic level of water pollution control. Through successive improvements in technology, eventually approaching a target of zero discharge, wastes to the environment would be progressively eliminated. To a large extent the raison d'etre of the program is a reliance on continuous advancement in abatement technology. The program is based on the assumption that any waste discharge to water would potentially upset the natural integrity of water; therefore, any increase in the removal of pollutants is desirable.

The statute also recognizes that the uniform application of technology at any one time cannot guarantee the appropriate level of control necessary to ensure that every water body would be adequately protected. Similarly, this statute also recognizes that over time the application of higher levels of technology might not be necessary to achieve the desired level of health and environmental protection. Thus, the concept of a control level dependent upon the desired in-stream beneficiary uses of a water body and criteria needed to support those uses is still maintained. Furthermore, the statute stipulates that if this beneficial use cannot be protected through the application of the technological standard established for that period, a higher level of treatment technology can be required. On the other hand, if over time it is found that it is not technologically feasible and/or economically practical to move to zero discharge, then the water quality standards, in conjunction with beneficial uses, can be used to determine a less stringent level of control.

There are two basic approaches for establishing levels of control in the United States. The first is effluent guidelines which function like performance specifications defining what treatment technology can do and when the technology can be implemented. The second is the intended water uses for a specific water body. Models and water quality analysis are the vehicles employed to determine the needed control levels to protect these uses. Thus, models and water quality analysis become the critical factors in determining the levels of control to be established on industrial and municipal dischargers.

There are two basic approaches contained in our statute for establishing levels of control, i.e., technology established limitations and water quality established limitations. There are different rationales and basic assumptions employed in each approach. The technology driven approach is based on the assumption that the problem cannot be solved without technology and, moreover, that a technological solution is administratively easier to implement. Cause and effect analysis, or water quality modeling analysis, is not needed in this case, rather a simple demonstration of the existence of technology that is adequate to establish a control level. The water quality analysis approach is predicated on the basis that in some instances technology may not be adequate to protect beneficial uses, while in other cases the application of technology might not be cost effective. A key weakness of this approach has been the historical difficulty in demonstrating cause and effect analysis.

Prior to the passage of the 1972 statutes, an analysis was performed of the total municipal and industrial control costs that would result if increasing levels of technology were applied. The figures in Table I indicate that a five-fold cost increase could result from going from secondary treatment to a zero discharge. An assessment of the cost of doing individual water quality analysis and tailoring the control level to the level needed to protect beneficial uses was not made. The concluding section of this paper does provide some insights into this issue.

Water Quality Analysis Approach

The first step in determining control levels using water quality analysis is to establish the goals or intended water uses for a specific water body. These goals, embodied in the term "water quality standards," are established jointly by the states and the federal government. An underlying objective employed in the establishment of these standards is the achievement of fishable and swimmable waters where attainable. There are several key tasks to be performed in determining the standard. The first involves an evaluation of the biological criteria needed to support a fishable ecosystem--a balanced system of fish, shellfish, and wildlife--and an analysis of the health criteria needed to permit swimming in that particular water body. This evaluation seeks to make a determination of specific biological and chemical criteria needed to protect the intended uses. This evaluation includes an assessment of the natural background conditions for this given water body. If there are higher natural levels of pollutant constituents that would preclude an intended use of fishable or swimmable waters, then a lower use can be established. Additionally, if there are technological, economic, or institutional factors that would preclude the attainment of a fishable or swimmable level, then a lower use can also be established. In order to evaluate the above relationship between implementation, feasibility, and intended water use, it is necessary to be able to first determine a control strategy, and secondly to be able to relate the effects of this control strategy with the level of water quality. Water quality analysis, including modeling consideration of alternatives, is the tool employed to provide the decision-makers (local, state, and federal officials) with the information needed to determine the attainable beneficial use and the necessary control program.

TABLE I. TOTAL NATIONAL COSTS* (Dollars in Billions)

Dischargers' level of removal	10-Year capital expenditures	20-25 Year operating costs	Total expenditures	Annualized costs in 1981
100%	94.5	220.0	316.5	21.1
95-99%	35.3	83.5	118.8	8.4
85-90%	17.6	43.2	60.8	4.1

*Excludes $12.0 billion costs for intercepting sewers.

Water Quality Analysis

The starting point for the analysis is water quality standard generally predicated on the level of fishable/swimmable water, unless natural background conditions precluded this use. The analysis would be performed for each pollution parameter which is considered important in order to protect the intended uses. The analytical procedure for performing this analysis is outlined in Figure 1. Each source contributing that certain pollutant to the water body should be identified and alternative structural and nonstructural corrective measures identified. The final treatment control strategy for this segment should reflect a combination of control methods which will meet water quality standards for all water quality parameters.

As noted in Figure 1, modeling is generally the appropriate method of ascertaining the effects of proposed alternative abatement strategies. To

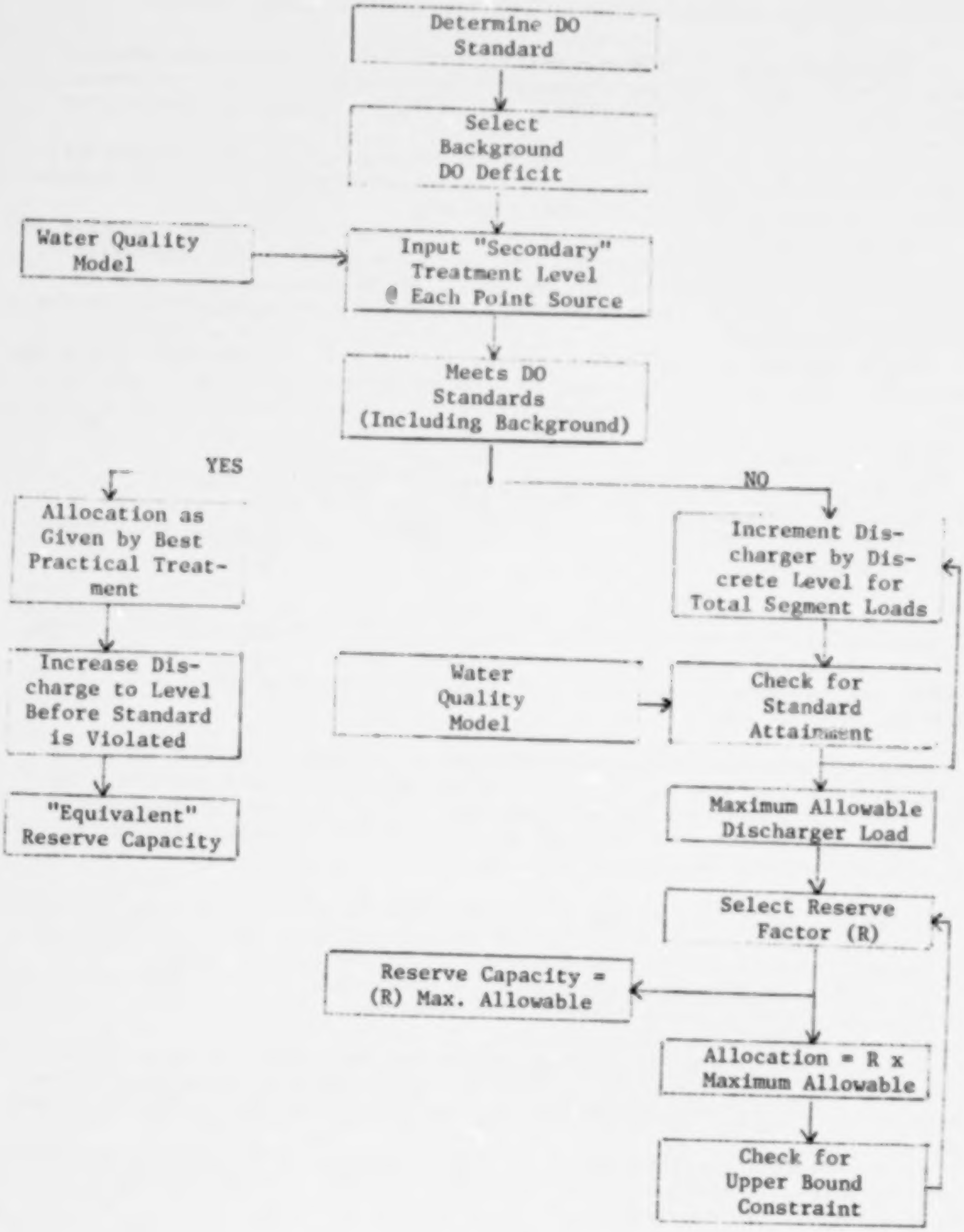
Determine DO Standard
Select Background DO Deficit
Water Quality Model
Input "Secondary" Treatment Level @ Each Point Source
Meets DO Standards (Including Background)
YES
NO
Allocation as Given by Best Practical Treatment
Increase Discharge to Level Before Standard is Violated
"Equivalent" Reserve Capacity
Increment Discharger by Discrete Level for Total Segment Loads
Water Quality Model
Check for Standard Attainment
Maximum Allowable Discharger Load
Select Reserve Factor (R)
Reserve Capacity = (R) Max. Allowable
Allocation = R x Maximum Allowable
Check for Upper Bound Constraint

Figure 1. Suggested Allocation Procedures (BOD-DO Example).

choose the proper analysis, modelers consider the following factors:

Physical Sytems: Rivers, lakes, and estuaries obviously have unique characteristics which must be addressed. Each system is to be represented as either one, two, or three-dimensional and in either dynamic or static time frames.

Complexity of Problem: Water quality concerns arising from dissolved oxygen depression, temperature, eutrophication, toxics, or biological contamination will each dictate a different approach.

Available Resources: A basic practical concern is the amount of time and money which is available for data gathering and analysis. The dependence of planning programs on modeling results requires time-phasing, adherence to schedules, and initial accurate estimates of what can be accomplished in the available time period.

Data: Acquiring a sufficient amount of data that is required to make the decision at hand must be determined. Collecting more data than is needed in order to make a decision is often a pitfall in the analysis. The problem that needs to be solved should be defined and limited before the data collection effort is initiated.

Environmental Risks/Potential Investment: In choosing the level of analysis, the consequences of making a wrong decision and the expenditure resulting from this decision should be considered in determining the actual level of analysis.

The above factors are to be considered in selecting the type and level of analysis to be performed. The basic objective of our program is to keep the analysis as simple as possible and to perform analysis only to the extent needed to support either the water quality standards decision or to determine the level of control.

The criteria summarizing the selection of an analytical technique are shown in Table II. The table lists each criterion mentioned above for the four levels of complexity and presents the type of problems in water bodies for which that level of analysis is appropriate, the planning complexity associated with that level, and the time required for the study.

Type A: Simplified analysis generally has relatively few waste sources, a lack of multiple stream effects, and the assumption is made that there are no changes in waste loading and hydrology with time (steady state).

Type B: Steady state linear kinetics has several sources which have overlapping stream effects, steady-state conditions still hold (i.e., no change in waste loading and hydrology).

Type C: Transient linear kinetics has multiple sources, which have overlapping stream effects. Waste loading and hydrology vary linearly with time (e.g., storm events, diurnal variations, and highly variable waste dischargers).

Type D: Time variable and nonlinear kinetics have multiple sources which have overlapping (two-dimensional) stream, lake, and estuary effects. The internal mixing process represents a major problem.

There are a number of issues related to the model application which are important and emphasize the need for judgment and understanding. There are no

TABLE II. CRITERIA FOR SELECTION OF TECHNIQUES

Model complexity	Water Quality Problems and variable	Water body	Planning characteristics	Time required for study
Simplified analysis (Type A)	D.O. (carbon & nitrogen)	One-dimensional streams & estuaries (completely mixed)	a) Low risk of capital and/or environmental quality degradation; b) No alternate strategies & control options available	Days to weeks
Steady state linear kinetics (Type B)	a) D.O. (carbon & nitrogen) temperature & nonpoint source; b) Anticipated or existing water quality problems	One or two-dimensional streams, estuaries, rivers lakes	a) Low to moderate risk to capital & environmental quality degradation; b) Alternative strategies & control options must be available	2 to 9 months
Transient linear kinetics analysis (Type C)	a) Time varying D.O. nonpoint source analysis & temperature. Simple eutrophication analysis. Full storm water overflow analysis; b) Water quality problems	Rivers, lakes, & estuaries. One or two-dimensional	a) Moderate to high risk of capital & environmental quality degradation; b) Alternative strategies & control options must be available	6 to 24 months
Time variable non-linear kinetics analysis (Type D)	a) Detailed eutrophication analysis et al.; b) Water quality problems; c) High growth of area projected	All bodies of water	a) High risk of capital and/or environmental quality degradation; b) Varying strategies, control be available	12 to 36 months

simple answers to these issues, most of which stand unresolved. However, these issues must be addressed either explicitly or implicitly before recommendations on specific control programs can be made.

Definition of the proper design conditions: The model design must account for extreme or critical stress conditions that will affect the recommendations of the analysis. For example, for stream analysis a low flow/high temperature period is usually chosen, such as the 7-day duration, 10-year recurrence low flow. In certain water bodies an analysis based on the 7-day, 10-year time period may not be protective of sensitive biological systems. If problems are associated with winter ice cover or seasonal discharge activity, then a different design flow would be appropriate. The wet weather design conditions are critical for storm water pollution analysis. Inasmuch as the pollution potential here is not only dependent on the intensity and duration of rainfall but also on the antecedent conditions, simple rules are elusive. A better approach to determine the appropriate decision rules for such an area would consider the treatment costs and the corresponding probabilities of a stream standards violation causing varying losses of beneficial uses.

Definition of mixing zones: If the discharge is of a different quality than the receiving water, then at the point of discharge there will be a zone of differential water quality called a mixing zone. The size of the zone may vary, but if the zone becomes too large, then the beneficial uses for the river could be impaired.

Definition of permit conditions: Once an individual effluent limit has been established, it is transferred to a permit which is legally enforceable. Treatment plant performance is based on a number of factors, and plant performance can be quite variable. The permit condition may be expressed as a maximum discharge concentration never to be exceeded, or as a daily or weekly discharge average.

The validity of the criteria: The goal of our efforts is to improve or preserve beneficial water uses. Sometimes it may be desirable to step back and consider the criteria which are assumed to justify use classifications. An example would be a coliform bacteria limit for swimming. If the limit is exceeded during rainfall periods, it might be reasonable to ask whether anyone would swim at this time anyway.

Should there be time variable effluent limits: In a case where effluent limits are based on water quality, it is reasonable to argue that the allowable loading should be based on the existing assimilative capacity of the water body. For some pollution constituents this assimilative capacity may vary considerably from season to season.

Definition of the safety factors: Some of the above-mentioned issues such as design conditions and permit expression can be considered in the context of safety factors. During the course of the analysis, numerous assumptions must be made and the question arises if in our concern for safety we have been too conservative in our estimates. That is, what level of risk are we willing to accept?

The above are some of the issues which may be addressed as decisions regarding specific enforceable effluent limitations made from model results.

Experience to Date with Water Quality Analysis

As we initiated this program, there was a realization that water quality analysis could not be performed simultaneously throughout the entire country if the legislative mandates contained in the statutes were to be achieved. As a first step, the nation's waters were divided into two kinds of segments: effluent limited segments, where established effluent limitations would achieve water quality standards; and water quality limited segments, where the effluent limitations alone would not achieve water quality standards. The most complex and challenging work is on the water quality limited segments. With the states providing the lead, some 2,000 water quality limited segments were identified. Our objective for these segments was to perform intensive water quality analyses, which were to establish the necessary levels of pollution control for these segments. The states have completed approximately 1,700 segment analyses since the initiation of this program. The remaining segments have more complex problems that require further time for analyses. Although the sophistication and technical expertise of the different states varies greatly, the last few years have seen an increased acceptance of the modeling approach for setting effluent controls.

Water Planning Division Case Studies

An analysis was performed using water quality models to set control levels for five water quality limited segments with severe dissolved oxygen problems. The purpose of the analysis was to study the effect of the differend methods of establishing the control levels including the effect of different waste load allocation procedures. Segments were chosen to represent a variety of hydrologic and waste-flow conditions. The segments are listed in Table III, along with the average yearly stream flow in cubic feet per second and the municipal and industrial contributions in millions of gallons per day.

The basic methodology employed in the analysis was to use a model verified for each stream segment (which in all cases was a Type B steady state, linear kinetic model) and perform a water quality analysis of the effect of daily waste-source inputs into historical stream-flow conditions. The analysis enabled us to identify the impact of various waste treatment plant options on the water quality levels.

The first step in the procedure was to determine the generalized daily variation of treatment plant effluents, derived from historical data from seventeen plants in Michigan and Texas. The treatment costs for these plants were based upon cost curves for treatment processes as a function of the waste flow. The costs included amortized construction, operation, and maintenance. The historical daily flow records for each segment were available from the Geological Survey for the last 22 years. Daily waste loads were generated and, with stream flows, information was used as inputs to the verified model to determine the impact on dissolved oxygen conditions. The next step in the analysis was to analyze different possible policies in terms of cost and effectiveness. The effectiveness of policies was evaluated in terms of the

TABLE III. SELECTED RIVERS FOR CASE STUDIES

Segment	Location of River	Cubic Feet Per Second	Municipal (million gals per day)	Industrial (million gals per day)
Chattahoochee	Georgia	3,543	108	--
Flint	Michigan	344	10	1
Cache La Poudre	Colorado	130	7	6
Reedy	South Carolina	84	15	7
Upper Mississippi	Minnesota	12,087	250	--

probability of meeting a stream standard of five milligrams per liter dissolved oxygen (DO). The three levels of control of waste sources were secondary treatment or best practical treatment (B.P.T.) for industry, best available treatment (B.A.T.) or a comparable level of municipal treatment, and a level of control that was determined by the necessity to meet the in-stream criteria of five milligrams per liter of DO. For each of the control levels, a total cost curve was derived.

From the results of the analyses, some conclusions may be derived. For these segments, the cost of B.A.T. treatment would be approximately equal to the total cost associated with determining the effluent control level based on the water quality standard. The total cost of the waste load analysis for these segments, based on a 7-day duration, 10-year recurrence critical flow condition, ranged from about 30% to 50% greater than the total cost of the secondary treatment effluent. At secondary treatment, the percentage of in-stream violations for these water quality limited segments range from 1.2% of the time on the Chattahoochee River to 18.8% of the time on the Flint River, while at the waste load analysis level, the percentage of in-stream violations is less than 0.5% in all cases.

A major limitation in the above study is that the analysis only considered the parameter of dissolved oxygen. It is realized that DO cannot be used as the sole criteria for determining the biological integrity of a water body. Aside from the fact that DO may not be the best indicator of the biological integrity, reliance on this parameter alone fails to indicate the associated control level of solids, metals, and other toxics that are derived from more advanced levels of treatment, and it is these parameters that are likely to cause many water quality problems.

National Commission on Water Quality Evaluations

The National Commission on Water Quality was established to assess the implementation of the nation's water bill. It also performed an analysis on the relationship between alternative control levels and water quality conditions. The Commission looked at 21 rivers covering approximately 4,600 river miles. At the present time, 380 river miles or about 8% of the total do not achieve the dissolved oxygen standard of 4 milligrams per liter. This mileage is decreased to 36 miles of 0.8% or the river miles studied through the application of secondary treatment or B.P.T., and 23 miles or less than 0.5% after the application of B.A.T. These results are demonstrated in Figure 2.

Summary and Conclusions

The above evaluation indicates the importance of water quality analysis in determining levels of control. Of the 5,500 water quality segments identified in the United States, approximately 2,000 were identified as having water quality problems. Models were applied in the majority of these cases and the control levels needed to meet water quality standards were established.

The case studies performed by the Water Planning Division and the findings of the National Commission on Water Quality confirm that higher levels of treatment control are not required in all places. Our estimate is that

35 to 40% of the nation's waters would require a higher level of control than provided by secondary treatment of base level industrial treatment requirements. The above case studies also indicate that where water quality problems exist, best available treatment (B.A.T.) will generally solve the problem.

Thus, we find that uniformly higher levels of waste treatment applied across the nation would not necessarily be the most effective way to proceed in the near term future. Appropriately applied modeling and water quality analysis will assist us in determining those areas where higher levels of treatment will be necessary as well as those areas where such treatment will not be required.

Concerning the above general conclusions, it must be noted that only dissolved oxygen was considered in the case studies, and it is generally recognized that reliance on dissolved oxygen alone may not be the best method for indicating the desirable level of control for a water body.

It should be further noted that the above case studies do not consider many of the nonpoint source problems, nor do they consider the problem of toxics and other heavy metals. The refinement of these tools and the expansion of our capability to handle both the transient problems introduced by storm water and wet weather flow problems, as well as the toxic elements, must be pursued if modeling techniques and water quality analysis are to be of further assistance to us in analysis. With explicit recognition of these additional problems, there is little doubt that we can enhance and improve our capabilities in this area.

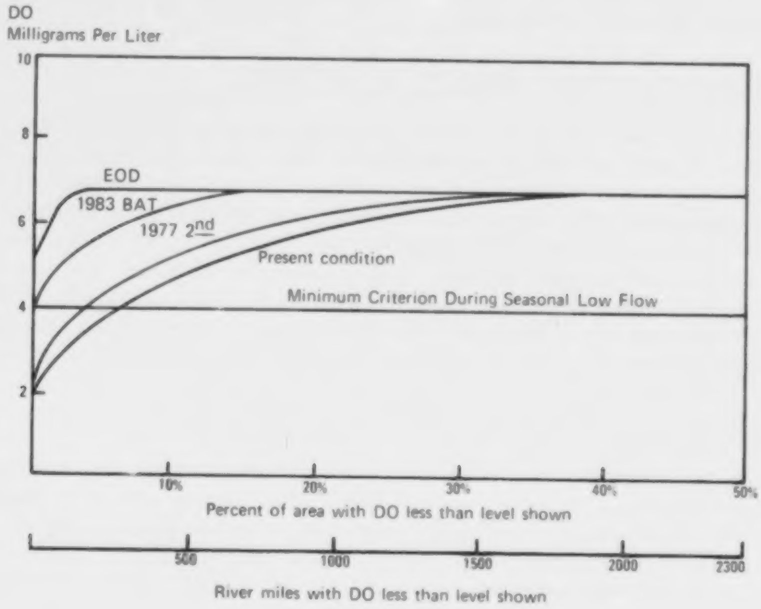

Figure 2. Dissolved oxygen improvement: 21 water bodies, 4600 river miles.

DISCUSSION

THOMANN: Mr. Pisano has just presented the various aspects of the researcher's responsibility to administrators. I want to use this tribunal to emphasize points which illustrate an administrator's responsibility to researchers:

1. Be sincere and listen to the recommendations we give you, even if you don't like them;
2. Relate to us with understanding. We also don't know everything, nor do we know how to do everything;
3. Be flexible in the use of your resources when introducing a policy recommended by the research community;
4. Realize that you are responsible for decisions made, not us;
5. Be magnanimous towards our mistakes.

DELUCIA: I want to add one point to Dr. Thomann's requirements: Give us enough time to fulfill the tasks before us. I want to remind Mr. Pisano, the economist, who has reproached researchers for the long continuance of their research, of the words of Mr. Pisano, the administrator, that a researcher should not err and his information should be sufficiently complete and detailed.

I would like to ask Mr. Pisano, the economist and administrator, to comment on the reasons for the improvement in water quality which has been observed over the last few years in the United States. In particular, I would like to hear about the role of descriptive, and especially prescriptive models. In addition, I would like to ask him to comment on the role of the United States' legislation and other aspects of influence emanating within the planning framework.

PISANO: I want to answer Dr. DeLucia's question from an administrative point of view. I already spoke about the complexity of the dilemma of choosing between the continuance of research and its infallibility. Everything in this world is relative; there are no absolutely good or bad decisions. All decisions, expressed when conducting determined reforms, possess a certain supply of stability regarding their relation to minor unexpected influences. Administrative personnel cannot understand all the nuances of modeling. Researchers should and can economize their time and efforts, indicating details and the basic results of decisions made, and determining the area of model application where the applied simplifications do not lead to mistakes. That is, the researcher should indicate to the administrator on what he should concentrate his attention when analyzing proposed decisions, what dependencies are significant in the model, and what are secondary.

In response to Dr. DeLucia's last request, the more we learn about the equations we are studying, the more we recognize the complexity of the problem of preserving the purity of the water environment and how insufficient are our efforts. Existing U.S. legislation, by special act of Congress, underwent an evaluation, from the point of view of effectiveness, using mathematical models. Dr. Thomann and Dr. DeLucia directly participated. I must note that we still know little about what is useful for such an evaluation. It is necessary to continue developments in this direction, and in the direction of changing the law.

Allow me, in conclusion, to express the hope that Soviet-American cooperation on this question will be useful for our country. I will emphasize several concrete areas of research which present considerable interest in this connection: development of optimization models, the fight against pollutants entering with unconcentrated flow, and evaluation of their influence.

SUKHORUKOV: I would like to make a few remarks on responsibility during the decision-making process. Each model is composed of three essential parts: 1) physical bases, 2) strict administrative limitations, and 3) criteria for evaluating the quality of the decisions. The responsibility of the researcher is dependent on the accuracy of his model. The administrator carries the responsibility for the limitations imposed by him. In addition, both must strive towards mutual understanding and implementation of the work.

Unfortunately, Mr. Pisano did not introduce into his presentation his classification of models by complexity. In the report, he proposed simple and quick models in small risk situations, but, for more complex and difficult cases, the preliminary research period can reach one to three years. It would be possible to develop the model's suitability for a specific case on the basis of the "information value method" which is well known in cybernetics.

BELOGUROV: In Mr. Pisano's report, two approaches to the problem of water protection were discussed. They both lead to a decrease in the discharge of polluting substances. In the Soviet Union, one more approach is being developed. It is connected with the redistribution of a given mass of pollutants dependent on the change in the assimilating ability of the water body. We call this water quality regulation. It is possible, for example, to regulate the discharge from tanks, to change the capacity of aeration facilities, and to change the graphics of discharge from water resources. It is even possible to change the level of wastewater treatment during the normal working processes of the treatment facility.

At VNIIVO, they are estimating the cost of decreasing the maximum concentrations of pollutants in a water body by decreasing the general mass of discharged substances and by redistributing the discharges. The results show that the latter is 4:11 times cheaper. In the Soviet Union, a lot of attention is given to this approach.

DETERMINING WATER QUALITY CRITERIA FOR WATER FLOW IN SOLVING THE PROBLEMS OF CONTROLLING WATER POLLUTION

Kh.A. Velner, V.I. Gurariy, A.S. Shayn

INTRODUCTION

In the Soviet Union a great deal of attention is paid to the protection of the environment from pollution, and in particular, to the problems of surface water quality. The improvement of water quality is considered one of the most important economic and political problems at the present stage of social development. The introduction of methods for the objective assessment of water quality could be an effective key factor in solving this problem.

To improve water quality, it is first necessary to define it quantitatively. The problems of organizing and introducing a state system of quality control that includes problems of planning, forecasting, optimization, and others must be solved by developing objective methods to evaluate water quality control.

Experience shows that information based only on water quality evaluation by individual properties does not provide an adequate solution to the problem. The necessity for developing a complex estimate of water quality is becoming more and more evident.

Laws, developed and promulgated in the Soviet Union, set maximum permissible concentrations (M.P.C. or water quality criteria) for harmful substances in impoundments and water courses, and define the water quality in a reservoir for drinking water, recreation, and fish breeding water uses (Cherkinskiy et. al., 1947, 1961, 1971, 1975).

However, some deficiencies have been found in these existing criteria for water quality assessment from the design and maintenance of water installations.

1. The M.P.C. is not a single determined value that corresponds to some threshold change in the quality of a substance, for example, the temperature threshold at which water boils. Therefore, the M.P.C. should be considered as an organizational artifact rather than a scientific criterion.

2. The M.P.C. has been defined for a large number of parameters, although they cannot be used to compare the qualitative conditions of water. Two qualitative conditions are identified within the limits of the criterion; for example, crystal-pure water and water at the limit of the criterion for acceptable water quality are very distinct, yet they have an identical quality

under M.P.C. concepts. However, two waters with similar water quality or with an index or parameter value outside the M.P.C. value are considered distinctly different.

M.P.C. criteria allows only opposing estimates: "yes" or "no," acceptable or unacceptable. The qualitative condition of water even slightly exceeding the M.P.C. in a single parameter is not considered. This can lead to unnecessary fiscal penalties for solving a number of economic problems for water pollution control.

3. In addition to limits for the value for each parameter, complex limits must also consider the effect of the combined presence of several substances. However M.P.C. concepts do not consider potential synergistic effects. These considerations are the most complex and require long-term complex investigations.

These considerations require the development of some new criteria, in addition to M.P.C., to guarantee the purity of water bodies. At the initiative of the Central Board of the Hydrometeorological Service of the Council of Ministers of the USSR (A. Israel), the problem has been formulated, and the development of the concept of a maximum pollutant discharge (M.P.D.) into watercourses and water bodies has begun. These M.P.D. (concurrently with M.P.C.) must be accepted later as state standards. It is evident that M.P.D. depends not only on environmental conditions, but to a great extent on technical economical prerequisites that may have particular problems pertinent to different pollution sources. The M.P.D. will be developed stage-by-stage on the basis of available technologies for pollution control in the state. The M.P.D. will have a complex character, i.e., to provide an integral evaluation of the quality of discharged waters.

Further, it is necessary to develop the maximum permissible biological or ecological degradation, the M.P.B., to evaluate the water quality of water bodies. The problem is complex and requires detailed investigation. At the discharge area for waste waters with primary treatment, the biological degradation in the water body should not exceed the permissible values $[\Delta B_i < (\Delta B_i)]$ so that the ecological equilibrium is maintained in the water body.

It must be emphasized that at present, the M.P.C. plays a very important role as a simple and convenient qualitative indicator of the permissible limit for contamination of water for sociolegal purposes. However, for a number of water economy problems (comparison of water quality, complex use and protection of water, water quality management, mapping of watercourses, and various scientific studies), it is necessary to develop additional criteria with a more comprehensive evaluation range.

The concepts of water quality criteria have a long history, but only recently both Soviet (Drachev and Bylinkina, 1964; Velner and Saava, 1970; Mitskene, 1974; and Gurariy and Shayn, 1974) and foreign authors have made attempts to evaluate water quality by a pollution index number to indicate total quality. The author is familiar with the publications of the American authors in the field (Brown, 1970, 1972; Harton, 1965; Prati, 1971; Markins, 1974; and others). Initially, Brown (1970) advocated developing a single global index of water quality; but he recently concluded that water quality indices should be developed for different water uses (1972).

The essential problem, when developing complex estimates of water quality, is the choice of the primary indicators that scale the indices and methods of aggregating estimates.

Depending on the problem formulation and the objectives of the investigators, sets of factors (5 in Drachev, 1964, to 13 in Prati, 1971) and systems of evaluation will differ. Bylinkina et al. (1964) were among the first to propose a complex evaluation of water quality in a water body by the physical, chemical, bacteriological, and hydrobiological pollution indicators. This evaluation has quantitative indicators developed in detail for each parameter. The proposed classification underlies the surface water purity criteria approved by the Council of Chiefs of Water Management Agencies of the CMEA in 1963.

Great attention is paid to the evaluation of water quality in Poland, Czechoslovakia, the German Democratic Republic, and Hungary. These evaluations are performed by using groups of categorized indicators (by the worst in a given group, by the average, etc.), and different types of classifications including from four to six classes.

The existing evaluation criteria represent a set of methods that are almost always badly substantiated, although effective in some cases. Their essential disadvantage is the subjectivism often exercised in the choice of the main indicators.

Therefore, a theoretical basis should be developed for complex evaluations of water quality to provide a unified approach to the problem and reduce the element of subjectivism as much as possible.

In general, the main problems of the criteria theory relate to:

1. describing the n-dimensional area Ω of acceptable water quality conditions $(X_1, \ldots X_n)$;
2. choosing the class of evaluation K (in the general case K is the set of vectors with a dimension less than n);
3. formulating the evaluation itself $U(\overline{X})$ depending on the kind of water usage V, i.e., determining the operator $U(\overline{X})$, which maps Ω into K, $\Omega \xrightarrow{U_V(\overline{X})} K$.

Criteria developed for evaluating water quality can solve the problem of comparing N qualitative conditions of water by n-indicators. Let us assume that each of the main factors can be evaluated during successive water quality examinations; then each qualitative condition of water can be determined as a point in n-dimensional space.

The problem therefore is to establish the best water quality conditions, the next best, and so on, to provide an ordering of the objects under investigation. To solve this problem, it is necessary to obtain a general estimate of the water quality on the basis of evaluations by separate factors (in other words, to produce an aggregation of estimates).

The solution is determined by using Fishburn's (1968) theory of utility. This approach assumes the existence of a single scalar value function $U(\overline{X})$ for condition evaluations by factors that can produce a complete ordering of objects. Function $U(\overline{X})$ substitutes a scalar estimate for each set of factors (water quality index, W.Q.I.). The ordering problem is then simply solved. In this present paper, as in previous studies, it is generally accepted that the function $U(\overline{X})$ exists for any formulation of the decision-making problem. At the same time, there are a large number of opinions and theories on this type of function, i.e., the method for aggregating estimates.

The application of separate methods for aggregating estimates is no substitute for the decision-making exercise, but is used only to acquire additional information. This fact is determined by the very character of the problem.

The purpose of aggregating the estimates is to attempt a rational solution of a problem that is difficult to formalize.

Generally speaking, the decision-making procedure is not purely mathematical in character, but has certain psychological attributes. Recent methods of aggregating estimates take these aspects into account (Byunayun et al., 1971). As their principle feature, these promising methods require that the person defining the problem and using the results (a person responsible for the decision) is engaged in some way in the process of aggregating the estimates.

Complex estimates of water quality are very complicated to determine, and their strict and complete formulation is impossible at this time. The immediate problem is to reduce subjectivism to a minimum at all stages in the development of a complex estimation.

Velner, Kask, and Saava, specialists at the Tallin Polytechnical Institute, together with members of the Ministry of Soil Reclamation and Water Management of the Lithuanian SSR (Daubaras and Mitzkene), proposed a system of pollution classification for water bodies that has been accepted for the Baltic region. The water quality evaluation is carried out by three groups of indicators: the first evaluates the general sanitary state of the water body; the second, the presence of harmful substances; and the third, the level of pollution by microbiological indicators. The water quality, determined during the rated low runoff conditions of a river, follows the accepted standards and the improved classification of river waters of the Baltic region, polluted mainly by organic substances without a marked toxic effect.

For each group of indicators, the critical parameter which furthest exceeds the M.P.C. is determined. The integral quality of water of a river (an index) is determined by the estimated factor of the critical group, taking into account the category (kind) of water usage (Table I). This "limiting condition method" gives a strict estimate of water quality. It was tested by the Ministries of Water Management of the Lithuanian and Estonian SSR in an evaluation of the Baltic region rivers.

TABLE I. APPROXIMATE CLASSIFICATION OF WATER QUALITY IN WATER BODIES

Pollutant group	Quality factors	Measurement units	Level of water quality				
			Clean 5	Slightly polluted 4	Moderately polluted 3	Polluted 2	Very polluted 1
Organic pollutants (I)	Dissolved oxygen	mg/ℓ	≥ 6.0	≥ 4.0	≥ 3.0	>1.0	<1.0
	Ammonia	mg/ℓ	≤ 0.1	≤ 0.2	≤ 0.5	<1.0	>1.0
	BOD at 20°C	mg/ℓ	≤ 2.0	≤ 5.0	≤ 8.0	<20.0	>20.0
	Dichromate oxidation	mg/ℓ	≤ 26.0	≤ 32.0	≤ 40.0	<65.0	>65.0
	Saprobic zones	--	mostly oligo-saprobic	mostly saprobic	from mesosaprobic to αmesosaprobic		from αmeso to poly-saprobic
Toxic substances (II)	Phenols, oil products, cyanides, mercury, etc.	ratio of actual to standard maximum permissible concentration	<0.1	<1.0	<5.0	<10.0	>10.0
Pollution introduced by micro-flora (III)	Coli-titre, number of colonies, etc.	units/mℓ	≥ 1.0 $<10^3$	$>0.1-1.0$ $<10^4$	$>0.01-0.001$ $<10^5$	>0.001 $<10^6$	≤ 0.001 $\geq 10^6$

Gurariy and Shayn (1974) in VNIIVO are conducting research on the development of methods for constructing water quality indices for separate kinds of water usage. The W.Q.I. for municipal water usage has been developed. Its analytical-expert method adequately eliminates the subjectivism in water quality estimation inherent in other methods and takes into account all available information on the values of the parameters (not only the worst indicator).

The development of the W.Q.I., as well as other complex quality estimations, is divided into two stages: 1) estimation of the simple properties; and 2) estimation of the complex properties and the quality as a whole.

The algorithm for obtaining the water quality index takes the form:

STAGE I

Determination of water usage conditions

↓

1. Construction of a hierarchical diagram of water indices necessary and sufficient for estimating its quality

↓

2. Designation of the measurements units for each index

↓

3. Selection of basic values of indices for comparison

↓

4. Determination of the type of dependence between the index values and their estimation

↓

5. Computation of estimates of separate indices

STAGE II

6. Selection of the dimension scale for a complex estimate

↓

7. Selection of method for determining the weighting of the indices

↓

8. Selection of method for aggregating estimates of separate indices to obtain complex quality estimates

↓

9. Computation of a complex water quality estimate

↓

10. Analysis of computed quality estimate and decision-making

Each operation of the algorithm was performed separately. In considering operations 1, 2, 4, and 7, the expert estimation method was widely used.

The complex estimate of water quality is constructed on the basis of two indices: a general sanitary index including general sanitary parameters essential for judging the water quality and, partially, the river regime, and an index of specific polluting substances that takes into account harmful substances present in the water.

VNIIVO's mathematical modeling laboratory distributed a questionnaire among experts (the inquiry was carried out in two steps: the second step corrected the results of the first). In the first questionnaire, a group of 50 water pollution experts from the country's leading institutions selected the most significant parameters, defining river water for municipal water usage, from a list of 38 parameters (general sanitary indicators and water body pollutants) and evaluated each according to a five-point system (5-max, 1-min).

The questionnaires were processed: The mean statistical value for each parameter, the frequency of possible maximum estimates, sum of ranks, and also the probability characteristics, variance coefficient, concordance coefficient, and the statistical significance of the level of agreement among the experts were determined. Proceeding from the mean statistical analysis, ten general sanitary factors (Table II) and oil and phenol--the most widespread pollutants--were chosen as the most significant indices.

TABLE II. WATER QUALITY ESTIMATES

		Point, W				
Parameter	Weight γ	5	4	3	2	1
Coli-index	0.18	0-100	100-1000	10^3-10^5	10^5-10^7	>10^7
Sign,points	0.13	0	1-2	3	4	5
BOD_5, mg/ℓ	0.12	0-1	1-2	2-4	4-10	>10
pH	0.10	6.5-8	6.5-8.5	5-9.5	4-10	<4>10
Dissolved oxygen, mg/ℓ	0.09	<8	8-6	6-4	4-2	<2
Color	0.09	<20	20-30	30-40	40-50	>50
Suspended substances, mg/ℓ	0.08	<10	10-20	20-50	50-100	>100
General mineralization, mg/ℓ	0.08	<500	500-1000	1000-5000	1500-2000	>2000
Chlorides, mg/ℓ	0.07	<200	200-350	350-500	500-700	>700
Sulphates mg/ℓ	0.06	<250	250-500	500-700	700-1000	>1000

The rank correlation coefficient (concordance coefficient) computed by the results of the first questionnaire (it is an index of the experts' agreement level) had the value $W=0.725$ and was indicative of good agreement ($W>0.5$). Confidence in the experts' agreement level was tested by the λ^2 criterion. Calculations showed that the estimates of the experts sufficiently coincided and the results of the inquiry were authentic.

The weight of the chosen parameters was calculated from the formula

$$\gamma_i = \frac{1}{S_i} \frac{1}{\sum\limits_{p=1}^{n} S_p^{-1}} \tag{1}$$

where S_i is the sum of ranks set by the experts for the i-th parameter.

$$S_i = \sum_{j=1}^{m} \gamma_{ij} \tag{2}$$

where γ_{ij} is the estimate rank set for the i-th parameter by the j-th expert, $j=1,2,\ldots,m$,
m is the number of experts.

The results from the second set of questionnaires confirmed the results of the first set and allowed the authors to establish the dependence of the water quality estimates on the pollutant concentration. The water quality scale is a five-point system: 5--corresponds conditionally to very clean water, 4--clean water, 3--moderately polluted, 2--polluted, 1--contaminated. Each interval of values in the columns is considered as a semi-interval of the type (a,b).

For normalized parameters, the superior limits of the interval corresponding to point 4 are the existing standards for municipal water usage.

To account accurately for parameter values exceeding standards (which corresponds to point values of $W_i=3$ and less), we also introduce the so-called "penalty" function

$$F = \prod_{i=1}^{n} \eta_i(W_i, \gamma_i).$$

The value η_i should decrease with the decrease of W_i. With equal quality levels W_i, the value η_i should decrease if the relative weight γ_i increases. In addition, the function should be normalized (the values should not exceed a unit). It is convenient to take function η_i as:

$$\eta_i = \begin{cases} \dfrac{\gamma_i}{\gamma_k} \dfrac{\sqrt{W_i}}{2} + \left(1 - \dfrac{\gamma_i}{\gamma_k}\right) & W_i \leq 3 \\ 1 & W_i > 3 \end{cases} \tag{3}$$

where γ_k is the relative weight of the coli-index maximum for all γ_i.

Thus, the formula for the general sanitary index T_a is of the form:

$$T_a = \left[\sum_{i=1}^{n} \gamma_i W_i\right] \prod_{i=1}^{n} \eta_i (W_i, \gamma_i)$$

$$0 < T_x \leq 5; \quad \sum_{i=1}^{n} \gamma_i = 1; \quad 1 \leq W_i \leq 5; \tag{4}$$

where γ_i is the relative weight of the i-th factor;
W_i is the point evaluation of water quality for the i-th factor;
n is the number of factors; and
η_i is calculated from equation 3.

In constructing the general sanitary water quality index, discrete quality estimates were used (the concentration range of the parameters was divided into intervals).

To account accurately for the polluting effect of impurities, the continuous dependencies of the pollutant level on the oil and phenol concentration were obtained. The curves were constructed by reference points with condensation points obtained from processing questionnaires and reference data.

$$q_H = \frac{3}{1 + 0.42\,(C_H - 1) + 0.02\,(C_H - 1)^2} \tag{5}$$

$$q_F = 1 - l_q C_F \tag{6}$$

where q_H, q_F are the point estimates of water quality depending on the concentration of oil and phenol;
C_H, C_F are the values of the concentration of oil and phenol in water.

Cherkinskiy (1971) showed that the effect of the combined presence of toxic substances in water, normalized by organoleptic properties, is equal to the sum of effects of each of them. Consequently, the pollution index takes the form

$$T_z = \begin{cases} q_{min} \sqrt[3]{\dfrac{q_H\, q_F}{5}} & q_H,\ q_F < 5 \\ q_{min} & q_H \text{ or } q_F = 5 \end{cases} \tag{7}$$

If the number of ingredients in the pollution index normalized by one toxic factor is equal to n, then generally,

$$T_z = q_{min} \sqrt[3]{\frac{\prod_{i=1}^{n} q_i}{5^n}} \qquad i = 1, \ldots, n \tag{8}$$

Thus, the estimation of water quality leads to the determination of two indices, a general sanitary index and a specific pollutant index, which are considered independent characteristics. The water quality index is represented by a two-dimensional vector. To compare different qualitative water conditions, it is necessary to solve the vector comparison problem. For this, the range of possible values of each component is divided into a number of intervals according to Drachev's classification (1964).

Five water quality classes are determined: very clean, clean, moderately polluted, polluted, and contaminated. Each class has its corresponding range of pollution and general sanitary indices and associated possibilities for using water for specific purposes. By using the independence of the indices, zones of possible values are divided into a number of rectangular areas (Figure 1); five water quality zones represent the transition from bad to better quality. The W.Q.I. is denoted by two numbers $T = (N, l)$. The first number N is the number of the zone in which the end of vector $\vec{T}(T_x, T_z)$ is found. It represents the possible water use; the second, l, is equal to the length of vector $\vec{T}$ and is necessary only for comparing water quality within a zone.

From the water quality estimates, it is possible to evaluate the potential use of water of the given class for other types of water utilization. Thus far, no W.Q.I.'s have been developed for these types.

An example (Table III) was constructed on the basis of existing standards and recommendations.

This complex evaluation of water quality can be conveniently used to determine the water preparation required for different types of water consumption, to estimate program efficiencies for water pollution prevention, to manage water quality, to map rivers, and to compare water quality of different water bodies.

Water quality management can be defined as the requirement to sustain the necessary level of the W.Q.I. for a given type of water usage. The water quality index for municipal water usage was determined by VNIIVO to evaluate the quality and map the Dnieper, Severskiy Donets, and the Moskva Rivers.

CONCLUSION

1. The development of criteria for water quality evaluation is an urgent social and national economic requirement.
2. At present, we lack a common system of water quality indexing.
3. The application of analytical-expert methods allows establishment of a common approach to the problem of developing water quality criteria for water management systems.

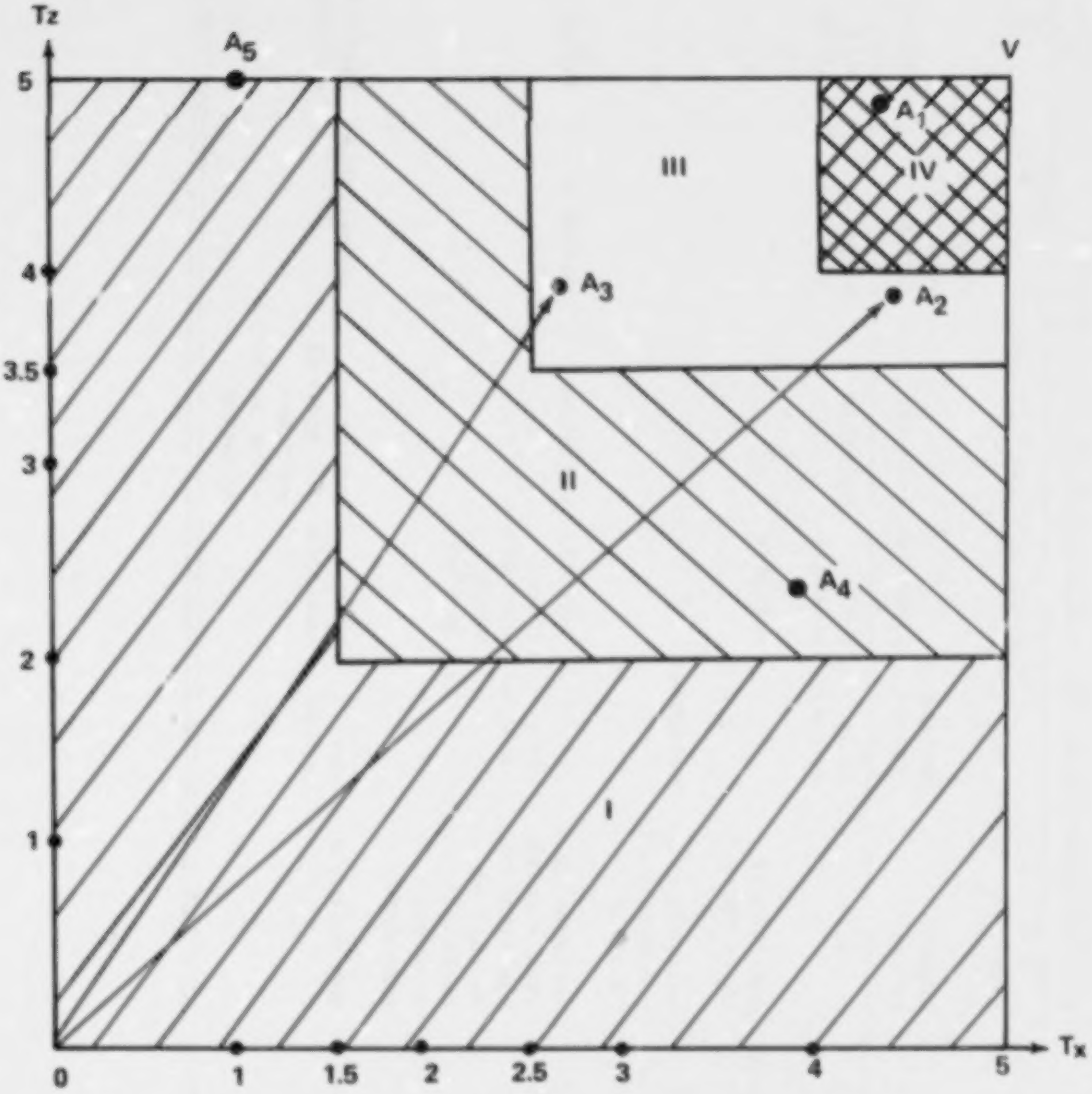

Figure 1. Water quality index (indicated by Roman numerals). Water quality indices are derived from a general sanitary index T_x, and a specific pollutant index T_z.

TABLE III. EFFECT OF POLLUTION OF THE POSSIBILITIES OF USING WATER

			Kinds of water consumption					
Water quality index	T_x	T_z	Domestic potable	Bathing sports	Fisheries	Industry	Transport port installations	Agriculture
Very clean 5	5	5	suitable	fully suitable	fully suitable	suitable	fully suitable	fully suitable
Clean 4	4-<5	4-<5	suitable if chlorinated	"	suitable	"	"	"
Moderately polluted 3	2.5-<4	3.5-<4	suitable after standard purification	suitable	"	"	"	"
Polluted 2	1.5-<2.0	2-<3.5	suitable only with special purification in case of technico-economic expediency	utilization doubtful	suitable except for valuable species	suitable	suitable	suitable
Contaminated 1	<1.5	<2.0	not suitable	not suitable	not suitable	suitable for special purposes after purification	undesirable	limited utilization

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DISCUSSION

PISANO: What criteria are used to establish the maximum permissible concentrations of substances in a water body and to determine the use of the water body? How do you determine the value of a certain parameters, or whether one parameter is equal to or is more important than another?

SHAYN: To establish a water quality index, we use an analytic-expert approach. Processing the expert's data and literature allows us to establish a ranking to the chosen indices. In developing criteria for evaluating water quality, we produce an evaluation which is applicable to an already available specific water use. The M.P.C., for specific types of water use, already exists in the USSR.

LOZANSKIY: I want to add the following: Determining the type of water use for a given water body or separate parts of it is an administrative decision. Authorized government agencies for water regulation, use, and protection from the USSR Ministry of Land Reclamation and Water Use are especially concerned with this. Unfortunately, these organizations have still not constructed a scientific basis for determining water use and for establishing limitations on certain water uses. They proceed from general impressions and the situation at a given water body. They try to satisfy all the requirements within a given water basin from the point of view of all the water users.

THOMANN: Why does your water quality index include BOD? Theoretically, we should examine the content of dissolved oxygen. Should the index be recalculated each time a new substance enters the water body?

BARTSCH: Is there any research in the USSR which is concerned with the problem of the supersaturation of oxygen in water used by the fisheries?

SHAYN: Most specialists agree that BOD must be included in the water quality index. Discussions have confirmed that this parameter is not superfluous. A limited number of parameters are water quality indices (ten general health and two more which characterize specific pollutants). Therefore, if a new pollutant appears in the river, the general method for constructing the index allows us to introduce this parameter into the index.

FUKSMAN: Did you calculate statistical characteristics for evaluating water quality?

SHAYN: Yes we did. Probability and statistical characteristics, their average statistical value, dispersion, and the correlation coefficient were calculated.

LOZANSKIY: The problem of oxygen supersaturation has not been identified in the USSR. Perhaps we have not sufficiently studied this question. Our American colleagues can obtain more detailed information from representatives of the Institute of the Biology of Inland Waters present at the symposium.

RODZILLER: I want to comment on several aspects of the report: First, the role of the water quality parameters--BOD. I was among those experts who answered VNIIVO's questions. They unanimously counted BOD among the most important water quality parameters. The health agencies and fish protection agencies gave it a high value, since BOD characterizes the potential for removing oxygen from a water body. In the USSR, when analyzing industrial waste waters, it is imperative that full BOD be determined, because any limited BOD measure is not adequate.

MATHEMATICAL MODELING OF EUTROPHICATION PROCESSES IN LAKE ONTARIO

Robert V. Thomann

INTRODUCTION

Purpose of Research

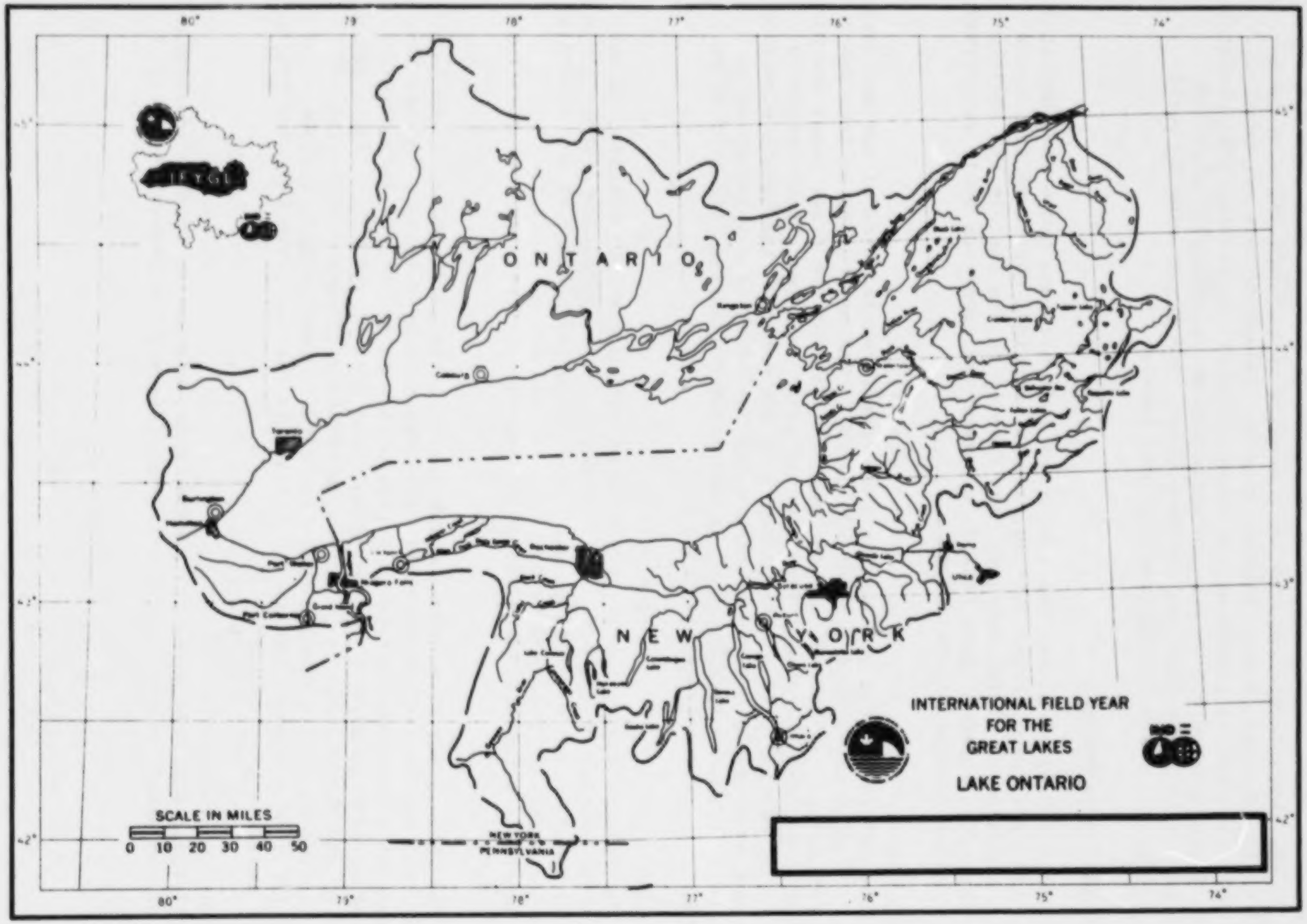
The overall purpose of this research is to structure a mathematical modeling framework of the major features of eutrophication in large lakes. Lake Ontario, the subject of intensive field work as part of the International Field Year for the Great Lakes (IFYGL), is used as the problem setting (Figure 1). The overall objectives of the research include:

1. determination of important interactions in lake eutrophication;
2. analysis of lake water quality and biological responses to natural and man-made inputs; and
3. formation of a basis for estimating the direction of change to be expected under remedial environmental control actions.

The problems of impairment of the quality of lake systems are magnified for "large lakes" such as the Great Lakes. The size of these lakes is such as to preclude any immediate improvement in quality after control actions are taken. Further, it is much more difficult to obtain reliable data on water quality, biological structure and hydrodynamic circulation, again because of the difficulty of sampling large lake systems. The general circulation itself may not be adequately known in relation to climatological and hydrological factors. In the biological area, measures of phytoplankton populations are temporally and spatially dependent and reflect complex interactions with nutrients, such as nitrogen and phosphorus, and upper trophic level predation. This report reviews work accomplished to date on development of a mathematical model of phytoplankton in Lake Ontario that hopefully will form a reasonable basis for estimating the effects of nutrient removal programs.

Scope of Research

The scope of this research is therefore lakewide and attention is directed primarily to the behavior of the phytoplankton for the lake as a whole. The smallest scale considered in the more advanced model (Lake 3) is on the order of 10-40 km. Detailed nearshore behavior on a lesser scale is not a part of this research. Furthermore, this research is concerned with phytoplankton dynamics as described on a time scale of weeks to months. Therefore the seasonal progression of the phytoplankton throughout a year and from year-to-year is the time scale of interest. Short-term fluctuations, in the order of hours or days, are not described here.


80°
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O N T A R I O
N E W Y O R K
NEW YORK
PENNSYLVANIA
INTERNATIONAL FIELD YEAR
FOR THE
GREAT LAKES
LAKE ONTARIO
SCALE IN MILES
0 10 20 30 40 50

Figure 1. General basin map of Lake Ontario.

Finally, the modeling structure developed as part of this research is aimed at the phytoplankton biomass as a measure of eutrophication and associated water quality. Rooted plants, such as cladophora, are not modeled. Particular emphasis is placed on the interaction of the passive phytoplankton biomass (as characterized by the concentration of chlorophyll) with the nutrients, principally nitrogen and phosphorus, and the grazing by herbivorous zooplankton and higher order trophic levels.

Lake Ontario--Background

With the Niagara River, the outflow of Lake Erie and hence the cumulator of all the upper Great Lakes outflow, as its principal source of inflow and the St. Lawrence River its outflow, Lake Ontario is the last in the chain of Laurentian Great Lakes. Lake Ontario's narrow and deep rock basin was formed by the action of glacial corrosion.

Morphometry--

Lake Ontario is the smallest of the Great Lakes in terms of surface area, 19,477 km^2 with a drainage area of 90,132 km^2. The mean depth of Lake Ontario, 90 meters, is second only to Lake Superior for the Great Lakes and can be considered one of its most important physical features. Lake Ontario's volume is 1,669 km^3 with a maximum depth of 244 meters. The lake's elevation above sea level is 74.01 meters which makes its depth of cryptodepression 170 meters. Lake Ontario's length is 307 km with a maximum width of 87 km. The lake's shoreline length is 1,380 km.

Hydrology--

The Niagara River, the major source of inflow for Lake Ontario, is the cumulative outflow of the other Great Lakes. The average flow of the Niagara River is 5,520 km^3/sec., and accounts for 84 percent of the flow discharged via the St. Lawrence River. The average annual precipitation on Lake Ontario's water surface is 83.28 cm and the average annual evaporation is 71 cm.

The thermal bar development which precedes the development of the full thermocline begins in late April or early May in Lake Ontario. The offshore movement of the thermal bar is such that within 17 to 28 days the nearshore ring of stratified water covers over half the area of the lake. The average depth of the thermocline is 17 meters, with its dissipation beginning in late September. The hydraulic detention time of Lake Ontario (volume divided by flow) is 8.1 years and is significant in that it gives some indication of the response time of the lake.

Nutrient Inputs--

Nitrogen and phosphorus are used as nutrients in the present configuration of the model, though silica will most likely be incorporated in the future.

Nutrient loads for Lake Ontario are shown in Table I.

TABLE I. NUTRIENT LOADS

Source	Phosphorus Kg/day	%	Nitrogen Kg/day	%
Niagara River	19,100	56	236,900	59
Tributaries	7,100	21	86,600	22
Municipal	7,300	22	32,900	8
Industrial	500	1	44,100	11
Total	34,000	100	400,500	100

Lake Ontario Model Structure

Theoretical Background--

The basic theory for the Lake Ontario model makes use of previous model structures (DiToro et al., 1971; Hydroscience, Inc., 1933; Thomann et al., 1974) and essentially represents a mass balance around each ecological or nutrient compartment and around physical space.

The basic structure consists of three parts:

1. transport and dispersion sub-system,
2. biological subsystem, and
3. chemical subsystem.

The latter two subsystems represent the kinetic behavior or the extent and complexity of interactions between relevant variables, while the first system represents the physical processes of water movement and mixing. The theory of each subsystem is presently developed to a different degree, with the transport and dispersion theory developed to a more advanced degree than the biological and chemical systems.

The general equation which results from applying the principle of the conservation of mass is:

$$\frac{\partial s_k}{\partial t} = -\frac{\partial}{\partial x}(us_k) - \frac{\partial}{\partial y}(vs_k) - \frac{\partial}{\partial z}(ws_k) + \frac{\partial}{\partial x}\left(E_x \frac{\partial s_k}{\partial x}\right) + \frac{\partial}{\partial y}\left(E_y \frac{\partial s_k}{\partial y}\right) + \frac{\partial}{\partial z}\left(E_z \frac{\partial s_k}{\partial z}\right) \pm S_k(x,y,z,t,s_\ell,s_k) \pm W_k(x,y,z,t): \quad k = 1..m, \; \ell = 1..m \qquad (1)$$

where s_k is the kth dependent variable (biological or chemical), u, v, w are the velocity components in the x, y, and z directions, respectively, E_x, E_y, and E_z are the dispersion coefficients in each respective spatial direction, S_k represents the kinetic interactions between the k variables and W is the direct inputs of the substance, s_k. It should be noted that Equation 1, as written, does not include any interaction between the variables s_k and the transport and dispersion regime. Thus, the physical system is separable, in the sense that it can be externally supplied and, once determined, it can be used for all variables. In vector form, Equation 1 is:

$$\frac{\partial s_k}{\partial t} = (\nabla . [E] (\nabla s_k) - \nabla . \vec{U} s_k) \pm S_k \pm W_k \qquad (2)$$

where [E] is a diagonal matrix of dispersion coefficients, (s_k) is a column vector, U is the velocity vector and

$$\nabla \equiv \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k}$$

In Equation 2, the first term in parenthesis on the right-hand side represents the dispersive and advective field in three dimensions, as given by the general circulation. In this work, it is assumed that the circulation is "known," either through observation or from output from a hydrodynamic model. The inputted circulation can be validated by utilizing Equation 2 for a tracer, such as dye or water temperature, to compare computed results to observed data. Water temperature has been used in some of this work and the results of that analysis are given in the Lake 1 model. Primary interest therefore centers on the reaction kinetics, their functional forms, and numerical values.

A finite difference approximation to Equation 2 is necessary to apply the equation to an actual water body. Thus, if the lake is divided into a series of finite, completely mixed volumes, n in number, and each with volume V_j, then it can be shown that:

$$[V] \frac{d(s)_k}{dt} = [A] (s)_k \pm (S)_k \pm (W)_k \qquad (3)$$

where [V] is an n x n diagonal matrix of volumes, $(s)_k$ is an n x 1 vector of the variable s_k; [A] is an n x n matrix of advection and dispersion terms; $(s)_k$ is an n x 1 vector of kinetic interactions; and $(W)_k$ is an n x 1 vector of inputs of variable s_k. For m interacting variables and n physical locations, Equation 3 indicates that a total of mxn equations must be solved. In general, the equations are both linear and non-linear.

When one recognizes that initially up to 10-15 interacting variables can be specified and that 50-100 initial spatial segments may be required, it is obvious that the computer program and data preparation necessary to obtain solutions are of a substantial magnitude. Accordingly, prudent practice would dictate a modeling strategy that allows one to proceed sequentially from simple models to the more complex. Such a strategy is utilized in this work and is illustrated in Figure 2.

As indicated, two parallel paths are being followed. The upper path involves the gathering of data on the lake geomorphology and transport and dispersion structure. The model is a single variable model of water temperature or chlorides both representing tracer variables that can be used to validate

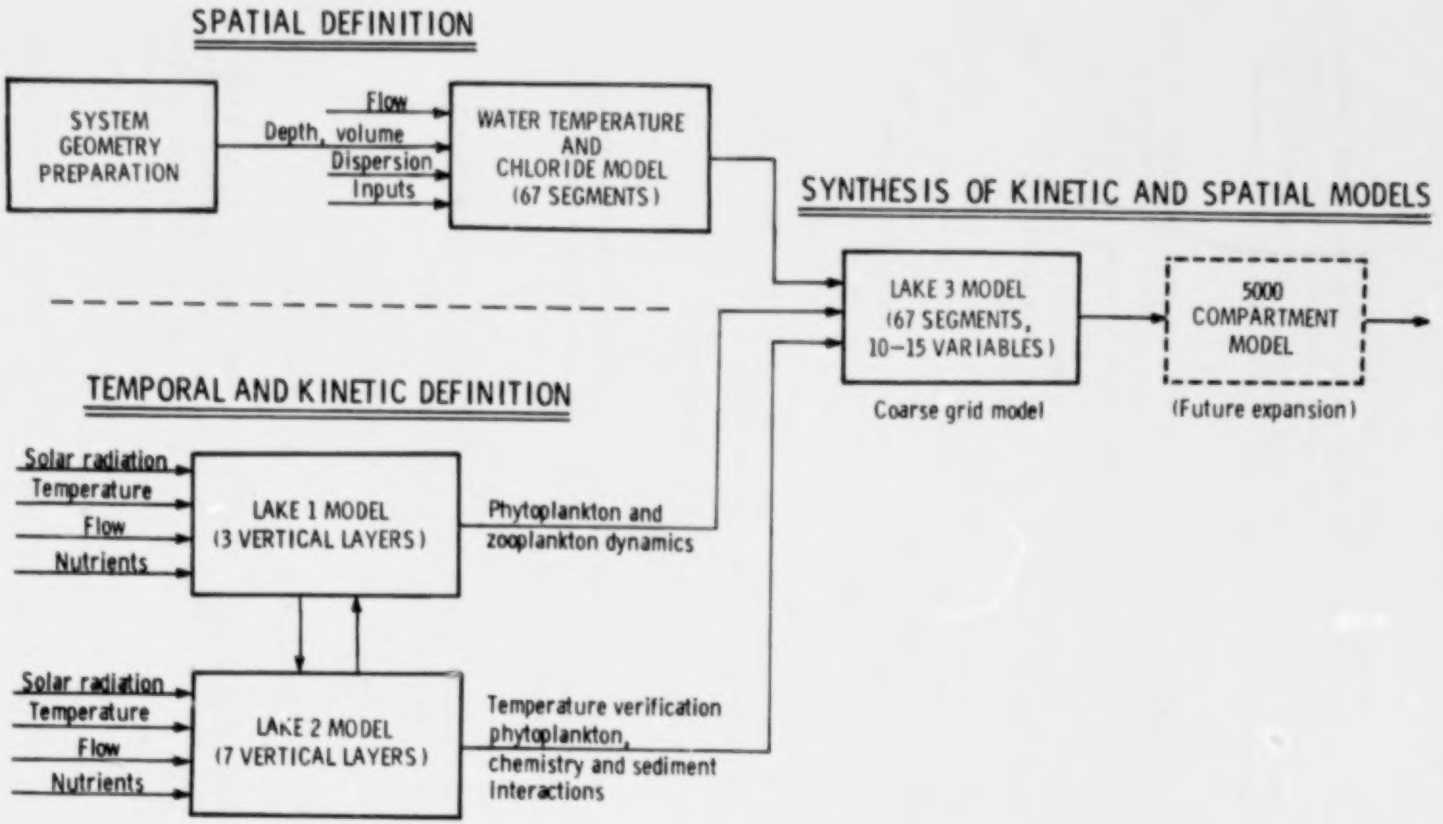
SPATIAL DEFINITION
SYSTEM GEOMETRY PREPARATION
Depth, volume
Flow
Dispersion
Inputs
WATER TEMPERATURE AND CHLORIDE MODEL (67 SEGMENTS)
SYNTHESIS OF KINETIC AND SPATIAL MODELS
LAKE 3 MODEL (67 SEGMENTS, 10–15 VARIABLES)
Coarse grid model
5000 COMPARTMENT MODEL
(Future expansion)
TEMPORAL AND KINETIC DEFINITION
Solar radiation
Temperature
Flow
Nutrients
LAKE 1 MODEL (3 VERTICAL LAYERS)
Phytoplankton and zooplankton dynamics
Solar radiation
Temperature
Flow
Nutrients
LAKE 2 MODEL (7 VERTICAL LAYERS)
Temperature verification phytoplankton, chemistry and sediment interactions

Figure 2. Eutrophication modeling strategy.

various sectors of the inputted circulation. A three-dimensional grid of 67 segments is employed.

The emphasis in the lower parallel path is on the subsystem kinetics with spatial detail held to a minimum. The lake is assumed to be horizontally completely mixed and is divided into a series of vertical layers. Attention is specifically directed to the interactions between phytoplankton, zooplankton, and water chemistry. Two models, Lake 1 and Lake 2, are explored in this path.

The synthesis of the spatial definition of transport and mixing with the kinetic interactions is accomplished in Lake 3. This model, a coarse-grid model, is three-dimensional and incorporates the major kinetic features of the Lake 1 and 2 models. With 67 segments and 10 variables, a total of 670 equations or "compartments" must be solved. Future expansion calls for an approximate order of magnitude increase in the number of compartments to 5000.

In this report, primary emphasis is placed on the Lake 1 model. As such, a detailed review of the kinetics employed in the models is given below for the biological and chemical subsystems. The systems diagram for the Lake 1 model is shown in Figure 3. The Lake 2 model includes those variables for Lake 1 and also includes a representation of the carbon cycle.

Phytoplankton Chlorophyll--

The basic kinetic interactions, for phytoplankton chlorophyll (a measure of biomass), is given for a single volume, j, as:

$$S_{P_j} = V_j \, (G_p - D_p)_j P_j - V_j \frac{w_{ji}}{H_j} P_j + V_i \frac{w_{ij}}{H_i} P_i \tag{4}$$

where P is the phytoplankton chlorophyll (μg/ℓ), G_p and D_p are the growth and death rate (1/day) respectively and w_{ji} and w_{ij} are the sinking velocities of the phytoplankton between segments j and i.

Phytoplankton Growth Rate--

The growth rate expression is similar to that developed previously (DiToro et al., 1971) and as used in this work is given by:

$$\begin{aligned} G_{pj} &= G_1 \, (G_{max},T) \cdot [I(I_a,I_s,k_e,P,H)] \cdot ([N'(K_{mn},N_2,N_3)] \, [(K_{mp},P_2)]) \\ &= \text{(Temperature effect)} \cdot \text{(Light effect)} \cdot \text{(Nutrient effect)} \end{aligned} \tag{5}$$

where G_{max} is the maximum growth rage, T is water temperature, I_s is light intensity at G_{max}, I_a is incoming solar radiation, k_e is the extinction coefficient, H is the depth of the segment, k_{mn} is the half-saturation constant for total inorganic nitrogen, N', N_2 is ammonia nitrogen, N_3 is nitrate nitrogen, K_{mp} is the half-saturation constant for phosphorus, and P_2 is the available phosphorus. The temperature effect is given by Eppley (1972) as:

$$G_1(T) = G_{max} \, (1{,}066)^T \tag{6}$$

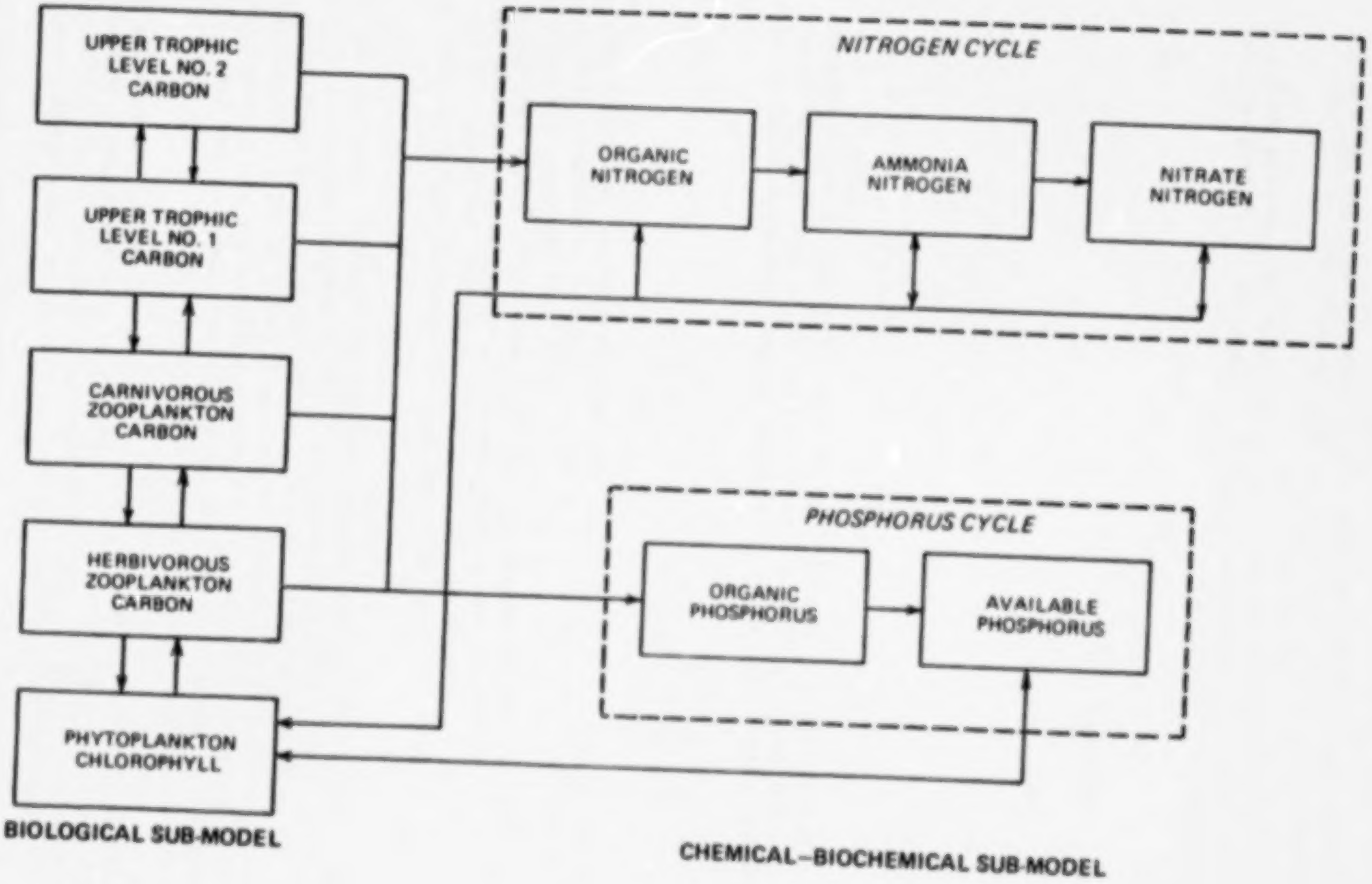
UPPER TROPHIC LEVEL NO. 2 CARBON
UPPER TROPHIC LEVEL NO. 1 CARBON
CARNIVOROUS ZOOPLANKTON CARBON
HERBIVOROUS ZOOPLANKTON CARBON
PHYTOPLANKTON CHLOROPHYLL
BIOLOGICAL SUB-MODEL
NITROGEN CYCLE
ORGANIC NITROGEN
AMMONIA NITROGEN
NITRATE NITROGEN
PHOSPHORUS CYCLE
ORGANIC PHOSPHORUS
AVAILABLE PHOSPHORUS
CHEMICAL–BIOCHEMICAL SUB-MODEL

Figure 3. System diagram, Lake 1 model.

where Eppley reports G_{max} at 0.587 (1/day). This form permits phytoplankton growth at low temperature and at 20°C results in a growth rate of 2.1/day.

The function form for the light effects is as given in previous work (DiToro et al. 1971) and incorporates vertical extinction of solar radiation and self-shading effects. The form is:

$$I = \frac{2{,}718f}{k_e H} (e^{-\alpha_1} - e^{-\alpha_0}) \qquad (7)$$

where $k_e = k_e' + 0.0088P + 0.054P^{0.66}$

$$\alpha_1 = \frac{I_a}{I_s} e^{-k_e H}$$

$$\alpha_o = \frac{I_a}{I_s}$$

and k_e' is the light extinction coefficient at zero chlorophyll and f is the photoperiod. All pertinent variables are functions of j, the segment location. The nutrient effect makes use of product Michaelis-Menten kinetics and is given by:

$$N\acute{p} = \frac{N_2 + N_3}{k_{mn} + (N_2+N_3)} \qquad \frac{P_2}{k_{mp} + P_2} \qquad (8)$$

The range of values for k_{mn} and k_{mp}, the half-saturation constants for nitrogen and phosphorus respectively, has been documented previously (DiToro et al. 1971). More recent other work (Caperon, 1972, and Hendrey, 1973) tends to support the original range; namely k_{mn} in the range 5-50 μg N/ℓ and k_{mp} in the range 1-10 μg/ℓ. There is an obvious species dependence of this constant as well as a possible dependence on the eutrophic state of the lake. In the Lake 1 model, k_{mn} is used at 25 μg N/ℓ, and k_{mp} of 1 and 10 μg P/ℓ is used as a range.

At present, silica limitation is not used in this model. Goering et al. (1973) and Paasche (1973) have estimated the half-saturation value for silica at about .02-.08 mg Si/ℓ for marine species. Under peak growing conditions in Lake Ontario, surface silica values are about 0.1-0.2 mg Si/ℓ, which tends to indicate that silica may not have a significant effect on the growth rate. Work is in progress, however, to include this nutrient since at the lower concentration and higher Michaelis levels, it could be important.

The complete growth rate expression, G_{pj} is then given by the product of (6), (7), and (8). Although there is no a priori reason for using a product formulation (Bloomfield, 1973), various experiments (DiToro, et al. 1971) have tended to support the approach (Ditoro, in press, and Hutchinson, 1967).

It can also be noted that the gross growth rate of the phytoplankton biomass, $G_{pj}P_j$ is equivalent to the daily average productivity in the jth segment. Thus:

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It can also be noted that the gross growth rate of the phytoplankton biomass, $G_{pj}P_j$ is equivalent to the daily average productivity in the jth segment. Thus:

$$\nu_j = a_{cc} G_{pj} P_j H_j \tag{9}$$

where ν_j is the average rate of carbon fixation (mgC/m^2-day), a_{cc} is the carbon to chlorophyll ratio (range 20-100 mgC/mg Chlor).

Phytoplankton Death Rate--

The expression for the death rate of the phytoplankton includes two primary effects: endogeneous respiration and predation by herbivorous zooplankton. The death rate is given by:

$$D_{pj} = K_2(1.08)^{T-20} + C_g Z_j \tag{10}$$

where K_2 is the constant (1/day), C_g is the grazing rate of the herbivorous zooplankton (liters/day -- mg zooplankton carbon) and Z is the carbon concentration of the herbivorous zooplankton. As noted below for the zooplankton observed in Lake Ontario, a filtering rate of .03-.06 liters/mgC-day-°C appears appropriate and at 20°C, represents about 2-6 ml, filtered per individual per day.

Phytoplankton Sinking--

In Equation 4, the sinking of phytoplankton from one segment to another segment can be a potentially significant sink or source of phytoplankton and associated nutrients. This phenomena is complex, incorporating vertical turbulence effects, the density structure, and the physiological state of different species of phytoplankton. For example, the generation of gelatious sheaths by phytoplankton has been shown to be of importance in settling (Hutchinson, 1967) and apparently the settling velocity of nutrient rich cells is somewhat less than cells that are nutritionally deficient (Yentsch, 1962).

Published values of the sinking velocity of phytoplankton, mostly in quiescent laboratory conditions, range from 0.07 - 18 meters/day. In some instances, the settling velocity is zero or negative.

Burns and Pashley (1974) have investigated the settling velocity profile of particulate organic carbon in Lake Ontario with an ingenious in situ measuring device. Their results show a general decrease of settling velocity with depth and a marked seasonal variation of settling velocity. For example, on 19 July 1972, settling velocity in the thermocline (about 10 m) was estimated at about -0.3 m/day, while at 50 meters depth, the settling velocity was about +1.3 m/day. But in March 1973, the velocity profile was positive ranging from 0.9 m/day at 20 meters to 0.1 m/day at 140 meters. Overall, the values given by Burns and Pashley varied between -0.4 and +2.0 m/day. These results and others tend to indicate that the settling velocity should be related in some way to the state of the phytoplankton biomass (e.g., actively growing versus declining phase) or vertical velocity variations. In this work, however, the vertical settling velocity has been treated as a constant. As such, this effect is only approximately modeled and, accordingly, the settling velocity has been treated as a constant over a range from 0.05 m/day to 0.5 m/day.

Zooplankton Carbon--

The measure of the herbivorous zooplankton population $Z^{(1)}$ is taken as the carbon content of the biomass and all sources of food for $Z^{(1)}$ are the phytoplankton chlorophyll. The general expression for a single volume j is therefore:

$$S^{(1)}_{zj} = V_j \, (G^{(1)}_z - D^{(1)}_z)_j \; Z^{(1)}_j \tag{11}$$

where $G^{(1)}_z$ and $D^{(1)}_z$ are the growth and death rates respectively of $Z^{(1)}$. The form of $G^{(1)}_z$ is:

$$G^{(1)}_z = a_{zp} \, C_{gzp} \, K_g \, \frac{P}{K_g + P} \tag{12}$$

where a_{zp} is the zooplankton carbon to phytoplankton chlorophyll efficiency, C_{gzp} is the grazing rate of the herbivorous zooplankton and K_g is the half-saturation concentration of phytoplankton at which the growth of $Z^{(1)}$ is half of maximum growth. This latter effect reflects the limiting growth of zooplankton at higher phytoplankton populations.

The mortality of the herbivorous zooplankton is given by two terms: endogenous respiration and grazing by the next highest trophic level. Therefore:

$$D^{(1)}_z = K^{(1)}_z \, (T) \; + \; C_{g21}(T) \, Z^{(2)} \tag{13}$$

where $K^{(1)}_z$ is the endogenous respiration rate as a function of temperature (1/day-°C), C_{g21} is the grazing rate (liters/day-mgC-°C) and $Z^{(2)}$ is the next highest trophic level, designated by superscript (2).

Expressions for all trophic levels above the herbivorous zooplankton arranged in a linear food chain are similar to Equations 13 and 14. Thus:

$$S^{(2)}_{Z_j} = V_j \, (G^{(2)}_Z - D^{(2)}_{Z\,j} Z^{(2)}_j)) \tag{14}$$

where

$$G^{(2)}_z = a_z \, C_{g21} \, Z^{(1)} \tag{15}$$

and

$$D^{(2)}_z = K^{(2)}_z \, (T) + C_{g21} \, (T) \, Z^{(3)}$$

These expressions are an obvious oversimplification of a complex food web and reflect only a "linear" type of food chain (see Figure 3). Indeed the effect of the zooplankton on the phytoplankton as formulated above is to always decrease the population, yet it has been shown (Patalas, 1969) that zooplankton can also increase phytoplankton population or have no effect. The mechanism for an increase in the population is apparently the passage of gelatinous green algae through the zooplankton gut with subsequent breakup of the colonies and regrowth after excretion. The smaller species are generally diatoms and Asterionella formosa, which are characteristic of Lake Ontario.

The zooplankton of Lake Ontario are dominated by the crustaceans specifically of Order Copepoda and Cladocera. Most hauls are made over 0-50 meter depth range. Principal Copepoid species given by Patalas (1969) are *Cyclops bicuspidatus* and *Tropocyclops prasinus* and the cladocerans, *Daphnia retrocurva* and *Bosmina longirostris*. In Patalas' work in 1967, these four species accounted for 91% of the total zooplankton and averaged about 60/liter from June - October.

Others (Glooschenko et al., 1972; McNaught et al., 1973; and Watson et al., 1974) generally show values in a similar range in some cases up to a maximum of almost 200/liter. For use in biomass computations, conversions must be made to equivalent carbon. Such conversion depends on age and accompanying weight of the animal and can therefore vary over a wide range. Watson et al (1974) in their conversions used weights of species to arrive at total biomass and equivalently averaged about 2.8 μg dry weight/individual.

The results, using an average of 40% carbon by dry weight, show peak values generally in August at lakewide averages of between 0.07 - 0.12 mg carbon/liter.

At the average dry weight of about 1-5 μg, individual grazing rates of about 0.8 - 2 ml/individual-day are appropriate (DiToro et al., 1971 and Kirby, 1971). Indeed the rate for *Bosmina longirostris*, a dominant species in Lake Ontario, has been given as 1-3 ml/individual-day (Hutchinson, 1967). Grazing rates expressed as liters per mg zooplankton carbon per day have been assumed to be linearly related to temperature and have been used in the range of 0.03-0.06 (1/mgC-day-°C), corresponding to the reported grazing ranges at 20°C.

Nitrogen System--

The basic equations for the nitrogen sub-system follow previous work (Hydroscience, 1933 and Thomann et al., 1974) and include non-living organic nitrogen, N_1, ammonia nitrogen, N_2, and nitrate nitrogen, N_3. The equation for the source-sink term for organic nitrogen is:

$$S_{N_{1,j}} = \frac{a_1}{a_c} \sum D_z Z + a_1 P_j \left(1 - \frac{a_{zp} K_g}{K_g + P_j}\right) C_{gzp} Z^{(1)} + a_1 D_p P - K_1 N_1$$

$$+ \frac{a_1}{a_c} Z^{(1)} (1 - a_z) C_{g21} Z^{(2)} \qquad (16)$$

where a_1 is nitrogen to chlorophyll ratio, and a_c is the carbon to chlorophyll ratio, and K_1 is the overall decay of N_1.

The first and third terms represent organic nitrogen released through endogenous respiration by the zooplankton and phytoplankton respectively and the second term represents the organic nitrogen of the grazed but unassimilated phytoplankton. The last term represents the decomposition, settling, and other effects that contribute to the decay of organic nitrogen.

For ammonia:

$$S_{N_{2,j}} = K_{12}(T)N_2 - K_{22}(T)N_2 - a_1\,\alpha G_p P \qquad (17)$$

where K_{12} is the rate of production of ammonia from organic nitrogen, K_{22} is the rate of oxidation of ammonia to nitrate, and α is a preference factor given by:

$$\alpha = \frac{N_2}{N_2 + K_{mn}} \qquad (18)$$

or is alternatively set equal to one-half for an "indifferent" uptake of nitrogen. The last term represents the uptake of ammonia for phytoplankton growth. For nitrate,

$$S_{N_{3,j}} = K_{23}N_2 - a_1\,(1-\alpha)\,G_p P \qquad (19)$$

where $K_{23} = K_{22}$, the production of nitrate by ammonia oxidation.

Phosphorus System--

The equations for the phosphorus cycle used in the models are as previously developed (Hydroscience, 1933 and Thomann et al. 1974) and include nonliving organic phosphorus and phosphorus available for phytoplankton growth. For organic phosphorus, the equation for the source and sink for p_1 is:

$$S_{P1,j} = \frac{a_p}{a_c} \sum D_z Z + a_p P_j \left(1 - \frac{a_{zp}\,K_g}{K_g + P_j}\right) C_g\, Z^{(1)} - K_1 p_1 + \frac{a_p}{a_c} Z^{(1)}\,(1-a_z)$$

$$C_{g21} Z^{(2)} + a_p D_p P \qquad (20)$$

where a_p is the phosphorus to chlorophyll ratio and K_1 represents a general decay of organic forms.

For the inorganic phosphorus, assumed as phosphorus available to the phytoplankton, the source-sink equation for p_2 is:

$$S_{P2,j} = K_{12}(T)\,p_1 - a_p\,G_p P - K_{22}\,(T)\,P_2 \qquad (21)$$

where K_{12} represents the rate of decomposition of organic phosphorus to forms available for phytoplankton utilization and K_{22} represents an overall loss of inorganic phosphorus.

LAKE 1 MODEL

Basic Structure

Because of the complexity of the overall modeling structure, a simplified version of the lake has been developed to explore the kinetic behavior in greater detail. This is indicated in the modeling strategy of Figure 2. The simplified model, designated Lake 1, assumes that Lake Ontario is well-mixed horizontally and gradients are allowed to develop only in the vertical dimension. Such a simplification obviously does not permit nearshore versus open-lake comparisons of the effect of the Rochester or Toronto discharges on the local lake environment. However, an inspection of the data on nutrients and chlorophyll does not appear to indicate substantial horizontal gradients although variations do exist in certain areas. (Lake Ontario can be contrasted in this regard to Lake Huron with marked horizontal gradients from Saginaw Bay or to Lake Erie with important horizontal gradients from the Western Basin to the Eastern Basin.)

Because of the complexity of the interactive systems and most importantly, the computational time involved in obtaining solutions, the horizontally well-mixed assumption appears to offer a reasonable starting point for understanding the dynamic behavior of Lake Ontario. Figure 4 is a schematic of the Lake 1 model and shows the three vertical segments that comprise the basic geometry. The principal physical features are 1) horizontal transport, 2) vertical dispersion between the epilimnion and hypolimnion, and 3) vertical settling of the phytoplankton. The lake shown in Figure 4 has a defined epilimnion depth of 17 meters and a hypolimnetic depth of 73 meters. The Lake 1 model is well-mixed in the winter and spring, stratifies during summer, and then goes through a fall overturn. The effects are simulated by control of the vertical mixing.

Results of "Final" Verification Analysis

Data were available for the years 1967-1970 collected by the Canadian Centre for Inland Waters for this final verification phase. These data were supplemented by other observations in the literature. Values for the various coefficients, parameters, and exogenous variables were obtained from published sources as discussed previously, and all values are used within reported ranges in the literature. More than 80 runs have been made of Lake 1 to examine the effects of various phenomena such as settling velocity, vertical mixing, zooplankton predation, and half-saturation constants for nitrogen and phosphorus. Out of these runs, a consistent set of parameter values has emerged to satisfactorily explain the observed data on a variety of different variables. A plausible explanation for the dynamic behavior of the phytoplankton is therefore possible and is discussed below.

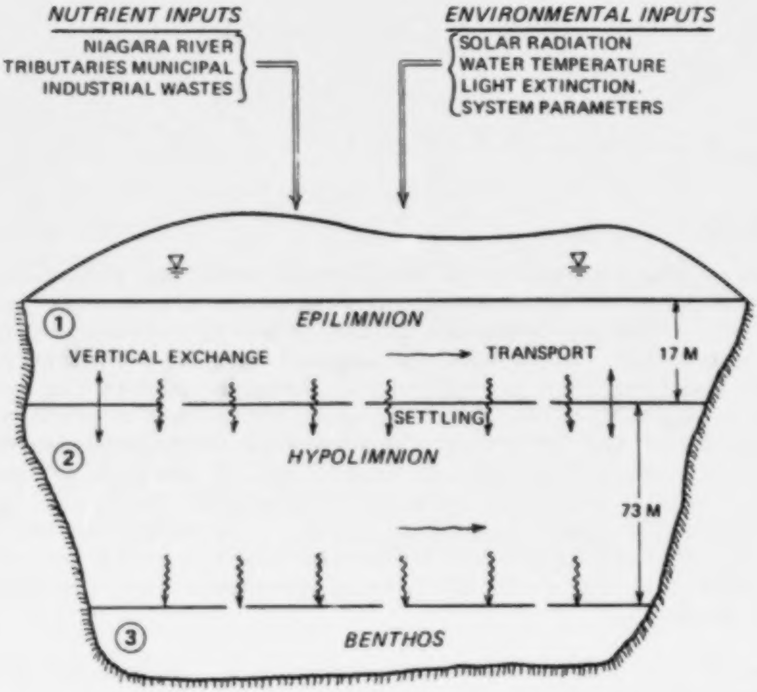

NUTRIENT INPUTS
NIAGARA RIVER
TRIBUTARIES MUNICIPAL
INDUSTRIAL WASTES
ENVIRONMENTAL INPUTS
SOLAR RADIATION
WATER TEMPERATURE
LIGHT EXTINCTION.
SYSTEM PARAMETERS
①
EPILIMNION
VERTICAL EXCHANGE
TRANSPORT
17 M
SETTLING
②
HYPOLIMNION
73 M
③
BENTHOS

Figure 4. Major physical features included in Lake 1 model.

Figures 5 and 6 show the verification of the Lake 1 model upper layer (0-17 meters), while Figure 7 shows the verification for the hypolimnion (data in range, 50-150 meters). Table 2 shows the principal coefficients used for the runs. As shown, the verification for plankton in the epilimnion is quite good and satisfactorily duplicates the spring peak, subsequent mid-summer dieoff, and fall bloom. Five other variables, in addition to the phytoplankton chlorophyll are satisfactorily verified in the comparison. It is generally not possible to obtain the verification shown in Figures 5-7 by arbitrary specification of the coefficients, so the results do not simply represent a "curve fitting" exercise. Rather, the results, based on the theory discussed earlier, are much more representative of the observed data than arbitrary statistical coefficients. Since the model permits computation of the components of the dynamic behavior, considerable insight can be gained by examining the influence of various phenomena on the growth and death rates of the phytoplankton.

Figure 8 shows the dynamics of the kinetic growth rate of the phytoplankton (G_p in Equation 4). As shown, maximum growth rate reaches a peak in mid-September in the Lake 1 model at about 1.8/day and then decreases rapidly due to the fall overturn. The reduction in the maximum growth rate due to light is substantial and shifts peak growth to early August. A further reduction in growth rate is due to the nutrient limitation so that the resultant growth rate, G_p, is lowest at the end of May and peaks about 0.25/day in August. The peak value represents an overall reduction of 90% from saturated growth condi-

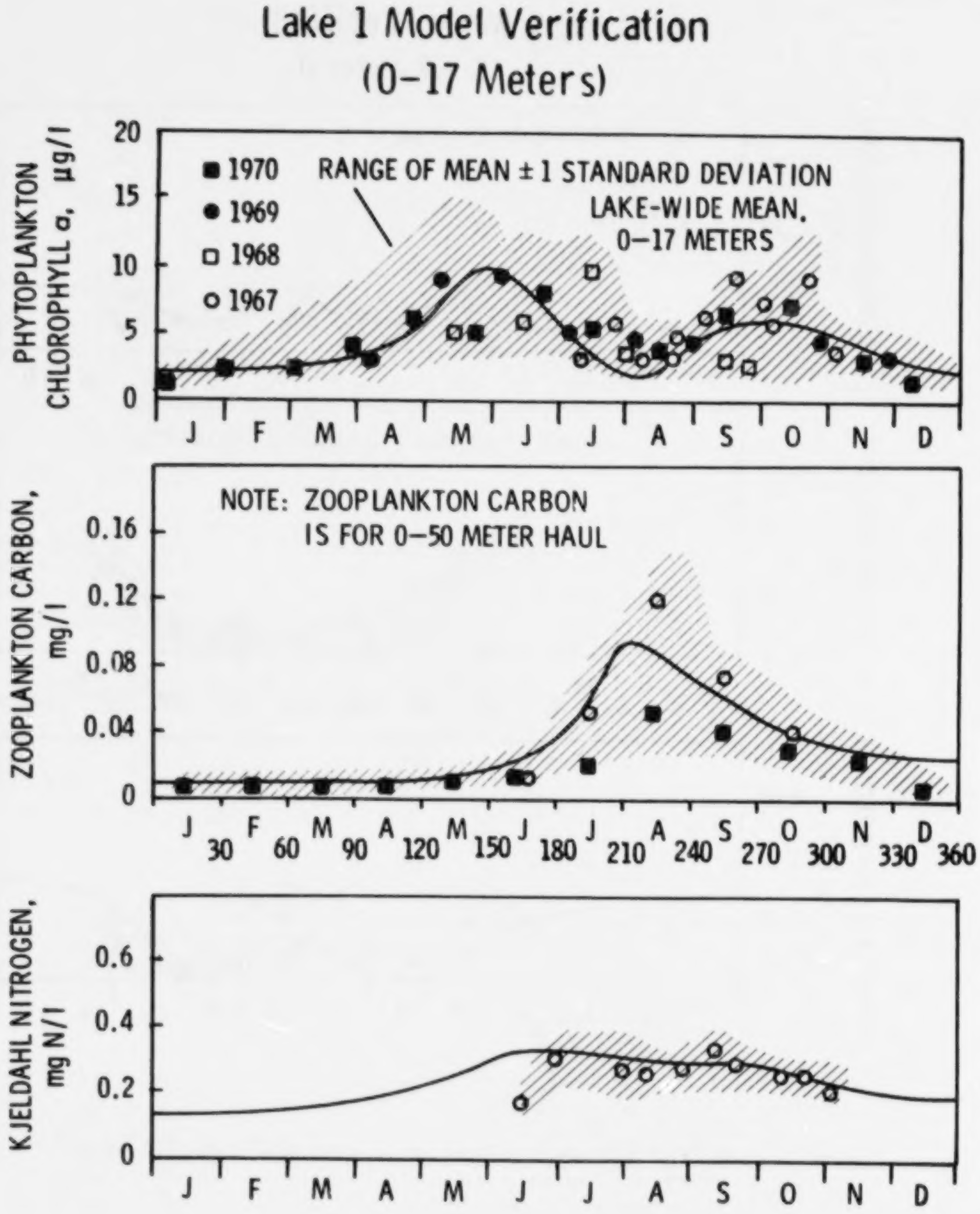


Figure 5. Lake 1 model verification, 0-17 meters.

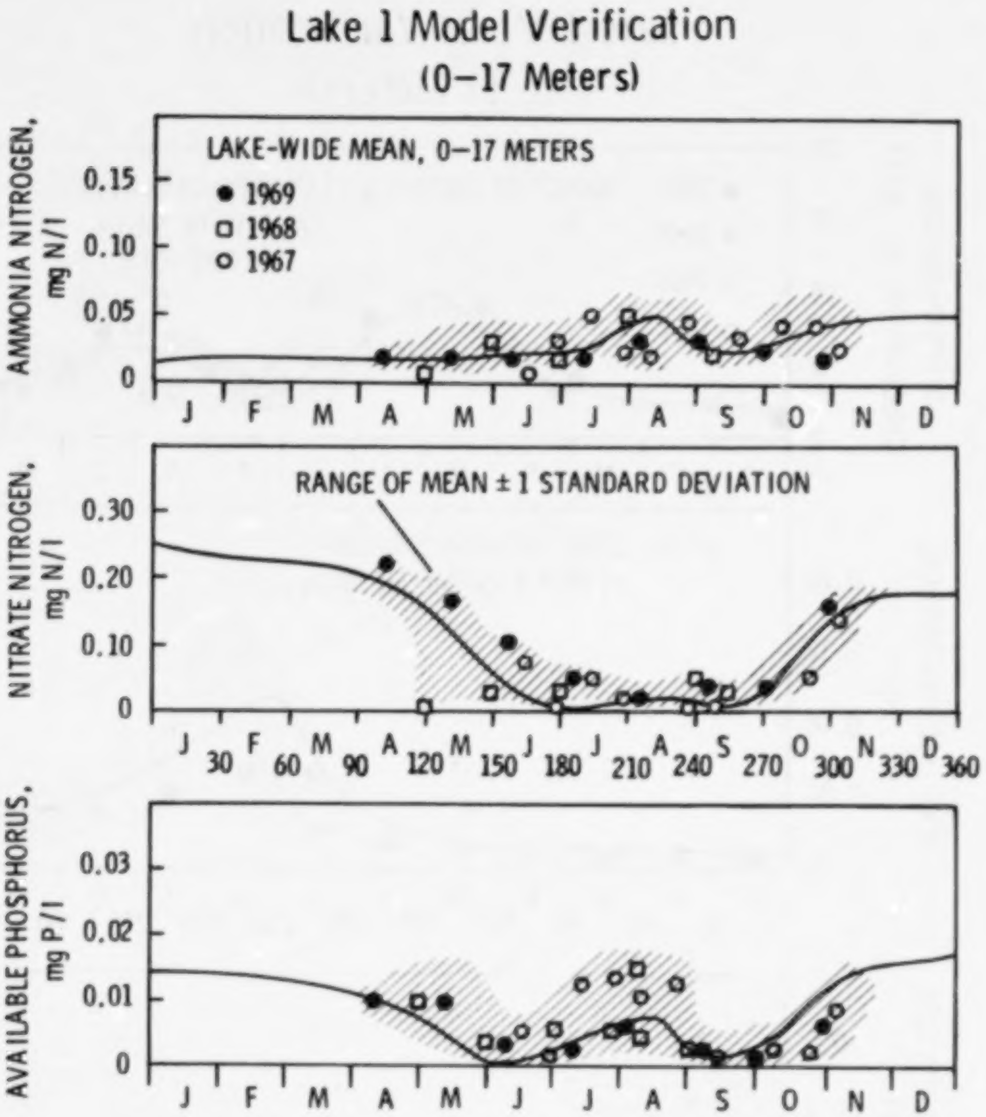


Figure 6. Lake 1 model verification (continued), 0-17 meters.

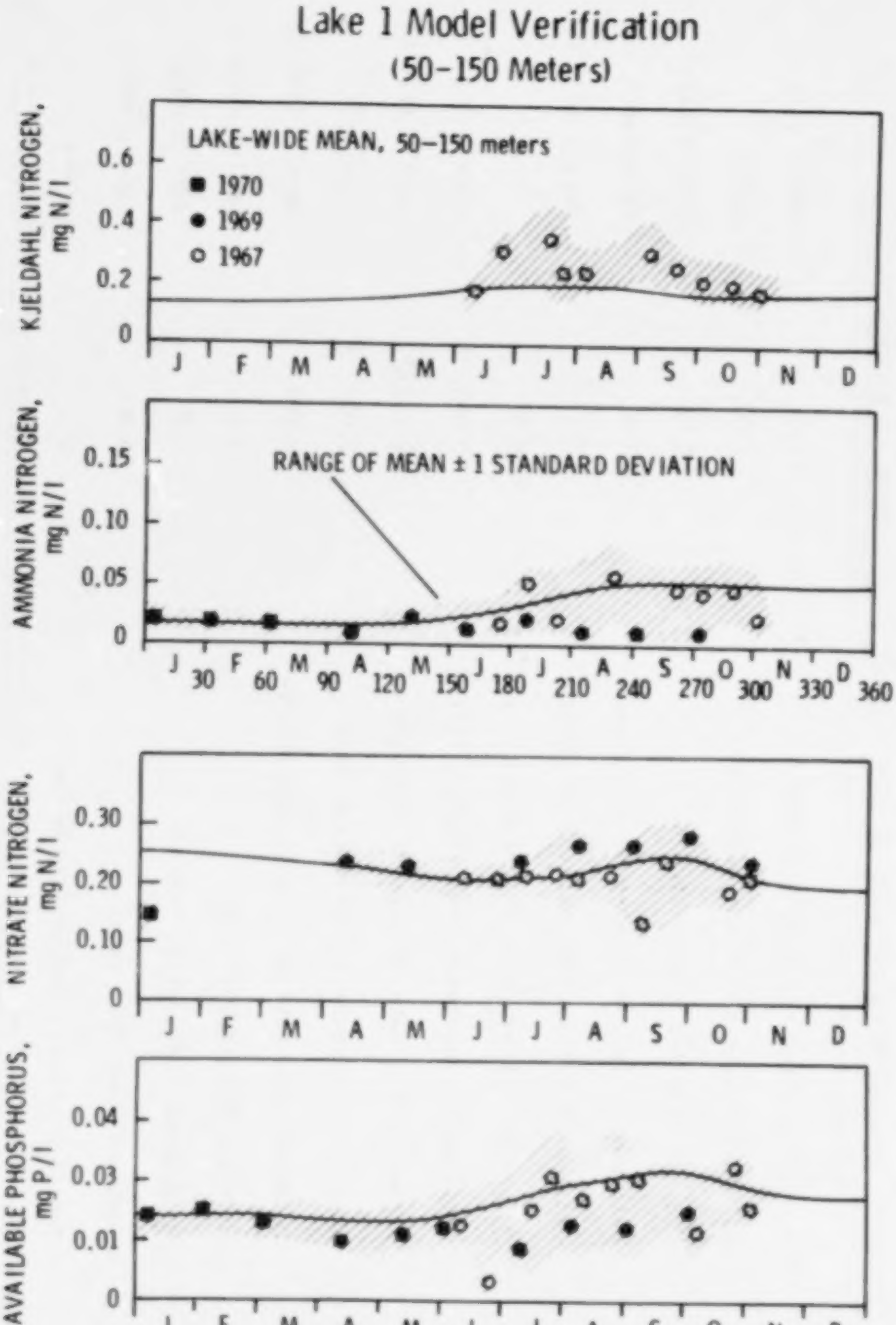


Figure 7. Lake 1 model verification, 50-150 meters.

TABLE II. PRINCIPAL PARAMETER VALUES USED IN LAKE 1 MODEL OUTPUT SHOWN IN FIGURES 5-7

K_{mn}	Half-saturation constant-nitrogen	25.0	$\mu gN/\ell$
K_{mp}	Half-saturation constant-phosphorus	2.0	$\mu gP/\ell$
C_g	Grazing rate for zooplankton	0.06	ℓ/mg C-Day-C°
K_g	Half-saturation constant-phytoplankton	10.0	μg chlor/ℓ
K_2	Endogenous respiration rate-phytoplankton	0.1	$days^{-1}$ (@20°C)
a_{zp}	Zooplankton conversion efficiency	0.6	---
K_z	Zooplankton endogenous respiration rate	0.001	$days^{-1}$-°C
K_1	Decomposition rate of organic nitrogen	0.00175	$days^{-1}$ (@20°C)
K_{22}	Ammonia to nitrate nitrification rate	0.002	$days^{-1}$-°C
K_{op}	Decomposition rate of organic phosphorus	0.007	$days^{-1}$ (@20°C)
a_1	Nitrogen-Chlorophyll ratio	10.0	$\mu gN/\mu g$ chlor
a_p	Phosphorus-Chlorophyll ratio	1.0	$\mu gP/\mu g$ chlor
a_c	Carbon-Chlorophyll ratio	50.0	$\mu gC/\mu g$ chlor
w	Settling velocity of phytoplankton	0.1	m/day

tions. Primary productivity values calculated from the growth rate and phytoplankton biomass, Equation 9, give peak values of about 600 mg carbon/m^2-day at the end of August and values of about 500 mg C/m^2 day at the height of the spring bloom. The former value is in general agreement with Glooshenko (1974) while the latter is somewhat lower than the Glooshenko's estimated spring values of 500-1100 mg C/m^2-day.

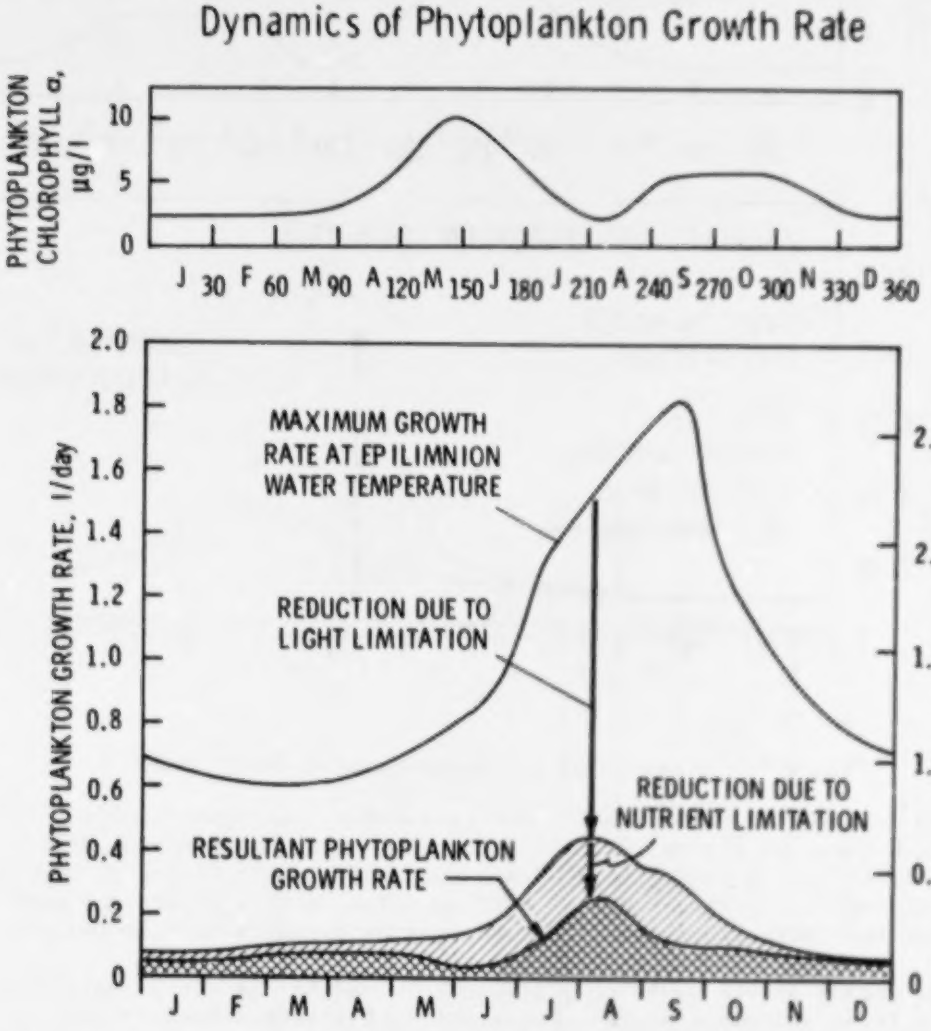


Figure 8. Dynamics of the phytoplankton growth rate.

Figure 9 shows the dynamics of the kinetic death rate of the phytoplankton, Dp in Equation 4. Three effects are noted: 1) phytoplankton settling, 2) water temperature effect on endogeneous respiration, and 3) effect of zooplankton grazing. Peak resultant death rate is 0.25/day at the beginning of August and is primarily due to zooplankton predation and quantitatively explains the mid-summer decline in phytoplankton biomass.

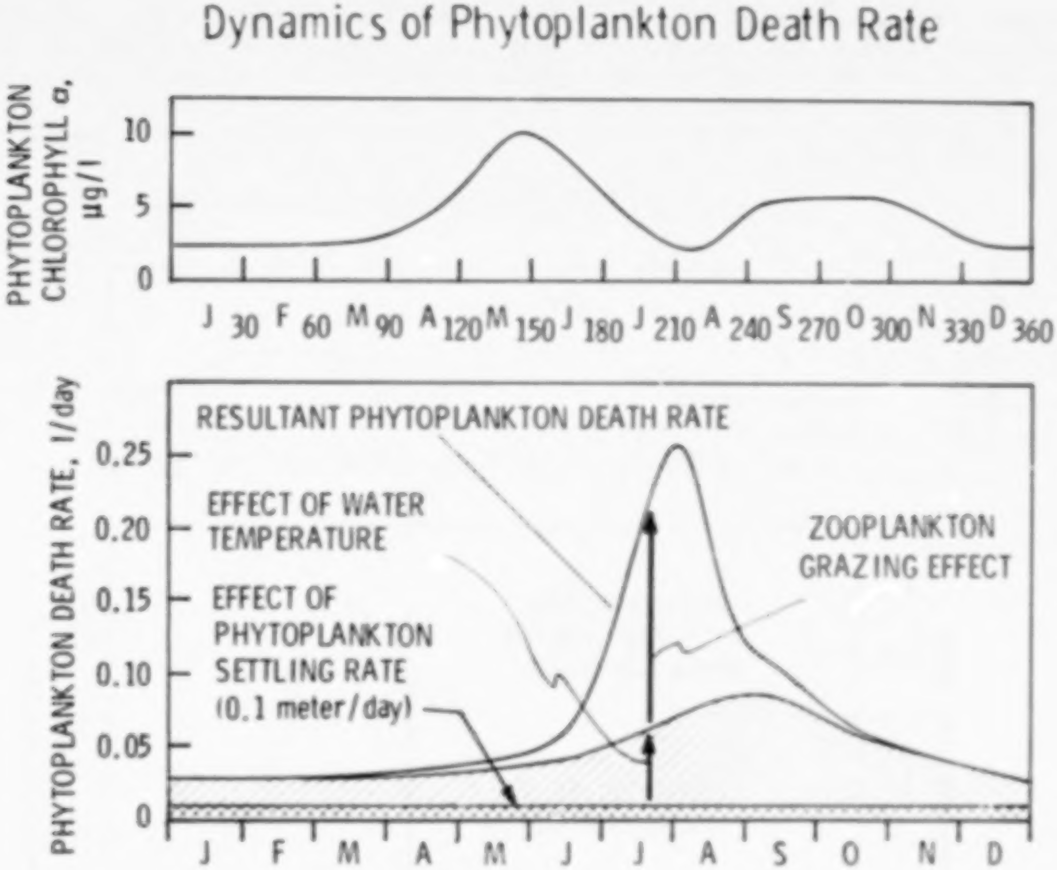


Figure 9. Dynamics of the phytoplankton death rate.

The dynamics of phytoplankton net production, i.e. chlorophyll/liter-day, dP/dt, are shown in Figure 10. Net production here includes the kinetic interactions of growth in Figure 8, death and predation in Figure 9, and the effect of vertical mixing and lake outflow. The results shown in Figure 10 summarize the basic hypotheses of phytoplankton dynamics in Lake Ontario.

The spring growth phase to approximately mid-May is due primarily to increasing light and temperature. There are little zooplankton as yet, so growth continues until nutrient limitation becomes significant in early June. (This is principally phosphorus limitation as discussed below.) The spring growth and spring peak are therefore simply described by a nutrient interaction effect with light and temperature dominating the growth and death terms. Following the spring peak however, the situation becomes considerably more complex and indeed a significant effort was devoted to unraveling the complex interactions which lead to a mid-summer decline and subsequent broad full peak.

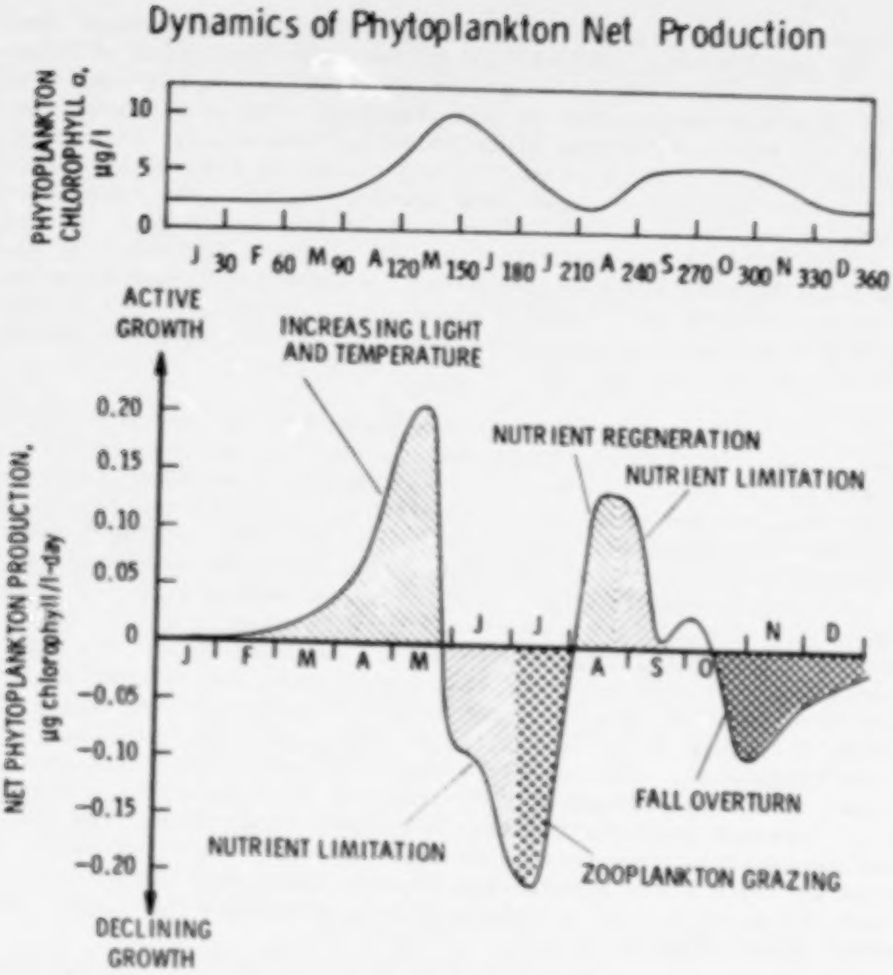


Figure 10. Dynamics of the phytoplankton net production

It is hypothesized as computed in Figure 10, that during mid-summer, and following the nutrient limitation effect, zooplankton grazing becomes increasingly important and accounts for the minimum values of biomass in July. However, at increased zooplankton grazing, biological recycling of nutrients becomes more significant. Nutrients are released back to the epilimnion through excretion as well as phytoplankton endogenous respiration. In late July therefore, growth exceeds death due to nutrient regeneration and an active growth phase begins again. Nutrients, however, are already at low levels so growth is not pronounced and proceeds slowly (as shown by the net production approaching zero in September but not yet entering a negative growth phase). The fall overturn of the lake then reduces the growth of phytoplankton biomass primarily because of mixing with the colder hypolimnion waters.

Figures 11 and 12 explore these dynamic effects at another level of detail. The model permits calculation of nutrient limitation effects as well as the flux of nutrients due to zooplankton excretion and release of nutrients due to endogenous respiration by the plankton. Figure 11 shows the nutrient effects. As seen, the spring bloom is halted essentially by a reduction in phosphorus that is quite pronounced, and growth is reduced by a factor of about 0.35 due to the low phosphorus levels. At the same time, nitrogen is not limiting and is at a level twice that of the phosphorus. Nitrogen, however, does become limiting during July and again in September at which time phosphorus also exerts a limiting effect on growth. Figure 11 shows therefore that the spring peak is primarily phosphorus controlled. The dynamics after the spring peak are, however, controlled by both nitrogen and phosphorus (as well as zooplankton predation).

Figure 12 shows the estimated sources of phosphorus recycled to the epilimnion by both biological effects and vertical mixing effects from the hypolimnion. As shown, during early spring the hypolimnion contributes almost twice as much nutrients to the epilimnion as from external inputs, and by early summer in June-July, biological recycling reaches almost five times the mass rate of external phosphorus sources due to waste residuals, Niagara River, and other contributors. Zooplankton excretion recycles almost twice the input rate and in late summer--early fall, biological recycling reaches another peak but then drops rapidly due to fall overturn. The flux of nutrients from the more nutrient rich hypolimnion during October is not sufficient to offset the drop in water temperature in the epilimnion. Biological recycling therefore drops rapidly in October, and the phytoplankton biomass declines (Figure 5). The results shown in Figure 12 indicate quantitatively the significance of biological recycling and vertical mixing relative to the level of external nutrient inputs. A reduced input phosphorus load therefore may not necessarily result in a concomitant reduction in biomass. Further, the time to reach a new equilibrium biomass level will extend beyond a single year due, in part, to the recycling effects discussed previously and long detention times in Lake Ontario.

Summary of Lake Responses to Nutrient Inputs

In addition to increased understanding of the behavior of lake phytoplankton, the model also provides a basis for estimating the direction of future changes due to various nutrient loading scenarios. A range of conditions on the external nutrient inputs has therefore been examined. However, to place the present loads into an historical perspective, a preliminary analysis was made of the nutrient inputs that might have existed at some distant time in the past. These loads were termed the Pastoral Loads. In addition, a review was made of nutrient reductions suggested by Vollenweider (1968) and those of the Great Lakes Water Quality Agreement (1973). A full review of all simulations is given by Thomann et al. (1975). The simulations were therefore prepared using a wide range of input nutrient loads with the Pastoral Loads as a baseline condition.

For phosphorus, the input range was from present conditions (34 metric tons/day) to Pastoral Loads (9.3 metric tons/day). The Water Quality Agreement and Vollenweider reduction loads were 73% and 63% respectively of

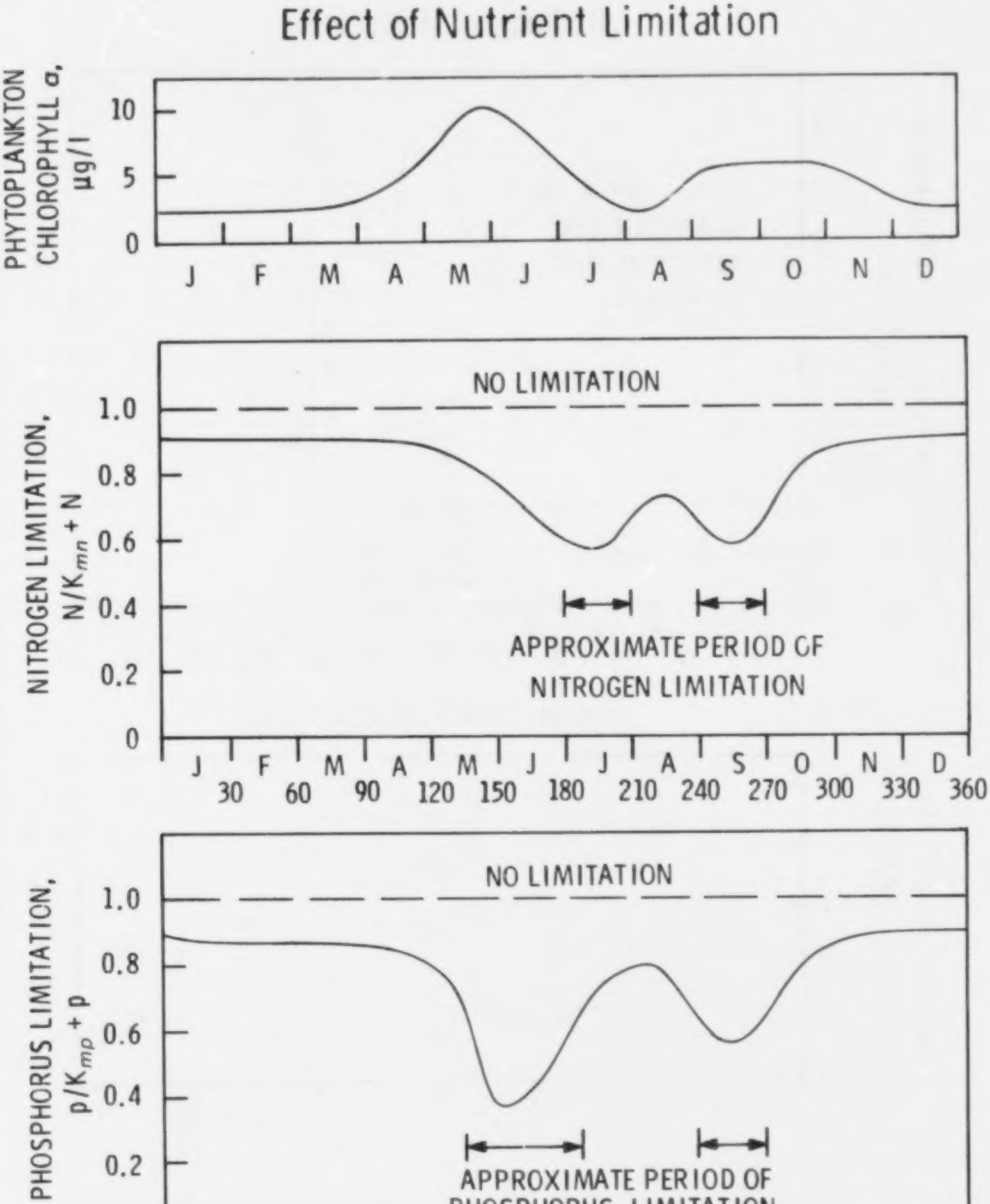


Figure 11. Dynamics of nitrogen and phosphorus limitations.

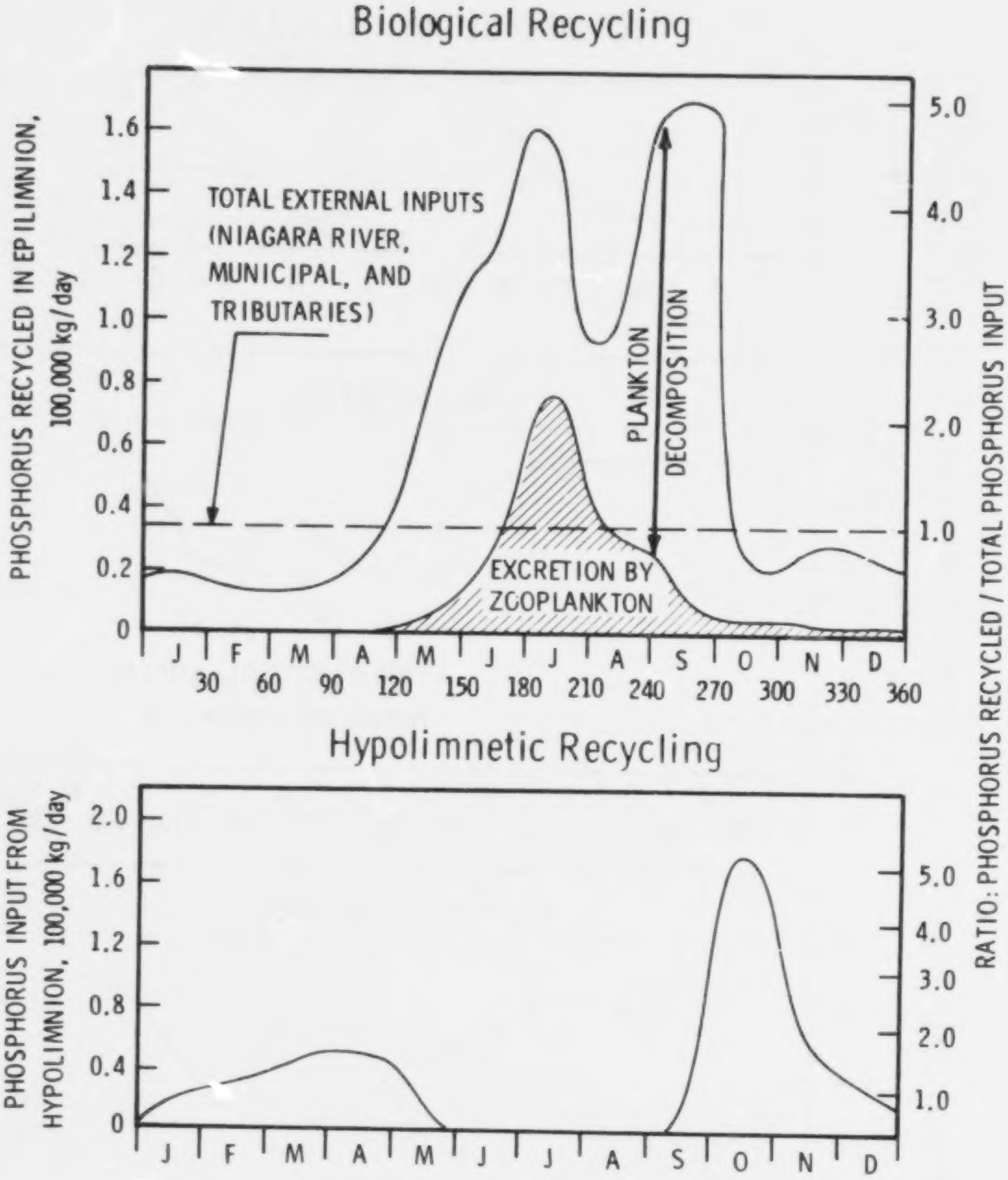


Figure 12. Biological and hypolimnetic recycling of phosphorus.

present inputs.

Figure 13 shows the results of the simulations under three conditions on the long-term loss of phosphorus. Using the "reasonable" set of kinetics, the results indicate that Lake Ontario may not be in equilibrium with the present nutrient inputs. As indicated in Figure 13, peak values may reach as high as 22 μg chlorophyll/ℓ under the pessimistic assumption and a continuation of present loads. The implementation of the WQA loads would reduce this "worst" case to only about 20 μg/ℓ. Under the reasonable kinetics that include organic decay, the change occasioned by the WQA would be less than 1 μg/ℓ. Reductions below present levels of biomass may therefore be difficult to achieve.

PRELIMINARY LAKE 3 MODEL

As shown in Figure 2, the Lake 3 model represents a synthesis of spatial geometry and transport structure with the biological and chemical kinetic interactions of the Lake 1 and 2 models. Lake 3 is therefore a three-dimensional time variable model which does permit some analysis of phenomena in the nearshore region as compared to the open lake region. The structure of Lake 3 represents a further step beyond the simple Lake 1 model, the extent of the step incorporating such factors as computational time for a one year run and availability of survey data.

The work reported here is of a preliminary nature since considerably more effort is required to assure the veracity of the computed results both in terms of the numerical output and the degree to which the model verifies observed data.

The basic segmentation used for the Lake 3 model is shown in Figure 14. As shown, 67 segments are distributed over five vertical layers. The upper two layers, 0-4 meters and 4-17 meters, are considered to represent the epilimnion during periods of vertical stratification. A "ring" of segments extending 10 km out from the coast is used to represent the nearshore environment.

All cross-sectional areas for interfaces in three dimensions between the segments were planimeters and volumes were computed and summed to within 10% of total lake volume.

The hydrodynamic circulation is considered to be externally supplied through inferences from observations and/or hydrodynamic model output. The final summer circulation pattern, as given by inter-segment velocity values can be seen in Figure 15.

Preliminary Phytoplankton Computations

Several runs have been made of the Lake 3 model using the transport and dispersion regime discussed previously and the system kinetics given in Section III.

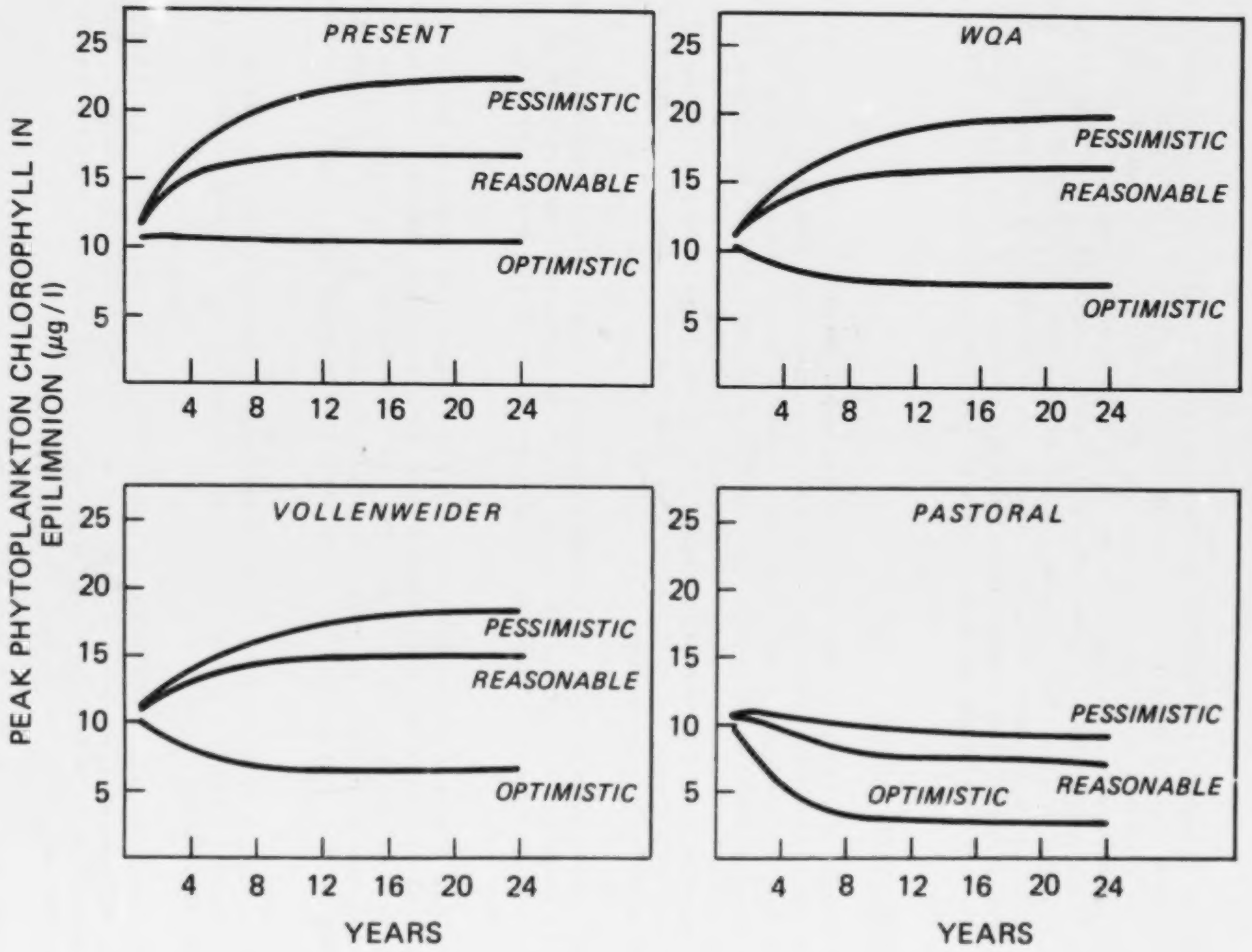
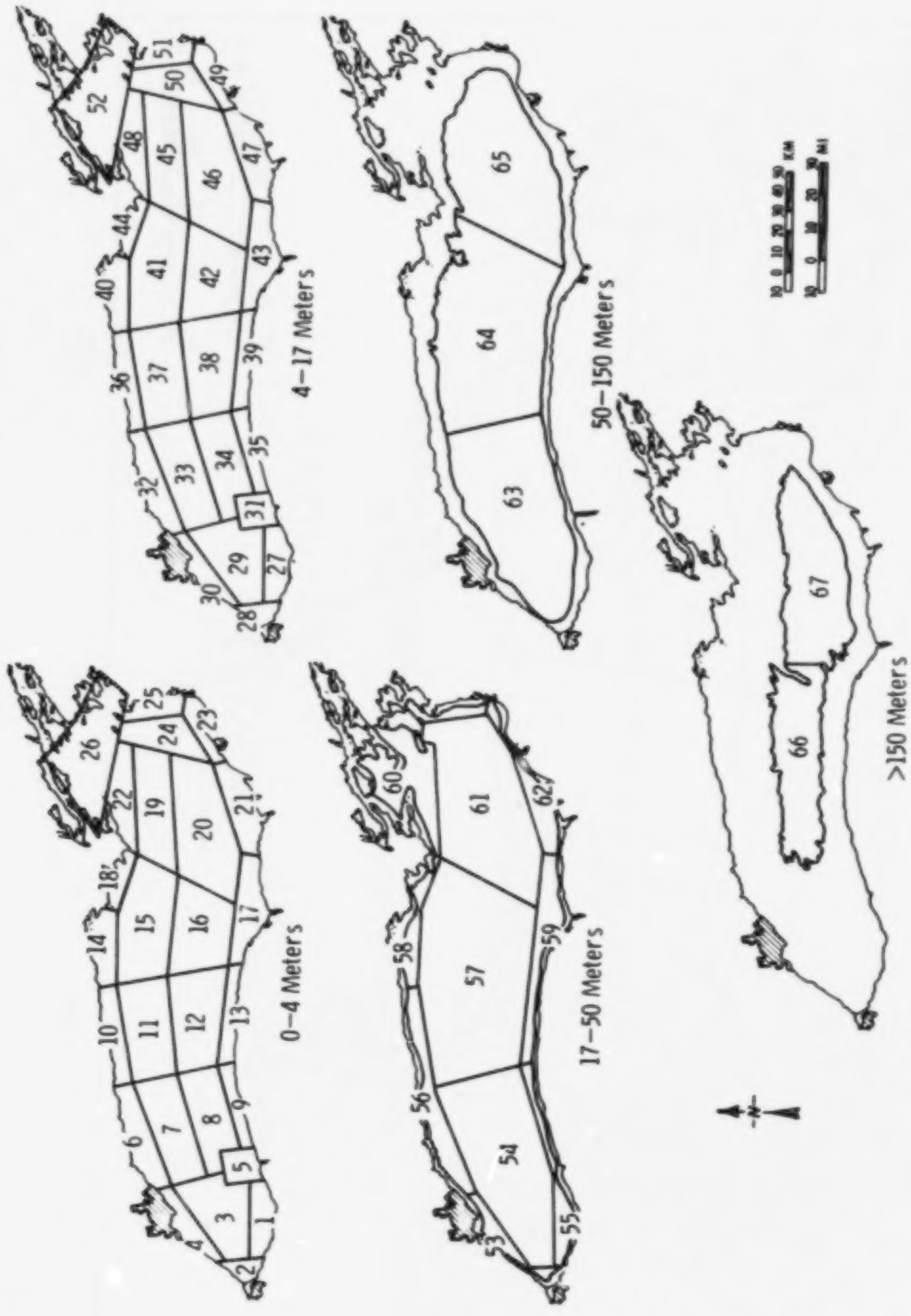


Figure 13. Summary of long term simulations.



0–4 Meters
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
4–17 Meters
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52
17–50 Meters
53
54
55
56
57
58
59
60
61
62
50–150 Meters
63
64
65
>150 Meters
66
67
N
KM
MI

Figure 14. Lake 3 segmentation.

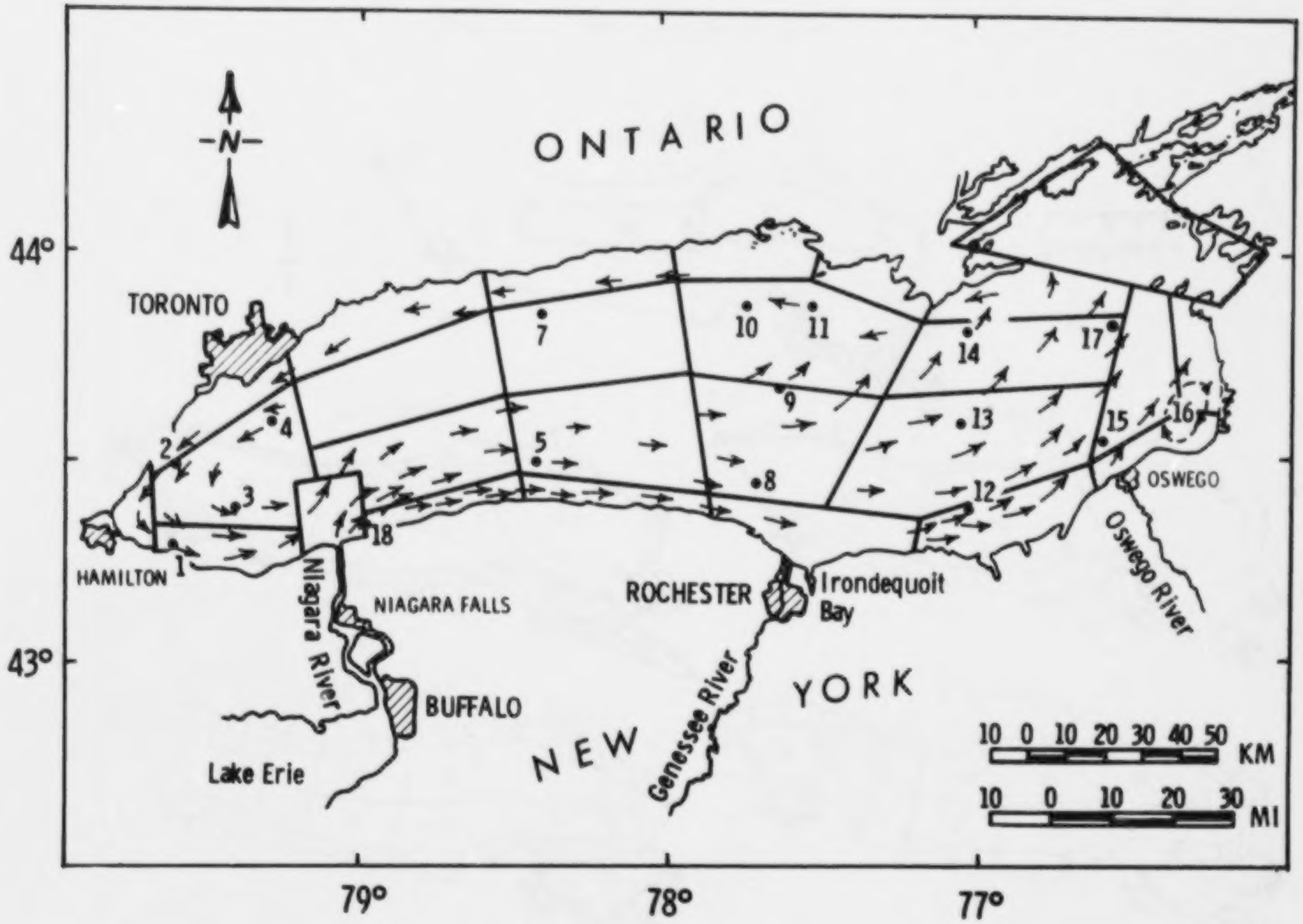

N
ONTARIO
TORONTO
HAMILTON
NIAGARA FALLS
Niagara River
BUFFALO
Lake Erie
NEW YORK
ROCHESTER
Irondequoit Bay
Genessee River
OSWEGO
Oswego River
44°
43°
79°
78°
77°
1
2
3
4
5
7
8
9
10
11
12
13
14
15
16
17
18
10 0 10 20 30 40 50 KM
10 0 10 20 30 MI

Figure 15. Assumed summer circulation for Lake 3 model.

Figure 16 shows early very preliminary results of phytoplankton biomass at three segments across the lake. This run did not include any vertical settling velocity. Segment #17 represents the Rochester embayment and as seen reaches peak values of almost 15 μg chlorophyll/ℓ at the end of May while the more open lake area (segment #16) lags by about one-half month. Peak values at the open lake segment are about half of the Rochester segment. At the end of May therefore, at day 150, a lateral gradient of chlorophyll of about 10 μg/ℓ is computed. This results primarily from the thermal bar effect simulated by variations in the horizontal dispersion. Gradients across the lake in the fall are not pronounced and reflect the general well-mixed character of the lake at that time.

Output Data Display

The Lake 3 model generates a substantial output totalling thousands of numbers that represent the time variable response of each variable at each segment. In addition, output on the growth and death kinetics, nutrient recycle, and nutrient fluxes are also obtained. The size of Lake 3 makes it difficult to fully comprehend the output so some effort is being devoted to further analysis and display of the output. In this way, it is hoped that further understanding and insight can be obtained on the behavior of the solutions that are generated.

Figure 17 shows the output flow diagram that is presently being utilized. Output from the model is written on tape in addition to hard copy and digital overplots. The latter plots are for general screening by the analyst. Plotting software is then employed on the tape output from a model run. Paper contour plots and three dimensional plotting routines are employed to provide further perspective on the output. Figure 18 shows a three dimensional plot of output of phytoplankton biomass in the 0-4 meter layer from a view looking down the lake from east to west. Plots such as these reveal the structure of the solution in a meaningful way and also permit scanning of the output to determine any strange or anomalous results that may indicate a flaw in the numerical computations.

Returning to Figure 17, the three dimensional plots are also displayed on a cathode ray tube, microfilmed, and prepared for a motion picture of the complete output over time. A screening of this film then reveals the dynamic and spatial behavior of the solution in a very unique way. At this stage, a film has been prepared of a preliminary run for a full year of the surface phytoplankton chlorophyll.

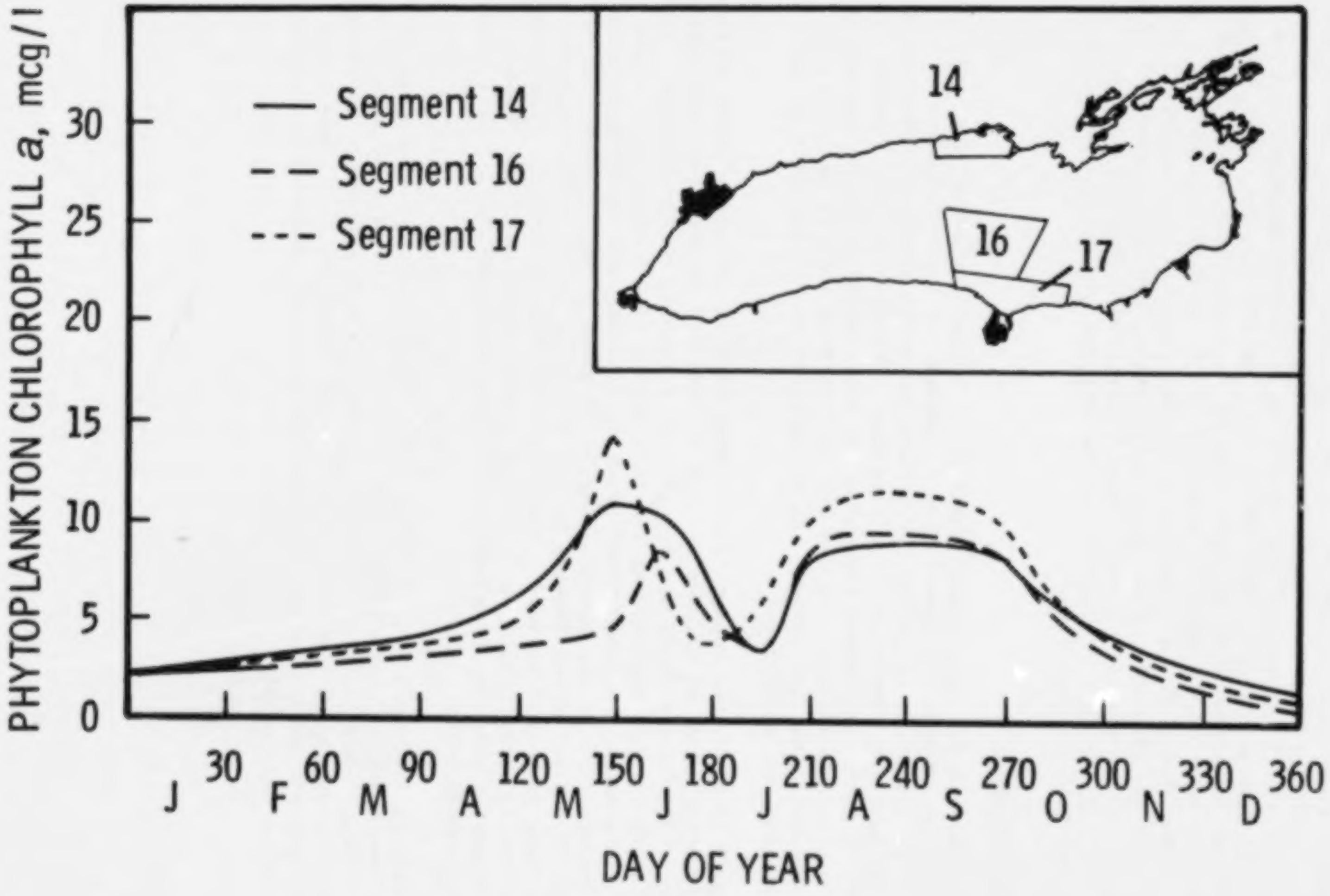


Figure 16. Preliminary results of Lake 3, 0-4 meters.

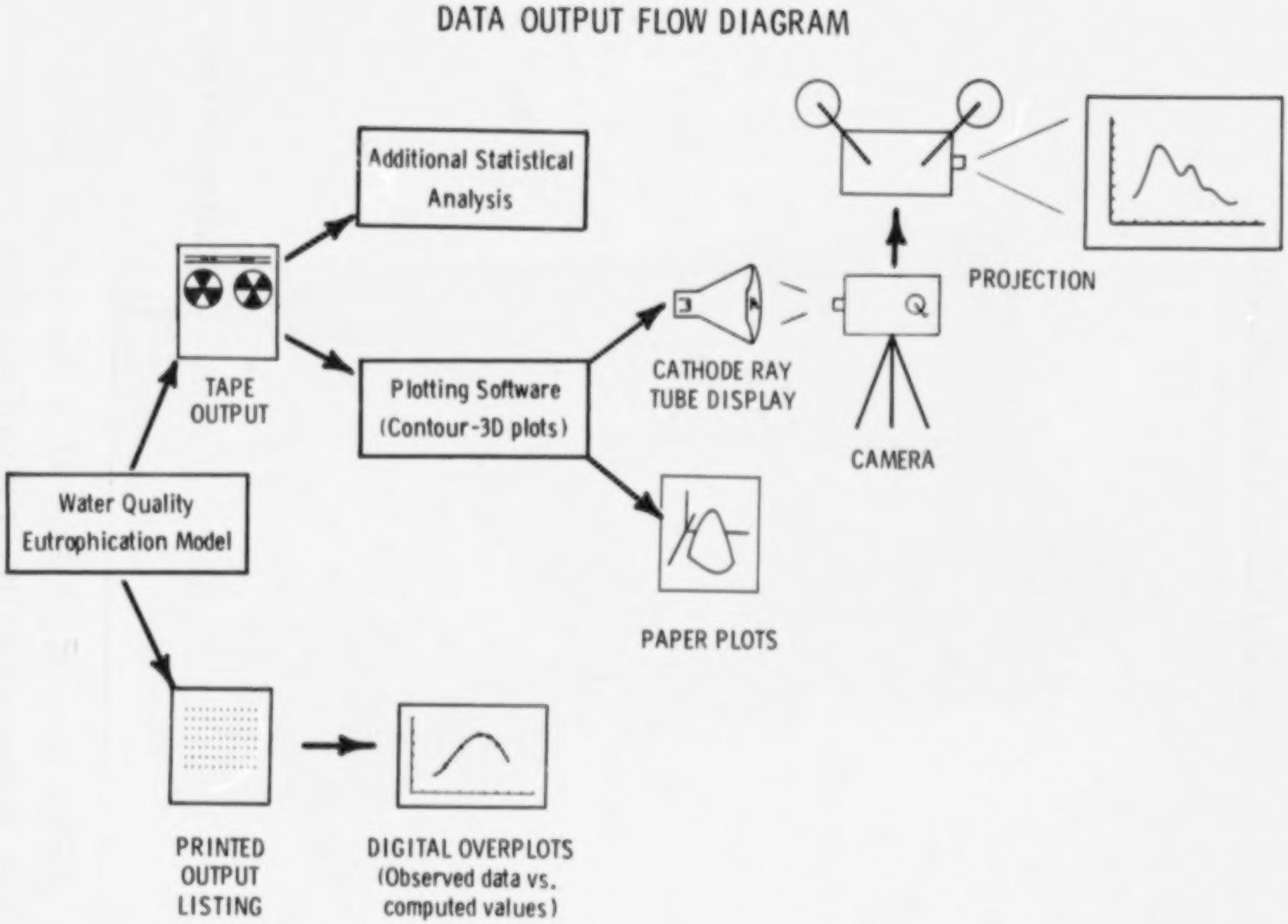

DATA OUTPUT FLOW DIAGRAM
Water Quality Eutrophication Model
TAPE OUTPUT
Additional Statistical Analysis
Plotting Software (Contour-3D plots)
CATHODE RAY TUBE DISPLAY
CAMERA
PROJECTION
PAPER PLOTS
PRINTED OUTPUT LISTING
DIGITAL OVERPLOTS (Observed data vs. computed values)

Figure 17. Data output flow diagram used for verification and display purposes.

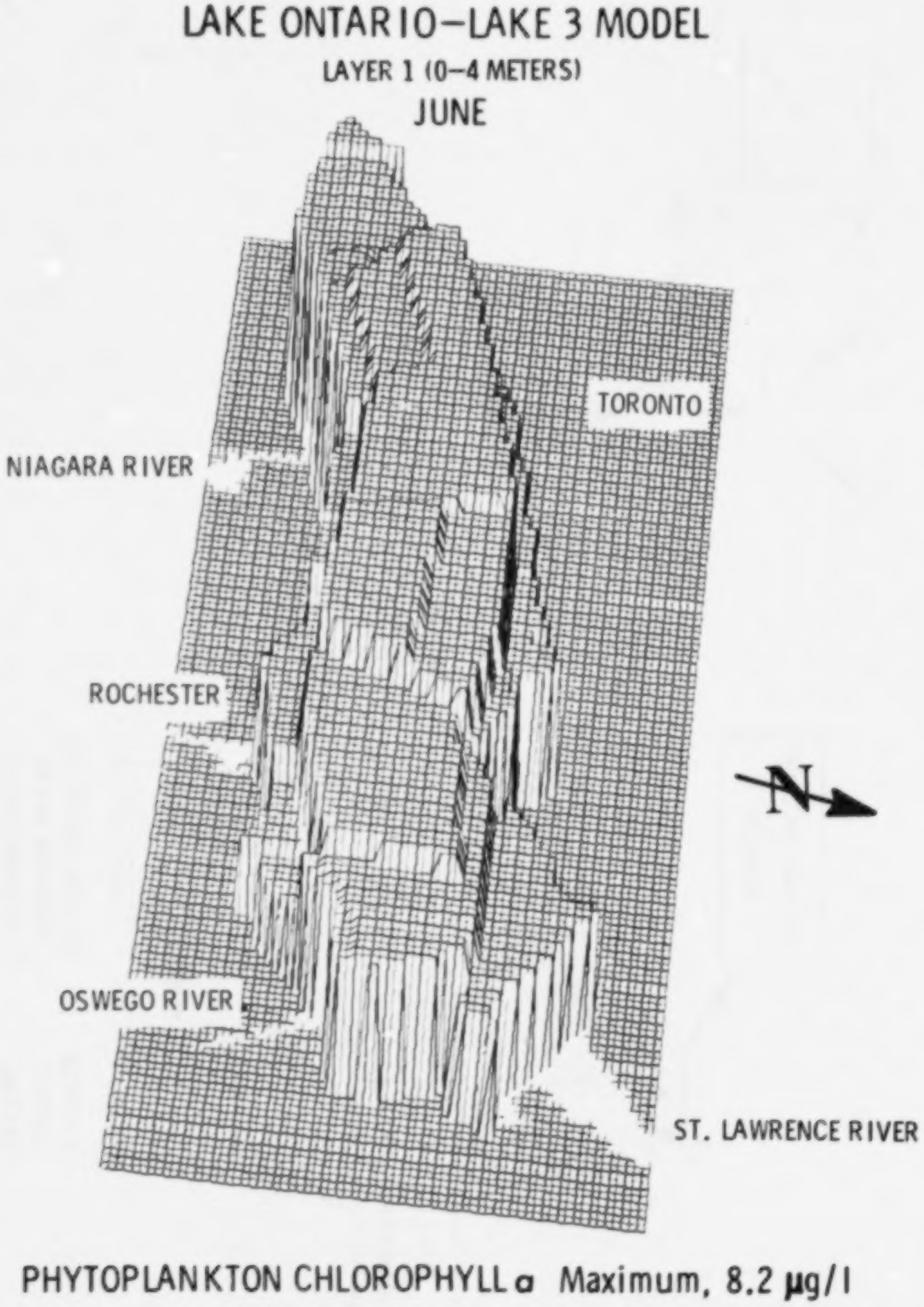


Figure 18. Three dimensional plot of phytoplankton chlorophyll calculated from Lake 3 model, June, 0-4 meters.

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DISCUSSION

VASILYEV: What is the ratio of the lake's volume to the total annual river flow to the lake? Did you consider current variability produced by wind effects? How significant were the diffusion processes and did they make an essential contribution to the individual compartments into which the lake was divided?

THOMANN: The ratio of the lake's volume to the annual flow is such that when the volume is divided by the flow, a hydrologic retention time of eight years is obtained. Vertical circulation changes under the influence of the wind. Tests were carried out to take seasonal changes into consideration. We assume that it is expedient to include the variability of the vertical effect. The three-dimensional model includes the influence of the diffusion processes. The influence of horizontal and vertical diffusion are included and their influence is variable.

YEREMENKO: If Dr. Thomann's three-dimensional model is based only on the concept of phosphorus as a limiting nutrient, could nitrogen become a limiting nutrient and if so, what is the influence of interference on eutrophication? Can the model state when the transition from one factor to another takes place? What kind of response or reaction takes place in the lake in response to a change in the phosphorus content?

THOMANN: The model considered both phosphorus and nitrogen and calculated complete interference of these factors. We calculated in time steps of one month over a period of 24 years. There are cases, it would seem, where the relationship of the content of limiting elements in the lake were displaced from organic phosphorus to organic nitrogen. Actually, transition to nitrogen limitation takes place in 10 years.

The model does not exclude the influence of various combinations of phytoplankton or rooted aquatic growths. It would be desirable to perfect a measure for evaluating lake eutrophication. This has only begun. We are beginning to attach models to estuaries, bays, and small lakes.

Our method using chlorophyll as the indicator for eutrophication is somewhat deficient because it is changed by the variability of chlorophyll in the phytoplankton. The model considers mass transfer from the sediment into water. The results of other works on lakes conclude that a general phosphorus regeneration from sediments comprises not more than 10% of all inputs. Therefore it is not a very important parameter and is considered only in general terms.

TULUPCHUK: Several species of phytoplankton change their buoyance. This causes an intense bloom. Have you modeled this?

THOMANN: The problem is very difficult and completely real in many areas. We have not modeled it. We have tried to construct the simplest model and even in this case we have obtained very complex equations. It is our opinion that when we have information on the actual lake condition, then everything will be clear and it will be possible to take all these elements into consideration. The model exists only as a model and the main point in this is that it is rarely connected with fact, which we actually observe. Our models have water quality management as their goal. Therefore I would say it is necessary to be brave enough not to consider all factors.

The process of evaluation of parameters begins with a comprehensive literature review for each parameter. We tried to distinguish as many parameters as possible, and parameters are introduced into the model. After four model runs, we can decide what parameters must be included. Eighty-two runs were devoted to the dynamics of the spring bloom and these details are shown in the graphs. We are now working with a three-dimensional model.

FUKSMAN: What kind of numerical values are assigned to the growth dynamics parameters and do these change from segment to segment?

THOMANN: In a three-dimensional model, numerical values of parameters are identical between different segments. In the future, we hope that some changes from segment to segment will be considered.

FUKSMAN: Were 700 equations actually introduced into the computer?

THOMANN: Yes, each specialist has a parameter which, in his opinion, is the main parameter. But it is necessary to choose the least possible number of equations because, given a large number of equations, the results can be unrealistic.

FUKSMAN: What is the largest size of a calculated system of equations?

THOMANN: From 1000 to 2000.

FUKSMAN: Did you initially evaluate the possibility of combining different vertical layers? Was a solution obtained?

THOMANN: We are at the stage of solving a three-dimensional model. We mean that we will be aggregating vertically but perhaps it will be successful to do it horizontally.

ROZHKOV: Do you take into consideration that there can be models which it is possible to adapt to the experiment?

THOMANN: Different methods which can describe the phenomenon are being examined. The basic approach is construction of a model based on the interrelationship of substances taking into consideration their internal kinetics. I include in our model data on flow in the lake which we can somehow control.

A three-dimensional model requires 40 minutes of machine time. This costs a lot of money. Each new variable introduced increases the cost of the computer run.

ZNAMENSKIY: The model described gives us good results. One of them is the conclusion that phosphorus retention in Lake Ontario continues for eight to ten years. This corresponds well with the period of complete water exchange. This confirms the fact that the basic factor is the water exchange factor. In our research on the "Volga Cascade," we came to the same conclusion. When we were trying to evaluate the contribution of different factors to the development of the chemical composition of water, in the most general approximation, we found that 80% of the control is derived from residence time of water, 10% from internal water body dynamic process, and all other factors the remaining 10%. Therefore it must be noted that in Dr. Thomann's model the most basic hydrological hydrodynamic parameters have been successfully determined. This allows us to obtain not only scientific conclusions but also practical recommendations.

TULUPCHUK: A model, not of a general ecosystem but of a specific water body, has been developed. The interpretation of the dynamics of seasonal changes, and in particular of algal productivity, is of great interest. This allows us to make a judgment about the assimilative capacity of a water body, and, on the basis of the model's reaction to the determined loading by nitrogen and phosphorus, to make recommendations for establishing loadings to a water body which will protect water quality.

YEREMENKO: I would like to comment on the role of the suggested models in a general system for monitoring water quality. The old models included both BOD and dissolved oxygen; these were preceded by models of algal biomass. Using the models, it is possible to calculate eutrophication processes. This is the significant value of Dr. Thomann's paper. Development of these models is being carried out on a large scale in the United States. We are trying to work in this direction and we've found difficulty in defining the constants which are introduced into the equations, and deriving their numerical expressions. We are trying to solve the problem of determining the constants and have not yet had satisfactory results.

THOMANN: I must fully acknowledge the role of limnologists, biologists, and other associates who have submitted a wealth of material for processing. I'm asking that all similar work begin with the simplest models because from them it is easier to move on to more complex models.

There is very little choice in determining parameters when constructing models. We chose the coefficients from data published in literature, then confirmed them in simulated laboratory conditions. This, unconditionally, is the greatest problem in the creation of models.

MODELING THE DISPERSION OF POLLUTION AND THERMAL POLLUTION IN WATER BODIES

O.F. Vasilyev

INTRODUCTION

This survey is compiled from the following eight reports:

1. Barannik, V.A. and Ye.V. Yeremenko (VNIIVO, Kharkov), "Calculations of Currents in the Nearshore Zone of a Deep Lake."
2. Barannik, V.A., Ye.V. Yeremenko, N.I. Selyuk, and B.S. Sinelshchikov (VNIIVO, Kharkov), "Calculations of Currents in Shallow Water Bodies with Estimated Wind Influences."
3. Vasilyev, O.F. and V.I. Kvon (Institute of Hydrodynamics, A. Sci. USSR, Novesibirsk), "Numerical Modeling of Thermal Pollution in Water Bodies."
4. Yeremenko, Ye.V. and V.I. Karas (VNIIVO, Kharkov), "Formation of Velocity Fields and Temperature under Wind Influence in a Water Body that Receives Heated Water."
5. Tolmazin, D.M. (Odessa Division of the Institute of Biology of the Southern Seas, A.Sci, Ukr, SSR, Odessa), "Calculations of Pollutant Dispersion in Shallow Basins (with the Kakhovka Water Reservoir as an Example)."
6. Kozhova, O.M., V.I. Gurman, G.I. Gershenhorn, and T.I. Vlasova (BGNII Irkutsk University Computer Center, Irkutsk), "Study of the Process of Bottom Pollution Dispersion with a Dynamic Model."
7. Bystrik, P.S. and V.I. Sinelshchikov (VNIIVO, Kharkov), "Transport of a Pollutant in a Shallow Turbulent Water Body."
8. Labzovskiy, N.A., A.M. Kryuchkov, and N.N. Filatov (Limnological Institute, A. Sci, USSR, Leningrad), and V.A. Rozhkov (Leningrad Division of the State Oceanograhic Institute), "The Analysis of Dispersion and Transformation in Polluted Waters in Large Lakes."

The problem of water pollution caused by discharge of industrial, agricultural, and municipal effluents containing various pollutants is very real. In some cases pollution can be linked with the discharge of heated water (thermal pollution).

Evaluation of the degree of influence of the pollution source on the water environment requires an evaluation of the resultant hydrodynamic and physical processes, as well as the chemical-biological processes of self-purification. Mathematical models of water quality have been developed to describe these processes. They are based on the application of hydrodynamic equations of turbulent liquid motion and include equations that describe the mass balance of miscellaneous pollutants as well as heat and oxygen. The major hydrodynamic question is the definition of diffusion coefficients when compiling these equations.

These reports studied separate aspects of the common problem of water pollution. The reports may be conditionally subdivided into three groups. This survey report is accordingly divided into the following three parts: 1) flow in water bodies; 2) hydrothermal processes in water bodies; and 3) convective diffusion. Special attention is given to numerical modeling of these phenomena in shallow water bodies.

The physical processes accompanying these phenomena are described by the following equations:

$$\frac{\partial U_1}{\partial t} + U_\alpha \frac{\partial U_1}{\partial X_\alpha} = -\frac{1}{\rho_o} \frac{\partial P}{\partial X_1} + \frac{\partial}{\partial X_\alpha} K_\alpha \frac{\partial U_1}{X_\alpha} + lU_2 \qquad (1)$$

$$\frac{\partial U_2}{\partial t} + U_\alpha \frac{\partial U_2}{\partial X_\alpha} = -\frac{1}{\rho_o} \frac{\partial P}{\partial X_2} + \frac{\partial}{\partial X_\alpha} K_{\alpha_2} \frac{\partial U_2}{\partial X_2} - lU_1 \qquad (2)$$

$$\frac{\partial P}{\partial X_3} = -g\rho; \quad \frac{\partial U_\alpha}{\partial X_\alpha} = 0, \quad \rho-\rho_o = -\beta (T - T_o)\rho_o \qquad (3)$$

$$\frac{\partial T}{\partial t} + U_\alpha \frac{\partial T}{\partial X_\alpha} = \frac{\partial}{\partial X_\alpha} K_{T_\alpha} \frac{\partial T}{\partial X_\alpha} + \frac{F}{C_p \rho_o} \qquad (4)$$

$$\frac{\partial S}{\partial t} + U_\alpha \frac{\partial S}{\partial X_\alpha} = \frac{\partial}{\partial X_\alpha} K_{S_\alpha} \frac{\partial S}{\partial X_\alpha} \qquad (5)$$

where t = time;
$X_\alpha (\alpha=1,2,3)$ = axes of a rectangular system of coordinates, the X_3 axis being directed vertically upward;
U_α = the components of the velocity vector;
P = pressure;
ρ = density;
ρ_o = a constant value;
T, S = temperature and concentration of the non-reactive pollutant, respectively;
T_o = the temperature at the density ρ_o;
$K_\alpha, K_{T\alpha}, K_{S\alpha}$ = coefficients, respectively, of the resultant viscosity, thermal conductivity, and diffusion (taking into account both the turbulent and the molecular exchange processes);
g = acceleration of gravity;
l = the Coriolis parameter; and
β = coefficient of thermal expansion of water. If the pollutant is not conservative, then the equation of pollutant transport contains an additional item. In particular, the equation for BOD is as follows:

$$\frac{\partial L}{\partial t} + U_\alpha \frac{\partial L}{\partial X_\alpha} = \frac{\partial}{\partial X_\alpha} K_{L\alpha} \frac{\partial L}{\partial X_\alpha} - KL \qquad (6)$$

where K is the mineralization coefficient, characterizing the rate of oxygen consumption due to oxidation of organic pollutants (the deaeration coefficient). Here the process of biochemical oxidation of the organic substances is approximately represented by the linear term KL which corresponds to the so-called monomolecular reaction (the Streeter-Phelps law). There are some hypotheses on the application of more complex (including non-linear) dependencies for describing the oxidation processes, which correspond, for example, to bimolecular or polymolecular reactions. If computers are used for calculations, then such complications are trivial.

The equations 1 to 6 describe motion and convective diffusion of heat and pollutants for a shallow water body. The Boussinesq approximation is implicit in these equations, which assumes that density variations can be neglected except in the terms representing buoyancy. In the case when a pollutant actively modified the dynamics of the current, the term connecting temperature and density is modified to include pollutant concentration,

$$\rho = F(T,S) \tag{7}$$

This can occur when calculating the intrusion of seawater into the mouths of rivers, or currents in estuaries.

To complete the system of equations 1 to 6, coefficients are defined to account for turbulent effects. In some of the works summarized here, constant coefficients are defined, in others, semi-empirical dependencies are used in the energy equations.

The contemporary models of water quality, in addition to the equations mentioned above, also contain an equation for dissolved oxygen. In each particular case, it is possible to add additional transport equations to such a system of equations. (If we are especially interested in the radioactive contamination of a water body, then an equation for this non-conservative pollutant can be added.) Besides, some components of the pollutant can interact. Therefore, when compiling equations for each components, these processes must be taken into consideration.

The specific versions of mathematical models similar to the one described above, especially the one-dimensional models, are now beginning to be widely used in engineering hydrology. It should also be noted that recently some research workers have made an effort to develop models that more completely describe the process of biochemical oxidation of organic substances (Vavilin, 1974), the process of photosynthesis, and several other processes that take place in the water ecological system (Umnov, 1971).

Flow in Water Bodies

The indispensible premise used in the study of pollutant dispersion is the quantitative determination of the velocity profile of flow in the water body. Therefore, the analysis of flow in the nearshore zone where treated effluents are usually discharged is of considerable importance. The problem of the constant density steady-state model has been considered by V.A. Barannik and Ye. V. Yeremenko (report 1). They used corresponding equations of the constant density. The terms for inertia and the terms for horizontal turbulent exchange were not calculated. The coefficient of vertical exchange is constant. The model for steady-state circulation in a deep lake has been solved using the Birchfield method (1972) for a lake having a bottom with a parabolic relief. The authors have generalized the approximation developed by Birchfield for the specific case of nearshore bottom relief with a rather well-defined underwater slope. Such is the case, for example, in Lake Baikal. The Ekman number has been accepted as a minor parameter. The problem actually was reduced to separate determinations of the pressure fields in the central and the nearshore parts of the water body; then these determinations were combined.

In accordance with the given wind velocity distribution over the lake, the pressure field is determined and then, by rather simple relations (Feltsenbaum, 1960), the velocity field of currents, characterizing the wind-induced circulation in the lake, is established. The velocity profile of currents calculated in this way at the free surface of Southern Baikal in its inner deep water area is shown in Figure 1a. The wind stress field at the lake surface is shown in Figure 1b. Such wind stress distribution corresponds to the southwest wind ("Kultuk") blowing along the lake. The circulation in the surface layer is such that water masses flow from the western to the eastern shore.

The dependencies allow us to calculate relatively simply, within the framework of the model, the steady-state currents in the nearshore part of the lake zone where water is sufficiently deep.

If the coefficient of the vertical eddy diffusivity is accepted as constant, it is impossible then to obtain the vertical velocity distribution of the wind-induced current that contains specific features of bottom and surface turbulent-boundary layers.

Barannik, Yeremenko, Selyuk, and Sinelshchikov, in report 2, attempted to describe these peculiarities of the current. They determined the vertical eddy diffusivity coefficient by the depth, as a third power polynomial:

$$K_3 = X_1 + X_2Z + X_3Z^2 + X_4Z^3 \qquad (8)$$

Its four coefficients, $X_i (i=1,2,3,4)$, were found on the assumption that the shear stress coefficient in the bottom and surface boundary layers were expressed in the form of:

$$K_3 = RU_*Z$$

where R is Karman's constant, U_* is the dynamic velocity in the bottom (or the surface) boundary layer, and Z is the distance from the bottom (or from the water surface).

The vertical eddy diffusivity coefficient was used for calculating the stationary wind-induced flow in a shallow water body.

As in the original equations, Barannik et al. used equations 1 to 3 written for steady-state flow. In this case, the water density and the coefficient of horizontal eddy diffusivity are accepted in the equations as constants. A drag coefficient was defined for the bottom and given the condition of pressure; the free surface, kinematic boundary conditions, pressure, and wind stress were defined and held constant. The problem was solved with a numerical algorithm similar to the one proposed by Mikhailova (1968) for calculating oceanic flow currents. Description of the numerical algorithm and an example of calculating the wind-induced flow in a rectangular water body of a constant depth were given.

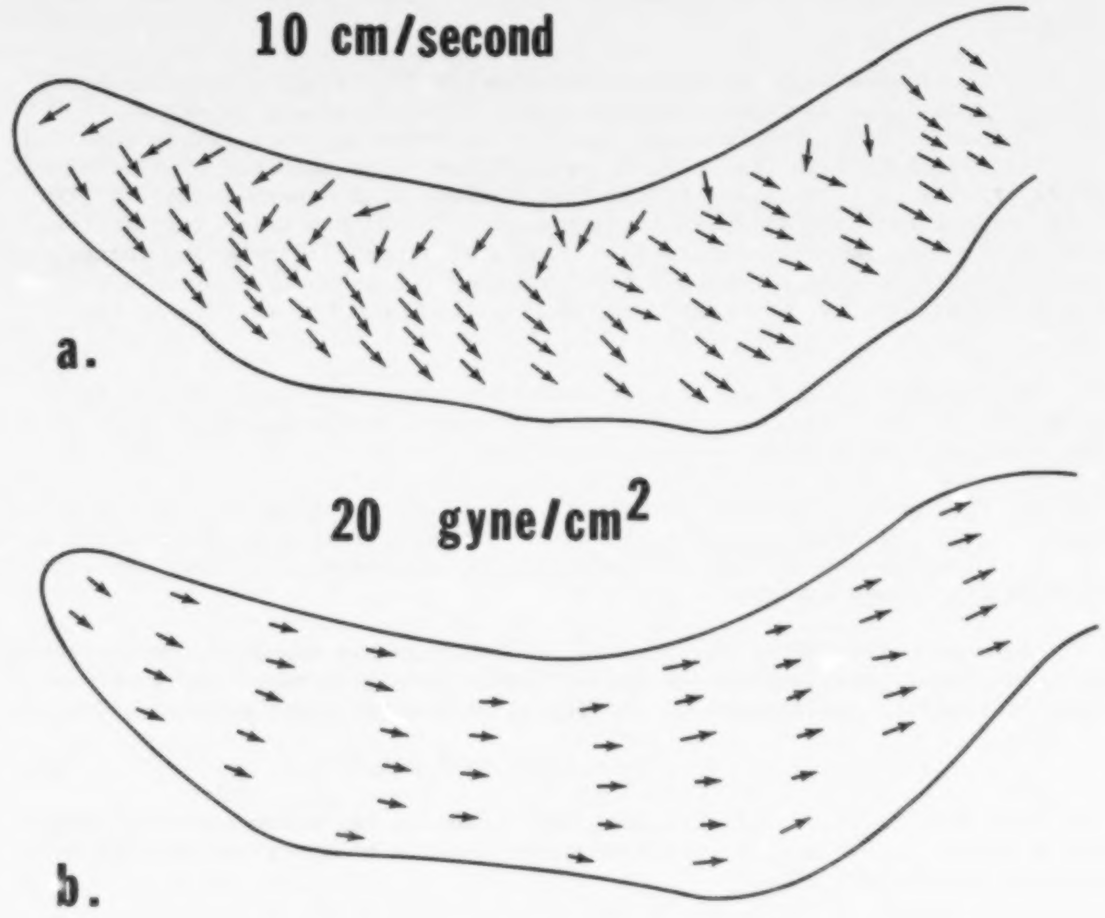

Figure 1. Diagram of calculated resulting flow velocity.
a. is the velocities activated on the surface layer of Southern Baikal by a southwest wind.
b. is the values of flow velocity and wind friction stress determined by vector lengths in the scales presented in the diagram.

Barannik et al. also considered the problem of the wind-induced flow in a water body with an irregular bottom. In this case, they used the motion equations which described conditions of dynamic equilibrium between the pressure force and the viscosity determined by the vertical eddy diffusivity change. Instead of the condition of adhesion at the bottom, the condition of sliding is accepted in the form of interconnection between the bottom friction stress and the local velocity. An algorithm of the numerical solution and the results of these calculations of a flow plan for a certain stretch of the Volga River are given in Figure 2.

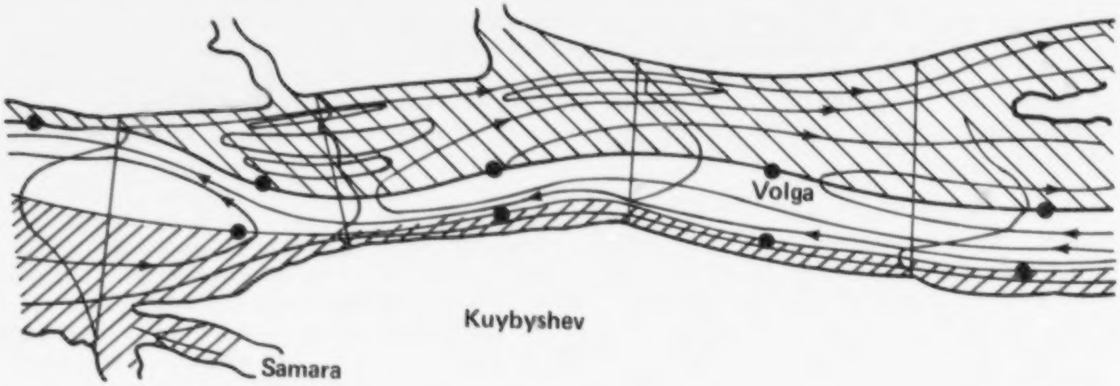

Figure 2. Plan configuration of flows in the channel with a wind direction opposite to river flow (arrows on the flow line show the direction of flow, circles designate the boundary flow line, flow function graphs are constructed on the cross-sections).

In this calculation, a wind direction was selected which produced a narrow zone along the left bank of the Volga River in which undesirable effluents were forced up the river. This picture of flow was formed in accordance with the integral stream function, which characterizes the flow averaged by depth.

For verifying the above method of assigning the coefficient of vertical turbulent eddy viscosity, the authors calculated the wind-induced longitudinally uniform flow in a flat trough. From the results of these calculations, it is possible to conclude that the vertical velocity profile of the wind-induced flow can be described sufficiently well by the proposed method of presetting the coefficient of vertical turbulent exchange.

Hydrothermal Processes in a Water Body

The hydrophysical problems connected with discharge of thermal water into water bodies when operating stream and nuclear power stations are becoming progressively more important. For solving these engineering, hydrological, and ecological problems, it is necessary to be able to forecast and regulate the temperature of water in water bodies. Dispersion of heat from a discharge in a water body is determined not only by the heat exchange processes but to a considerable degree by the character of specific hydrodynamic phenomenon occurring in the water body (for example, the appearance of a density stratification of the water body).

The density of thermal effluent water is temperature-dependent. And, although the temperature equation actually coincides with the inert pollutant transfer equation because of the interconnection of temperature and density, the velocity field and the temperature field are interconnected. As a result, the velocity field acquired an essentially different character than the flow of a homogenous liquid with constant density. Such plumes, entering the cooling ponds of thermal and nuclear electric power stations, are considered in the report of Vasilyev and Kvon (report 3). Two specific conditions are considered: hydrothermal phenomena in a shallow-water body of arbitrary shape; and a temperature stratified flow in a relatively narrow water body of oblong shape.

To solve the first (three-dimensional) problem, equations 1 to 4 are used. In these equations, the terms characterizing horizontal exchange and heat exchange may be omitted, if we assume that for flow in the cooling ponds, the coefficients of the turbulent viscosity and thermal conductivity are scalar quantities or that their values in different directions are of the same order. However, this assumption is not always correct; for example, the horizontal turbulent exchange is essential in mixing zones, when streams are spreading on the surface.

In such water bodies as lakes and reservoirs, the flow velocities are low and, in the equations of impulse flow, it is therefore possible to neglect the non-linear inertia terms.

In the authors' numerical model of temperature-stratified flow in the cooling pond, these simplifications of the problem are used; in particular, the non-linear inertia terms, as well as terms characterizing the horizontal exchange, are omitted. Then the equations 1 to 4, after simple transformations, may be written as follows:

$$\frac{\partial U_1}{\partial t} = g\frac{\partial}{\partial X_1}\left(Z + \frac{1}{\rho_o}\int_{X_3}^{Z_o} \rho\, dX_3\right) + \frac{\partial}{\partial X_3} K \frac{\partial U_1}{\partial X_3} + l U_2 \qquad (9)$$

$$\frac{\partial U_2}{\partial t} = -g\frac{\partial}{\partial X_2}\left(Z + \frac{1}{\rho_o}\int_{X_3}^{Z_o} \rho\, dX_3\right) + \frac{\partial}{\partial X_3} K \frac{\partial U_2}{\partial X_3} - l U_1 \qquad (10)$$

$$\frac{\partial U_\alpha}{\partial X_\alpha} = 0; \quad \rho-\rho_o = -\beta\rho_o\,(T-T_o) \qquad (11)$$

$$\frac{\partial T}{\partial t} + U_\alpha \frac{\partial T}{\partial X_\alpha} = \frac{\partial}{\partial X_3} K_T \frac{\partial T}{\partial X_3} + \frac{F}{C_p \rho_o} \qquad (12)$$

When developing these equations, it was assumed that the deviation of the level of free surface Z from its undisturbed level Z_o was insignificant (in comparison with the depth).

The water body is considered limited by the lateral cylindrical surface consisting of a hard-shore section and two liquid sections, (one through which water flows into the water body and the other through which water flows out).

The following boundary conditions are defined: the shoreline is impervious. At the water discharge section, normal velocity component and temperatures are given, and in the water intake sections, the normal velocity component is given. Kinematic conditions and tangential wind stress on the free surface are given, as well as the thermal flux. For calculating such important meteorological factors as dynamic wind action and heat exchange with the atmosphere, the connection between the tangential stress and velocity near the bottom is given as a condition on the bottom. The heat exchange through the bottom and the lateral walls is considered insignificant in comparison with the heat exchange through the open free surface, and is therefore not considered. When solving a non-steady-state problem, it is necessary to specify the boundary conditions, velocity distribution, temperature, and the state of the free surface in the beginning of the simulation.

The coefficients of turbulent exchange are determined by applying the equation of energy balance of turbulence. The problem is solved numerically by a method of incremental steps with an implicit difference scheme.

Formulation of the two-dimensional problem is similar. When studying flow and heat exchange processes in relatively narrow water bodies, the problem, in many cases, may be simplified by transition to a two-dimensional representation averaging the width of the water body. This method of representing stratified flow can be useful when examining the flow in water bodies which are narrow and shallow relative to the longitudinal dimension.

In this model, the channel is relatively straight and has only minor cross-sectional changes. Since there is density stratification, the dynamic and kinematic character of flow can be averaged in the longitudinal direction in each transverse section and thus can eliminate one spatial variable, reducing the problem to a one-dimensional case.

To evaluate the drag caused by friction against the side surfaces of the bed, the authors used the hydraulic formula for the tangential stress on the wall. This, added to the differential equations of the conservation of momentum and heat, provided initial and boundary conditions similar to those given for the three-dimensional problem. This model differed from the three-dimensional problem by setting the conditions at the input and outflow of the current in the region under study as well as defining the conditions at the bottom. In this case, the rate of flow was defined at the inflow and outflow cross-sections of the channel, or at the water surface level and temperature at those points where water flows into the area under consideration. The dynamic condition at the bottom was given the condition of adhesion, which may be considered as the extreme case of the more general condition described above.

This system was used for several numerical calculations of non-steady-state situations. The results reproduced the actual conditions observed during the experiment.

Yeremenko and Karas (report 4) modeled the wind-driven currents in a shallow water body affected by a thermal discharge with a two-dimensional model scheme. It was solved by complete thermodynamic equations of turbulent motion of an incompressible liquid with a scalar coefficient of turbulent viscosity. The coefficient of turbulent heat conductivity (eddy conductivity) was accepted as equal to the coefficient of turbulent viscosity. The coefficient of turbulent exchange was determined by using its known empiric dependence on the Richardson number. The differential equations of motion were written for vorticity and the function of flow. These equations were completed by the addition of initial and boundary conditions. The initial conditions corresponded to an immobile liquid at a particular temperature. The boundary conditions balanced the inflow and outflow of water to the water body and determined distribution of the velocities and temperature at the thermal discharge site, uniformity of the tangential stresses at the air-water interface, constant heat exchange between water and air at the free surface, and absence of heat exchange at the bottom of the water body.

Using Fromm's method (1967), the authors obtained a numerical solution to the equations for vorticity and temperature by using the explicit three-layer method, and solution of the equation of flow functions by using the explicit iterative method. The results of the calculation of several versions of horizontal flow are given, from which the authors concluded that wind action, thermal water discharge, and heat exchange considerably influence the character of flow. Depending on interrelationships among these factors, qualitatively different velocity and temperature profiles are formed.

Convective Diffusion of Non-Reacting Components

Dispersion of non-reactive pollutants in a water body is due to its advective transfer by flow and by turbulent diffusion. Since the character of the distribution of the passive substance does not influence the dynamics of the flow, the motion equations in this case may be solved independently of the pollutant diffusion equation, and then, using the velocity field, the equations of pollutant diffusion may be solved.

The problem of pollutant distribution in the Kakhovka water body is described by Tolmazin (report 5) when affected by the discharge of mining effluents. The calculations of water flow were made by the author using the stationary model of a shallow sea, developed by Feltsenbaum (1960). With this model, calculations of a flow were reduced to the determination of the flow integral function from elliptical differential equations and subsequent calculation of the horizontal components of velocity by the analytical dependencies. The integral function of flow Ψ is determined through the components of elementary flow Q_x and Q_y by the following formulae:

$$Q_x = -\frac{\partial\Psi}{\partial y}, \quad Q_y = \frac{\partial\Psi}{\partial x}$$

where x and y = the coordinate directions in the horizontal plane. The forces of inertia and the influence of the salt content of the discharged effluents on the flow dynamics and processes of horizontal turbulent exchange are not accounted for by the model. The effects of wind and of a river flowing into the water body on the currents are considered in the model by means of the wind stress on the free surface and the river flow at the side boundaries. At the liquid boundaries of the water body, the condition of free flowing liquid is accepted. In accordance with this condition, it is supposed that the elementary flows at the liquid boundary are directed perpendicularly to this boundary. Mathematically this condition is written as:

$$\partial\Psi/\partial n = 0$$

where n is the direction of the normal to the liquid boundary. By analysis of the results of calculations of the flows, the author concludes that the steady state currents, calculated on the basis of this model, are in agreement with the observed data.

The problem of diffusion of the pollutant was solved in two stages by Tolmazin. At the first stage, convective diffusion for the pollutant near the point source was examined. The pollutant was discharged into a flow moving at a velocity varying only with depth. This problem was solved with simplified non-time dependent equations for the convective diffusion of the pollutant in which there are no terms of vertical transfer of the pollutant by the flow and no terms of turbulent diffusion in the direction of the flow. Velocity distribution with depth and the coefficients of the vertical and cross turbulent diffusion were accepted as power dependencies on the vertical coordinate. Analytical solution of the pollutant dispersion near the point source was obtained by Tolmazin.

In connection with the solution of the convective diffusion of the pollutant, Tolmazin determined the coefficient of vertical eddy diffusivity by using the momentum and energy equations. However, the results of this analysis have no application for solving the main problem of pollutant diffusion.

At the second stage, the mean concentration of the pollutant was calculated in the region of discharge, outside the discharge mixing zone where the mining effluent was already completely mixed with the water medium.

Kozhova, Gurman, Gershenhorn, and Vlasova (report 6) studied the processes of pollutant dispersion in the nearshore zone of a water body. The problem was reduced to solving the pollutant diffusion equation (6). The right side of the equation contains a term which represents the presence of a source of pollutant. The current field was predetermined. There was no exchange of pollutants through the water surface or into the bottom, and the zero values for concentrations at the lateral boundaries of the region of the water body were accepted as boundary conditions. The problem was solved numerically by a method of incremental steps. The coefficients of eddy diffusion were constant but differed in direction. The results of a series of numerical experiments and different physical conditions are described in this paper.

Bystrik and Sinelshchikov (report 7) considered the problem of selecting the optimum algorithm for numerical solution of non-reactive pollutant dispersion in water bodies. The algorithms constructed on the basis of the explicit and implicit differential approaches were compared.

The basic idea of the comparison is to evaluate the limitations for a time step with Δt for the explicit approach. The authors assumed that the explicit scheme was optimal if: a) the limitations of the Δt time step for the convective and diffusive transfer are of the same order; and b) the Δt time step is of the same order as spacing characteristic for a non-stationary process. If not, the authors recommend the implicit method (i.e., the method based on the principle of separation) or a combination of the methods explicit for processes of one type and implicit for processes of another type, depending upon the completion of some characteristic inequalities.

In the work of Labzovskiy, Kryuchkov, Filatov, and Rozhkov (report 8), results of field measurements of the water quality indicators in the Onega and Ladoga lakes were statistically analyzed. They also studied processes of dispersion and dilution of effluents from pulp and paper plants.

Water quality measurement values for lignine, dissolved oxygen, calcium, pH, electrical conductivity, and turbidity were grouped by geographic proximity and water depth. This data showed dependent random values correlate highly with the main water pollution indicator, the concentration of lignine in water.

The influence of the dynamics of hydrological processes on the dispersion and dilution of effluents was studied. These processes, when statistically analyzed, are not steady-state and are non-uniform in space. Regularities of the hydrological processes were established on the basis of frequency with time, two-frequency auto-and mutual-spectral densities. The values of these statistical parameters were calculated and analyzed for a set of long term field measurements (six months) to determine the degree of this equilibrium and non-uniformity of the thermodynamic state of Lake Ladoga, the degree of anisotropy, and direction of the maximum flow variability.

Vetrov and Dekin conducted a field determination of effluent dispersion in deep water lakes with a radioactive tracer. For such a large water body as Lake Baikal, ordinary calculations (e.g., the equation of the steady turbulent diffusion) are complicated due to time-space changeability of the flow field and the necessity to consider temperature stratification and complex bottom relief, etc. In such conditions, the data required for production of pollutant dispersion in a lake can be obtained with the so-called tracer method by using a radioactive indicator. This experiment was conceived as a field study of the dispersion of a conservative pollutant in the lake by discharge of quantities of the isotope in the effluent. Relatively short-lived isotopes with a large yield of gamma rays are usually used as radioactive indicators. The concentration of the indicator in the water body is measured directly with the help of a radiometric probe-transmitter lowered to the required depth from a ship. In 1972-1975, the Institute of Applied Geophysics of the Hydrometeorological Service carried out a series of experimental discharges of a radioactive indicator from the Baikalsk pulp and paper works into Lake Baikal to determine the direction and velocity of the pollutant transfer, and the degree of dilution of the effluents at different distances from the point of discharge and during different hydrological seasons.

For some years in the USSR, mixing processes have been studied by following the release of fluorescent dyes into water bodies and monitoring the dispersion from discharges with towed fluorimeters.

The majority of research on the turbulent diffusion of non-reactive pollutants was connected with determination of the spacial characteristics of the field of averaged concentration. However, some problems, such as calculating marine effluent discharge, also require determination of other probability characteristics of concentration of fixed points in the water body. The work of Aitsam, Astok, and Juhat (1973) addressed this problem. On the basis of analysis of the existing mathematical models of turbulent diffusion, the authors concluded that these models could evaluate the distribution density of pollutant concentration probability with satisfactory accuracy only in the case of high concentrations. At low concentrations, there was a discrepancy in the results of the experimental observations.

Experimentally determined densities of the dispersion of pollutant concentration probabilities are given in Figure 3. The graphs show that the probability densities are of bimodal character. The authors explained the bimodal character of the curves by the influence of perturbations of different scales. They assumed that the probability density at a fixed point of a turbulent flow may be presented as the sum of two probability densities y(s) and f(s). The first describes the turbulent fluctuations of the pollutant concentration inside the plume, the second reflects the influence of the low-frequency perturbations, deflecting the plume axis. From the results of the experimental observations, the authors assumed that each of these probability densities can be approximated by the Gaussian distribution curves.

But, since the pollutant concentration cannot acquire negative values, the approximation is achieved through the single-sided normal truncated distribution.

The total probability density of the pollutant concentration is expressed in the following way:

$$P(S) = \frac{n}{\sqrt{2\Pi}} \left\{ \frac{\exp\left[-\frac{(S-\bar{S}_y)^2}{2D_y}\right]}{\sqrt{D_y}\left[0.5 + F\left(\frac{\bar{S}_y}{\overline{D_y}}\right)\right]} + \frac{2}{\sqrt{D_f}} \exp\left[-\frac{(S-S_f)^2}{2D_f}\right] \right\} \qquad (13)$$

where $\bar{S}_y$, $\bar{S}_f$, D_y, D_f are respective components of the mathematical expectation and dispersion of the substance concentration, F(x) is Laplace's function;

$$F = \frac{f}{\sqrt{2\Pi}} \int_0^{\alpha} e^{-\frac{x^2}{2}}\, dx \qquad (14)$$

where f is determined from the condition of normalization. In this case $f=0.5$.

The results of comparing the supposed distribution (equation 13) with the experimental data are given in Figures 3 and 4.

From the formula 13, it follows that the probability density distribution of the pollutant concentration is determined by the values of $\bar{S}_f$, $\bar{S}_y$, D_f and D_y. $\bar{S}_f$ is the "background concentration." In practical calculations for sea conditions, we propose accepting $\bar{S}_f=0$. The values of S_y, D_f and D_y can be determined from semiempirical equations describing changes of these characteristics.

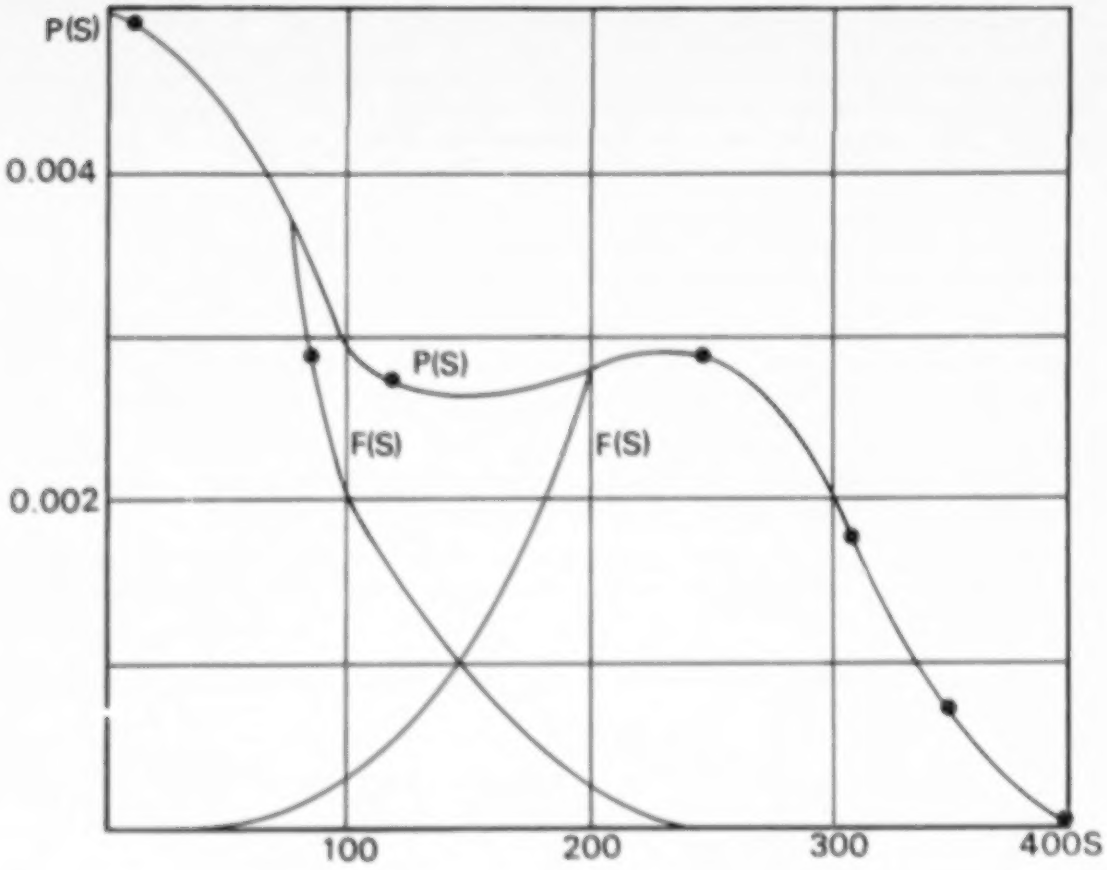

Figure 3. Probability density of the pollutant concentration dispersion in hydraulic flow at a distance of 1.5 m from stationary point source.

This report also gave some results of field study related to probability characteristics of the field of velocity in the Gulf of Finland, in particular dispersions and spectral densities of the turbulent fluctuations of velocity at the depth of 14 m from the level of the free surface.

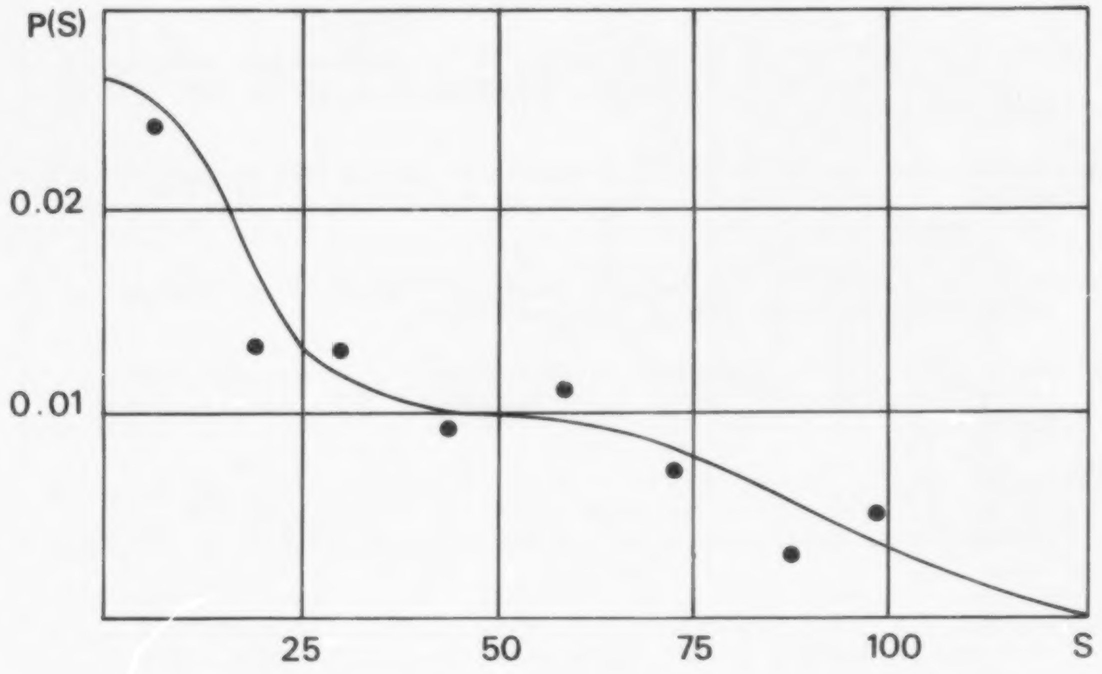

Figure 4: Probability density of the dispersion of dye concentration in a nearshore marine site.

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DISCUSSION

BOYCHUK: What methods did you use for identifying the coefficients of the mathematical models?

VASILYEV: Common heuristic methods were used in the models. The Hydrodynamics Institute has begun work on applying contemporary identification methods when determining hydrodynamic system parameters. For the future, a wider use of identification methods has been proposed for application to the model we have discussed. Our French colleagues have some interesting examples: They have used identification methods for determining hydrodynamic and hydrophysical characteristics of motion in the upper sea layer. They included the well-known corriolis parameter and obtained excellent results.

THOMANN: Please discuss the model verification.

VASILYEV: Professor Yeremenko can reply to this question for his paper. The heuristic method is used when applying mathematical models of water quality in rivers and modeling thermal pollution in water bodies. We are making the first steps towards transferring models to real situations. For the most part, we are relying on our experience and intuition. Ecological aspects open a new area for us; we will be relying on the experience of our American colleagues.

YEREMENKO: The calculations for currents in Lake Baikal were verified by an integral evaluation of the water body between middle and southern Baikal. The results were compared with Dr. Znamenskiy's field data. The characteristic circulation currents were described for characteristic wind situations on Baikal. These characteristics coincided with our calculations. The calculation of turbulent viscosity for depth of flow for given wind currents, allows us to determine the numerical value of polynomial coefficients used for calculating the velocity profile. A comparison of reference data with laboratory research gave excellent results.

MORTIMER: Professor Vasilyev has compiled an analysis of a large quantity of work. This is a great service. Steady states are rarely observed in nature. In all stratified flows, if the Richardson number is lower than the critical value, a certain situation takes place. The Richardson number expressed the balance between the terms which characterize the gravitational forces and turbulence. The Richardson number is a very important parameter. This number was included in Karas' and Yeremenko's model. I have three questions for the speaker in reference to this: (1) Was the Richardson number included in Professor Vasilyev's model? (2) Does Professor Vasilyev agree with me that modeling flow without the Richardson number is like playing Hamlet without the prince himself? and (3) Have non-sloping (non-discharge) streams been studied in Russia?

LOBZOVSKIY: Over the last few years the Limnological Institute has studied the dispersion of inflowing polluted waters in lakes. They have searched for methods and have familiarized themselves with the behavior of the polluted water. Polluted lake water does not dissipate but behaves in a complex manner. Heavy particles fall to the bottom forming a bottom flow. The lighter particles are spread along the surface, and behave as stream currents. The polluted stream is dispersed along the surface for up to 15-20 kilometers, assuming the temperature of the environment and having a density somewhat higher than clean water. The polluted water descends downward and forms lenses at various levels. We have presented this problem as a random, non-uniform vector process (Rozhkov method). This is one of many directions which can be taken to study the problem of polluted flow dispersion in lake water.

LICK: I agree with Dr. Mortimer that steady-state models are not suited to deep lakes. It is possible to apply them only to shallow lakes where a steady state is reached in 1 to 2 days. We reviewed the case when the inflowing water was warmer or colder than that in the lake. In the case where it was warmer, the inflowing water rose. And in the case where it was colder, the same thing took place but at the bottom.

VASILYEV: We have already discussed the role and meaning of steady-state problems. We have to consider that it is necessary to know how to solve steady-state problems in order to explain the characteristics of the flows studied. However, quasi-stationary flow systems can occur in practice. In regard to Richardson's number, Dr. Mortimer and Shakespeare are correct. Dr. Yeremenko does take Richardson's number into consideration. Dr. Tolmazin did not observe any stratification phenomena in his work on the Kakhovka Reservoir (a shallow, well-mixing reservoir). Therefore, density stratification is not considered in his work.

Regarding the Hydrodynamic Institute's model, for the sake of brevity in the paper, we were not able to fully describe the solution used. The system of equations in the report is supplemented by the turbulent energy equilibrium. This equation considers turbulent energy diffusion, turbulent energy inertia, the Archimedean Force, and turbulent energy dissipation; Richardson's number is concealed in the equation. However, we can separate it if we combine two terms--the turbulence energy generation and buoyance force.

Regarding Dr. Labzovskiy's statement, at present Hydrodynamics' specialists are studying jet flow. I can cite corresponding references. These specialists have discovered some good approaches to the physical and mathematical formulation of such problems.

IDENTIFICATION OF ECOSYSTEM MODELS BY FIELD DATA (INVERSE PROBLEMS OF ECOLOGY)

V.B. Georgiyevskiy

INTRODUCTION

We propose constructing ecosystem models to be identified in this paper on the basis of conventional differential equations (dynamic models) and in partial derivatives.

Identification is the determination of numerical values of coefficients for use in differential equations by firsthand observation of biotic and abiotic components of an ecosystem in a certain region.

In ecology, problems of this type are examined by identifying objects of automatic control (Prober, 1971; Beck, 1974; and Leaky and Skog, 1972). The usual identification method is to select numerical values for the model parameters, which minimize the mean square difference between the observed and calculated trajectories. That is, an execution of the procedure:

$$\min_{z} \sum_{j=i}^{n} [H^j - \tilde{H}^j]^T p^j [H^j - \tilde{H}^j], \qquad (1)$$

where $H^j (\tilde{H}^j)$ represents the j-th realization of the vector of observed (calculated) biotic and abiotic components of an ecosystem, p^j is a positively defined weighting function, T is a transpose index, and z is a vector of unknown parameters. These methods of ecosystem identification are based on the solutions of direct boundary conditions $\tilde{H}$.

Nevertheless, ecological problems possess some characteristics which essentially hamper this approach and sometimes render solutions impossible. The characteristics of inverse problems in ecology, which necessitate identification without using solutions of the direct boundary conditions, are as follows:

1. The ecosystem models may be of a very complicated type: the equations may be non-linear, of partial derivatives, and the number of equations may be large. The identification in this case may be realized, e.g., by using quasilinearization methods, integral transformations of Laplace, Fourier, Mellin, Kantorowitz, etc. But in all cases, the problem will prove to be multidimensional and quite complicated.

2. In ecology the information to be interpreted is essentially stochastic in character. Two circumstances are important here:

   a. The functions measured in a real ecosystem tend to be masked by variability. This property has an objective nature. It demands application of calculating procedures which smooth down the random "noise," i.e., mean square minimization, integration, etc. The procedure is determined by this property.

b. The boundary conditions where H^j is observed have not been obtained experimentally; in general cases they have a stochastic nature. The divergence between H^j and H^j are solutions of different boundary characteristic problems. The use of B.1 in this case leads to systematic errors caused by the method of defining parameters and not by objective reasons. The value of errors may be quite large since the boundary characteristic definition may prove to be an improperly posed problem. A similar situation is discussed in the next paragraph.

The circumstance just mentioned is one reason that necessitates the identification of ecological systems without solving direct boundary characteristics problems.

3. The third essential characteristic is the necessity to define the type of model more accurately in the course of identification. In a general case, the structure of ecological models is not initially known. When constructing a model, it is necessary to assume a priori every possible trophic link and the influence of as many parameters as possible, and when treating experimental results, the hypotheses concerning the type of links should be verified.

This problem cannot be solved by the standard methods of identification based on the utilization of theoretical solutions, in particular B.1. Moreover, the designation of model type can prove to be an improperly posed problem.

If we assume that both processes of diffusion and elimination exist in a steady-state regime,

$$\nabla^2 S - \gamma S = 0 \quad \text{at} \quad S\big|_{z=\infty} = 0, \tag{2}$$

where

$$\nabla^2 = \frac{1}{z}\frac{\partial}{\partial z}\left(z\frac{\partial}{\partial z}\right),$$

we find γ by measurements in two points:

$$\frac{S(z_1)}{S(z_2)} = \frac{K_0(\sqrt{\gamma}\, z_1)}{K_0(\sqrt{\gamma}\, z_2)} \tag{3}$$

But if there is really no elimination and the process is described by the equation $\nabla^2 S = 0$, then we will inevitably find by B.3 a parameter which differs as much as it can from the true one.

It is evident that similar errors are possible when designating trophic links and when studying the turnover of organic and biogenic substances.

The example discussed above is quite a simple one, and it would be easy to show a procedure here which would help correctly solve the hypothesis on the type of ecological mode. But in a general sense, this cannot be done. It should be noted that it would be possible to illustrate that the boundary characteristic scheme, discussed in the previous paragraph, is incorrect.

These characteristics of ecosystem identification lead us to conclude that it is expedient to find identification methods which would not use the solution of direct boundary conditions. This method can be based on integral transformations of differential equations of ecological models (Georgiyevskiy, 1971), aimed at excluding the experimental functions differential operation.

With such an approach, the identification problem changes from a problem of mathematical physics into an algebraic problem. Some results of the research in this direction will be outlined below. Identification methods are reduced to constructing an optimum set of integral transformations (in the sense of the number of arithmetical operations).

It is important to note that the procedure for defining parameters is deterministic. The presence of only high-frequency masking variability or "noise" is assumed. More complete information on "noise" ought to be used when studying the qualitative evaluations of the desired parameters.

Identification based on integral transformations enables us to avoid these difficulties:

1. Very complex models can be identified, since the difficulties in solving boundary condition problems are not important in this method;
2. The errors implicit in the designation of boundary characteristic schemes are excluded; and
3. Hypothesis on the type of models is formally solved, as only members with coefficients that differ considerably from zero are left in the model.

The identification of differential equations by integral transformations allows us, with a great degree of objectivity, to approximate real and highly complicated ecosystems by generalized model parameters. Identification of this type also allows us to define certain rules for ecological observations. It should be noted that the inverse problems of ecology are considered here from the view of interpreting hydrobiological information.

The Type of Differential Equations of Water Ecosystem Models

The identification method is determined by the type of differential equation terms, subjected to integral transformations. A widely used dynamic model of ecosystems is:

$$L = \frac{dC_i}{dt} - C_i(\alpha_i - \sum_j^n \beta_{ij}C_j) = 0 \quad i=1,2,\ldots,n \qquad (1.1)$$

Fairly generalized models, reflecting the dynamics of spatially heterogeneous water ecosystems, are described by differential equations in second order partial derivatives:

$$L = \mu\frac{\partial C_i}{\partial t} - \mathrm{div}(D\,\mathrm{grad}\,C_i) - \mathrm{div}(c\overline{v}) - C_i(\alpha_i - \sum_j^n \beta_{ij}C_j) = 0, \quad i = 1,\ldots,n \qquad (1.2)$$

In most cases, it is expedient to consider the equations with constant parameters, linear relative to these parameters, as this essentially simplifies the problem of excluding the experimental function derivatives. Finding the variable parameters is proposed generally in parametric form, for instance, by finding the constants for extension to a certain series. Thus, the problem of defining variable parameters is reduced to the problem of defining constants, which is a natural corollary of the general principle, i.e., rejection of solutions of boundary conditions.

In paragraph 4.1, a case of variable parameters, identification without parametrization will be discussed.

Model Parameters as Systemic Characteristic of Water Biocenosis

An important characteristic of inverse problem solving is the fact that the parameters defined by field data are a certain systemic characteristic of water biocenosis.

Real, complicated hierarchical ecosystems with extremely ramified links are approximated, when modeling, by generalized groups of producers, consumers, and degraders. The parameters which identify a complicated ecosystem with a generalized model in a specific area may differ substantially from practical coefficients of reproduction, feeding, death rate, migration, selection, and other ecosystem components. For this reason, the use of parameters obtained under laboratory conditions proves to be quite difficult for constructing models, and also accounts for the fact that model parameters obtained by field observations for specific ecosystems may differ substantially even under identical biocenosis.

The systemic characteristization of biocenosis by a certain vector of generalized model parameters makes it possible to reduce the study of waterbody pathology (toxic effect, pollution, eutrophication) and ecosystem condition monitoring to operative observation of the model parameters. The last circumstance defines the importance of inverse problem solution by field observations which are stochastic in nature.

Method of Intergral Transformations

The method of excluding experimental function derivatives is described in the following section, i.e., replacement of a differential equation by an integral algorithm which serves as an algebraic equation relative to the desired parameters.

The exclusion of derivatives may be achieved by a different type of integral transformation. Transformations differ in terms of the efforts expended and in limitations necessary to their structure. One method which can make it possible to solve the problem of identification is the M-method (modulating functions method) (Loeb 1965; Takava 1968). However, it is not the optimal method algorithmically.

The method of integral algorithms which is optimal in the sense of expended efforts is cited below. Its algorithms can be divided into two classes: (1) minimum smoothing algorithms using minimal required experimental information; and (2) maximum smoothing algorithms using all the experimental information. The smoothing of information is done with random weight, and the algorithms under such conditions require minimum effort.

The structure of integral transformations modified for exclusion of derivatives is:

$$L = \mu\frac{\partial H}{\partial t} - D_{\nabla}{}^2 H - \overline{v}\,\mathrm{grad}\, H - \gamma\,(C_H - H) = 0, \qquad (6)$$

where H is any of the ecosystem components and C_H is a certain constant.

The structure of integral transformation is defined by the necessity to exclude the higher derivative from the equation. The integral transformation of the differential equation L = 0 is done term by term and in succession relative to each independent coordinate. An important feature of the method is the fact that the integral analog of various expressions (table of images) may be calculated beforehand and the algorithm in each case can be obtained by a formal operation.

The method consists of the substitution for a differential equation by an integral one, and algebraic exclusion of the values not obtained experimentally. Further in the text, we will consider the argument x - any argument for a second order equation (e.g., x,y,z), and the argument t - any argument for a first order equation.

Minimum Complexity Intergral Transformation I for an Equation in Cartesian Coordinates

A. Image with respect to x. Let us consider the expression

$$L = \frac{\partial^2 H}{\partial x^2} - f = 0 \qquad (7)$$

where f represents all members along some coordinate except the second derivative. Transforming equation 3.2,

$$\int_{x_0}^{x}\int_{x_0}^{x} \left(\frac{\partial^2 H}{\partial x^2} - f\right) dzdz = 0$$

we obtain

$$H(x) - H(x_0) - \frac{\partial H(x_0)}{\partial x}(x - x_0) - \int_{x_0}^{x}\int_{x_0}^{x} f\, dxdx = 0.$$

Out of the last equation we compose a system of two algebraic equations taking $x = x_i$, $i = 1,2$ and excluding from the system the member conditioned by the value $\partial H(x_0)/\partial x$, which is not obtained experimentally. The integral analog obtained as a result of the operation is the differential equation L=0 minimum smoothing representation with respect to the argument x. It will be designed via $I_x^{*}(L)$,

$$I_x^{*}(L) = (x_2 - x_1)\int_{x_0}^{x_1}\int_{x_0}^{x} L\, dzdx - (x_1 - x_0)\int_{x_1}^{x_2}\int_{x_0}^{x} L\, dzdx. \qquad (8)$$

We put various expressions in 3.3 instead of L and obtain the table of minimum complexity images for various terms of differential equations by which algorithms for parameter determination can be formally obtained according to the kind of ecosystem model equations.

$$I_x^* \left(\frac{\partial^2 H}{\partial x^2}\right) = H(x_1)(x_2-x_0) - H(x_2)(x_1-x_0) - H(x_0)(x_2-x_1),$$

$$I_x^* \left(\frac{\partial H}{\partial x}\right) = \int_{x_0}^{x_1} Hdx(x_2-x_1) - \int_{x_0}^{x_2} Hdx(x_1-x_0),$$

$$I_x^* (H) = \int_{x_0}^{x_1} Hdx \cdot x_0(x_2-x_1) + \int_{x_0}^{x_1} xHdx(x_1-x_2) + \int_{x_1}^{x_2} Hdx \cdot x_2(x_0-x_1)$$
$$+ \int_{x_1}^{x_2} xHdx(x_1-x_0);$$

$$I_x^* (C_H) = \frac{1}{2}(x_1-x_0)\ (x_2-x_0)\ (x_1-x_2)\ C_H \tag{9}$$

B. Image with respect to t. Let us now consider the expression:

$$L = \frac{\partial H}{\partial t} - f = 0; \tag{10}$$

it is evident that

$$I_t^* (L) = \int_{t_0}^{t_1} Ldt, \tag{11}$$

where I_t^* is the inverse image of differential equation L=0 minimum smoothing with respect to argument t.

Consequently:

$$I_t^* \left(\frac{\partial H}{\partial t}\right) = H(t_1) - H(t_0),$$
$$I_t^* (H) = \int_{t_0}^{t_1} Hdt,$$

$$I_t^* (C_H) = (t_1-t_0)\ C_H \tag{12}$$

The Intergral Transformation of Complete Smoothing for Equation with Cartesian Coordinates.

A. Inverse with respect to x. The expression 3.2 should be transformed to

$$\int_{x_0}^{x} P_x \int_{x_0}^{x}\int_{x_0}^{x} \left(\frac{\partial^2 H}{\partial x^2} - f\right)\ dzdzdz = 0$$

Internal double integration excludes the derivative, external integration averages H, with the random weight P_x.

Fixing $x = x_i$, i=1,2,3, we exclude from the last expression, the terms dependent upon the value $\partial H(x_0)/\partial x$, (this value is not measured) and upon $H(x_0)$ so that instead of separate values of the functions, only mean values will be included into the algorithm. The resulting integral analog will bear the name of the differential equation L=0 inverse with respect to argument x under complete smoothing and it will be designated via $I_x(L)$. That is:

$$I_x(L) = \int_{x_0}^{x_1} P_x \int_{x_0}^{x}\int_{x_0}^{x} Ldxdxdx \cdot Ix + \int_{x_1}^{x_2} P_x \int_{x_0}^{x}\int_{x_0}^{x} Ldxdxdx \cdot IIx + \int_{x_2}^{x_3} P_x \int_{x_0}^{x}\int_{x_0}^{x} Ldxdxdx \cdot IIIx, \tag{13}$$

where $I_x = \int_{x_1}^{x}{}^{2} P_x dx \int_{x_2}^{x}{}^{3} P_x x dx - \int_{x_1}^{x}{}^{2} P_x x dx \int_{x_2}^{x}{}^{3} P_x dx;$

$$II_x = \int_{x_2}^{x}{}^{3} P_x dx \cdot \int_{x}^{x}{}^{1} P_x x dx - \int_{x_2}^{x}{}^{3} r_x x dx \int_{x_0}^{x}{}^{1} P_x dx;$$

$$III_x = \int_{x_0}^{x}{}^{1} P_x dx \int_{x_1}^{x}{}^{2} P_x x dx - \int_{x_0}^{x}{}^{3} P_x x dx \int_{x_1}^{x}{}^{2} P_x dx.$$

If we put various expressions in 3.8 instead of L, we will obtain a table of inverses of different types of ecosystem model differential equation terms. The table helps obtain the parameter determination of algorithm (see sections 10 and 11 [Georgiyevskiy 1971]).

B. Image with respect to t coordinate. Expression 3.5 should be transformed into:

$$\int_{t_0}^{t} P_t \int_{t_0}^{t} \left(\frac{\partial H}{\partial t} - f\right) dt dt = 0$$

Here internal integration excludes the first derivative, the second integration averages the experimental information with random weight P_t. Excluding the term dependent upon $H(t_0)$ from the last expression, we obtain the differential equation L=0 inverse with respect to argument t under complete smoothing.

$$I_t(L) = \int_{t_1}^{t}{}^{2} P_t dt \int_{t_0}^{t}{}^{1} P_t \int_{t_0}^{t} L dt dt - \int_{t_0}^{t}{}^{1} P_t dt \int_{t_1}^{t} P_t \int_{t_0}^{t} L dt dt = 0 \qquad (14)$$

Intergral Transformations of Complete Smoothing for Equations in General Curvilinear Orthogonal Coordinates

The integral transformations described above are expedient for the cases where the experimental area is formed by a parallelepiped. When interpreting information in a complex-form area, it would be inefficient to limit that information contained in the parallelepiped. Therefore, it is necessary to construct integral transformations for equations in the general curvilinear orthogonal coordinates.

Let us consider a differential equation of the second order with respect to the independent arguments x_1, x_2, x_3. The structure of integral transformations with respect to x_1, x_2, x_3 is uniform.

Similar to 3.2, we obtain $I_x(L)$, the inverse equation L=0 with respect to x_1 under complete smoothing:

$$I_{x_1}(L) = \int_{x_1^0}^{x_1^1}{}^{1} P_{x_1} \int_{x_1^0}^{x_1}{}^{1} \frac{h_1}{h_2 h_3} \int_{x_1^0}^{x_1}{}^{1} h_1 h_2 h_3 L dx_1 dx_1 dx_1 \left(\int_{x_1^1}^{x_1^2}{}^{1} P_{x_1} dx_1 \int_{x_1^2}^{x_1^3}{}^{1} P_{x_1} \int_{x_1^0}^{x_1}{}^{1} \frac{h}{h_2 h_3} dx_1 dx_1 - \int_{x_1^1}^{x_1^2}{}^{1} P_{x_1} \int_{x_1^0}^{x_1}{}^{1} \frac{h_1}{h_2 h_3} dx_1 dx_1 \int_{x_1^2}^{x_1^3}{}^{1} P_{x_1} dx_1 \right) + \int_{x_1^1}^{x_1^2}{}^{1} P_{x_1} \int_{x_1^0}^{x_1}{}^{1} \frac{h_1}{h_2 h_3} \int_{x_1^0}^{x_1}{}^{1} h_1 h_2 h_3 L dx_1 dx_1 dx_1 \cdot \left(\int_{x_1^2}^{x_1^3}{}^{1} P_{x_1} dx_1 \int_{x_1^0}^{x_1^1}{}^{1} P_{x_1}\right.$$

$$\int_{x_1^0}^{x_1}\frac{h_1}{h_2h_3}dx_1dx_1 - \int_{x_1^2}^{x_1^3}P_{x_1}\int_{x_1^0}^{x_1}\frac{h_1}{h_2h_3}dx_1dx_1 \cdot \int_{x_1^0}^{x_1^2}P_{x_1}dx_1\Bigg)$$

$$+ \int_{x_2^1}^{x_1^3}P_{x_1}\int_{x_1^0}^{x_1}\frac{h_1}{h_2h_3}\int_{x_1^0}^{x_1}h_1h_2h_3Ldx_1dx_1dx_1\Bigg(\int_{x_1^0}^{x_1^1}P_{x_1}dx_1\int_{x_1^1}^{x_1^2}P_{x_1}\int_{x_1^0}^{x_1}\frac{h_1}{h_2h_3}dx_1dx_1$$

$$- \int_{x_1^0}^{x_1^1}P_{x_1}\int_{x_1^0}^{x_1}\frac{h_1}{h_2h_3}dx_1dx_1\int_{x_1^1}^{x_1^2}P_{x_1}dx_1\Bigg) = 0 \qquad (15)$$

where P_{x_1} is a random weighting function for averaging experimental function values; h_1,h_2,h_3 are the Lamay coefficients; x_1^0,x_1^1,x_1^2,x_1^3 are the values of the x_1 coordinate in points 0,1,2,3.

The transformations I_{x2}, I_{x3} are obtained from I_{x1} when x_1,h_1,h_2,h_3 is substituted accordingly for x_2,h_2,h_3,h_1 or for x_3,h_3,h_1,h_2.

With the help of 3.10, we can obtain a table of images of the ecosystem model equations terms. In particular:

$$I_{x_1}\left\{\frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial x_1}\left(\frac{h_2h_3}{h_1}\frac{\partial H}{\partial x_1}\right)\right]\right\} = \int_{x_1^0}^{x_1^1}P_{x_1}Hdx_1 \cdot I_{x_1} + \int_{x_1^1}^{x_1^2}P_{x_1}Hdx_1 \cdot II_{x_1} + \int_{x_1^2}^{x_1^3}P_{x_1}Hdx_1 \cdot III_{x_1},$$

$$I_{x_1}\left\{\frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial x_1}(h_2h_3H)\right]\right\} = \int_{x_1^0}^{x_1^1}P_{x_1}\int_{x_1^0}^{x_1^1}h_1Hdx_1dx_1 \cdot I_{x_1} + \int_{x_1^1}^{x_1^2}P_{x_1}\int_{x_1^0}^{x_1}h_1Hdx_1dx_1 \cdot II_{x_1} + \int_{x_1^2}^{x_1^3}P_{x_1}\int_{x_1^0}^{x_1}h_1Hdx_1dx_1 \cdot III_{x_1},$$

$$I_{x_1}[H] = \int_{x_1^0}^{x_1^1}P_{x_1}\int_{x_1^0}^{x_1}\frac{h_1}{h_2\cdot h_3} \cdot \int_{x_1^0}^{x_1}h_1h_2h_3Hdx_1dx_1dx_1 \cdot I_{x_1} + \int_{x_1^1}^{x_1^2}P_{x_1}\int_{x_1^0}^{x_1}\frac{h_1}{h_2h_3}\int_{x_1^0}^{x_1}h_1h_2h_3H$$

$$dx_1dx_1dx_1 \cdot II_{x_1} + \int_{x_1^1}^{x_1^3}P_{x_1}\int_{x_1^0}^{x_1}\frac{h_1}{h_2h_3} \cdot \int_{x_1^0}^{x_1}h_1h_2h_3Hdx_1dx_1dx_1 \cdot III_{x_1} \qquad (16)$$

For particular cases of equations in cylindrical and spherical coordinates, the table of images is given by Georgiyevskiy (1971) for minimum and complete smoothing algorithms. The image with respect to argument t remains in the form 3.9.

I is the Method for Equations of n-th Order

The method described above is equally valid for the equations of the n-th order (Georgiyevskiy 1971)

P as a Method

When the argument values are equidistant, an image of differential equation L=0 under minimum and complete smoothing of a certain recurring method may be obtained (Georgiyevskiy 1971).

M as a Method (Loed 1963, 1965; Takava 1968)

The M method may be considered as a particular case of the n method under the special selection of weighting functions P. The M method is not algorithmically optimal, but it is very convenient because it is compact. The table of images based on the M method is of the following form:

$$M_x\left(\frac{\partial^2 M}{\partial x^2}\right) = (-1)^i \int_{x0}^{x1} M \frac{d^i y}{dx^i}\, dx, \quad i=0,1,2,\ldots,n \tag{17}$$

where

$$\left.\frac{\partial^i y}{\partial x^i}\right|_{\substack{x=0\\ x=x_1}} = 0, \qquad i=0,1,\ldots n-1.$$

Example of Algorithms

The algorithm for defining the ecological equation parameters is the equation inverse for several areas defined by number of the desired parameters if constant, or by the number of expansion terms if variable:

$$I_t I_x I_y I_z (L)_i = 0, \tag{18}$$

i=1,2,... is the number of integral transformation areas.

It is possible to define the algorithm for determining parameters for the dynamic system (1.1). For this case the derivatives of the first order with respect to t are present in 4.1, so it is quite sufficient to use 3.9:

$$I_t(L) = \int_{t0}^{t_1} PC_i dt \int_{t1}^{t_2} Pdt - \int_{t1}^{t_2} PC_i dt \int_{t0}^{t_1} Pdt - \alpha_i \left(\int_{t0}^{t_1} P \int_{t0}^{t} C_i dT dt \int_{t1}^{t_2} Pdt - \int_{t1}^{t_2} P \int_{t0}^{t} C_i dT dt \int_{t0}^{t_1} Pdt\right)$$

$$+ \sum_j^n \beta_{ij} \left(\int_{t_0}^{t_1} P \int_{t0}^{t} C_i C_j dT dt \int_{t1}^{t_2} Pdt - \int_{t1}^{t_2} P \int_{t0}^{t} C_i C_j dT dt \int_{t0}^{t_1} Pdt\right) = 0 \tag{19}$$

i=1,2,...n.

It should be noted that the parameters α_i, β_{ij} are established from the i-th analog in 4.2 independently of the other analog parameters; i.e., when establishing parameters the system 1.1 is "disintegrated."

The necessary number of independent algebraic equations according to the number of desired parameters α_i, β_{ij} may be obtained in the i-th analog 4.2 by changing the integration intervals t_1, t_2 or the weighting function P. At this point the spacially heterogeneous ecosystem model 1.2 identification algorithm is identified.

For the reasons shown in previous examples, the index i is omitted. For simplicity, only one space coordinate is considered. Assume that div V = 0, D = constant, and using the minimum smoothing algorithm 3.4, 3.7, given $x_0=0$, $t_0=0$, we obtain:

$$I^*(L) = \mu\{x_1 \int_{x1}^{x_2}[c(x,t_1) - c(x_1,0)]\,dx - x_1x_2\int_{x1}^{x_2}[c(x,t_1) - c(x,0)]\,dx + (x_1-x_2)$$

$$\int_0^{x_1} x[c(x,t_1) - c(x,0)]\,dx\} - D\{x_2\int_0^{t_1} c(x_1,t)\,dt - x_1\int_0^{t_1} c(x_2,t)\,dt - (x_2-x_1)$$

$$\int_0^{t_1} c(0,t)dt\} - v\{(x_2-x_1)\int_0^{t_1}\int_0^{x_1} c(x,t)\cdot dxdt - x_1\int_0^{t_1}\int_{x1}^{x_2} c(x,t)dxdt - \alpha\{(x_1-x_2)\int_0^{t_1}\int_0^{x_1}$$

$$xcdxdt - x_1x_2\int_0^{t_1}\int_{x_1}^{x_2} cdxdt + x_1\int_0^{t_1}\int_{x1}^{x_2} xcdxdt\} + \sum_j^n \beta_j\{(x_1-x_2)\int_0^{t_1}\int_0^{x_1} xcc_jdxdt - x_1x_2$$

$$\int_0^{t_1}\int_{x_1}^{x_2} cc_jdxdt + x_1\int_0^{t_1}\int_{x1}^{x_2} xcc_jdxdt\} = 0 \qquad (20)$$

Determination of Variable Parameters

By parametrization, for example, by expansion into a certain series, the problem of determining variable coefficients may be reduced to the problem of determining constant coefficients as described above. The number of expansion parameters during identification should be sufficiently large, but only parameters which differ significantly from zero should remain in the final result. It is also possible to obtain integral transformations of differential equations with variable parameters; thus an integral equation relative to the desired parameters will be obtained.

We are limited by a case of one independent variable, because the distribution to a case of several arguments will be quite similar. Assume that

$$\frac{d}{dx}\{k(x)\frac{dH}{dx}\} = f(x) \qquad (21)$$

Multiplying this equation by a system of linearly independent functions $\Psi_i(x)$, i=1,2 with the conditions $\Psi_i(0) = \Psi^{(1)}(0) = 0$, and double integration within the limits (0,x), for determining k(x), we obtain a system of two Volterrian integral equations:

$$H(x)\Psi_i(x)k(x) + \int_0^x H(z)\,[(x-z)\Psi_i^{(2)}(z)] - 2\Psi_i^{(1)}(z)]\,k(z)dz + \int_0^x H(z)[(x-z)$$

$$\cdot\,\Psi_i^{(1)}(z) - \Psi_i(z)]\,k^{(1)}(z)dz = F_i \qquad i = 1,2 \qquad (22)$$

To determine k(x) from the equation

$$k(x)\frac{d^2H}{dx^2} = f \qquad (23)$$

a system of three Volterrian integral equations must be constructed:

$$H(x)\Psi_i(x)k(x) + \int_0^x H(z)\,[(x-z)\Psi_i^{(2)}(z) - 2\Psi^{(1)}(z)]\,k(z)dz + 2\int_0^x H(z)\,[(x-z)$$

$$\Psi_i^{(1)}(z) - \Psi_i(z)]\ k^{(1)}(z)dz + \int_0^x H(z)\ [(x-z)]\ \Psi_i(z)k^{(2)}(z)dz = F_i(x)$$

$$i = 1,2,3 \qquad (4.7)$$

The accuracy of numerical solutions to integral equations will be determined by the statistical characteristics of experimental functions when establishing parameters by expansion into a certain series.

Determination of Dependent Parameters for Ecosystem Models

Of all the coefficients in the ecosystem model differential equations, only one is dependent in an algebraic sense. The rest are independent. By using the previous methods and only differential equations, it is possible to determine only independent coefficients, since one of the terms in the integral analog must be free. However, a problem may arise in establishing all coefficients of the equation's terms, for example, μ, D, v, γ in 1.2.

It should also be noted that several ecological parameters are included in the boundary condition values (meteorological factors, substrate effects, exchange between the epilimnion and the hypolimnion, advective flows). Consequently, the process of identification in general must provide methods to determine both independent and dependent parameters of ecological models simultaneously, and to use any information on boundary conditions values to simplify the integral transformation and increase the accuracy of identification (however the boundary condition value of the derivative may remain).

The general method for determining the whole complex of dependent and independent parameters is: the differential equation (for the ecosystem model) should be replaced by an integral one, and one of the terms, dependent on the experimental function derivative, should be determined by information on the boundary condition. In the expression derived, all the parameters will prove to be independent. It should be noted that after the first integration, a continuity equation for a finite volume is often guaranteed. Sometimes it is simpler to construct an equation of this type on the basis of physical considerations.

It should be emphasized that in the process of obtaining parameter determination algorithms, only part of the information on boundary conditions is used. This information is insufficient for solving a boundary condition problem. Therefore, the algorithms obtained make it possible to interpret complicated forms of motion (multidimensional, non-steady-state regime, law of random movement, etc.). They prove to be much more generalized than the theoretical solutions in which complete sets of boundary conditions values are used, but they are not as generalized as algorithms for determining independent parameters where boundary conditions values are not used.

This statement is illustrated with a simple example. Suppose that an ecological situation is described by the equation:

$$L = \mu\frac{\partial c}{\partial t} - D\frac{\partial^2 c}{\partial x^2} + R$$

and the value

$$D \frac{\partial c(C,t)}{\partial x} = Q(t) \tag{24}$$

is measured experimentally, then the identification algorithm μ, D, R has the form:

$$I_{T_t} I_{T_x} [\int_0^x L dx]_i = 0 \tag{25}$$

under 5.1.

Here I_{T_t} and I_{T_x} are integral transformations, their structure is derived from 3.9, then the variable t is the argument in I_{T_t}, and in I_{T_x} it is the variable x.

The content of 5.2 necessitates integral transformation, with calculation of the boundary condition. This approach may be easily generalized for a multidimensional case and for various coordinate systems.

Construction of the Algebraic Equations Systems

In essence, the outlined integral algorithm method reduces the problem of determining parameters to an algebraic problem, that is, substitution of algebraic equations for differential ones relative to the parameters. The solution of these equations requires the application of all the technology which has been developed to the solution of the algebraic problems.

The main requirement is to work out well-specified systems of equations (the choice of integration intervals and weighting functions, orthogenalization of matrixes, etc.), to verify the condition number $A = \|A\| \, \|A^{-1}\|$, and to evaluate the error of the solution.

After complete algebraic reduction of the problem, the concept of observation and identification are applied to the integral algorithms procedure (Li 1966).

Calculation of Intervals

The problem is reduced to the use of the Newton-Cotes quadrature and cubic capacity formulas, Gaussian and Tchebysheff's quadratures with tabulated coefficients and node, and also to the approximation of certain special experimental functions. The details are given by Georgiyevskiy (1971).

Parameter Determination When Inadequate Information Is Available on Some Part of the System

When interpreting ecological observations, the information on some coordinates may be insufficient. For instance, the information on process dynamics is often abundant but the number of sampling points is relatively small. For such cases, the following approach is recommended: the presence of derivatives in the differential equation on coordinates with a "large" amount of information should not be considered a complication. However, assuming the exclusion of these derivatives by a certain integral transformation, then the derivatives on the coordinates with "little" information should be excluded by using a priori information on the type of movement, by solving boundary condition problems, or by choosing special approximation laws (Georgiyevskiy, 1971).

For example, the algorithm for determining parameters (both dependent and independent μ, D, γ) for the equation describing an ecological situation which has cylindrical symmetry, is written as:

$$L = \mu \frac{\partial C}{\partial t} - D\nabla^2 c - \gamma c = 0; \qquad \nabla^2 = \frac{1}{z}\frac{\partial}{\partial z}\left(z \frac{\partial}{\partial z}\right) \tag{26}$$

given a small quantity of information on z.

In integral form, the minimum smoothing algorithm follows:

$$I^*_{T_t} I^*_{T_z} \left[\frac{1}{z}\int_{z_0}^{z} zLdz\right] = 0 \tag{27}$$

when

$$D \cdot \partial C(z_0,t)/\partial z = Q/2\Pi z_0,$$

where $I^*_{T_t}$ and $I^*_{T_z}$ are the integral transformations of 3.7 with respect to t and z.

Choosing the approximation $c = B(t) + D(t)\ln z$, and finding B and D according to measurements at two points, we obtain:

$$\mu\{[c(z_1,t_i) - c(z_1,0)] + [c(z_2,t_i) - c(z_2,0)]\} \ln \frac{z_2}{z_1} - 1)\} - D \cdot 4\frac{\ln\frac{z_2}{z_1}}{z_2^3}$$
$$\cdot \int_0^{t_i} [c(z_1,t) - c(z_2,t)]dt - \gamma\int_0^{t_i} [c(z_1,t) + c(z_2,t)\cdot(\ln\frac{z_2}{z_1} - 1)dt$$
$$= \frac{2}{\Pi}\left(\frac{\ln\frac{z_2}{z_1}}{z_2}\right) \cdot \int_0^{t_i} Qdt, \qquad i = 1,2,3 \tag{28}$$

Statistical Model

It is expedient to construct a statistical model when random values are defined by the measurement errors. Evaluation of the accuracy of these ecological parameters is a problem of evaluating the probability properties of a random value formed as a result of nonlinear operator influence on a random field. This problem can be solved by existing methods.

Numerical Examples

A direct method of verifying the identification integral algorithms consists of parameter determination from the results of analytical or numerical solutions of boundary condition problems with preset parameters (test problems). Georgiyevskiy (1971) states that such calculations are given for determination of the parameters (3.1) and for the equation

$$\alpha\left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2}\right) = \frac{\partial H}{\partial t} \tag{29}$$

in Cartesian and cylindrical coordinates by I. M methods; for

$$\beta \frac{d^2H}{dx^2} - \delta H = 0 \tag{30}$$

for: $$\beta_k \frac{1}{z} \frac{d}{dz}\left(z\frac{dH_k}{dz}\right) - \gamma_{k-1}(H_k - H_{k-1}) - \gamma_k(H_k - H_{k+1}) = 0, \quad k = 1.2, \ldots .n \tag{31}$$

For these equations, examples of parameter determination from field observation data are given by Georgiyevskiy (1971).

The variable parameter was determined in the equation:

$$\frac{d}{dx}\left(\int_0^H k(z)dz\frac{dH}{dx}\right) = 0 \tag{32}$$

The parameter: $k(z) = k_0 + k_1 z + k_2 z^2 + k_3 z^3$ (33)
was derived from data sets which had a great deal of "masking" variability (for solution of the equation 10.4, the variability was superimposed from the random numbers table). Up to a certain level of interference, the function regenerates steadily.

The basic model equation for interpreting ecological information is:

$$L = \mu\frac{\partial c}{\partial t} - D\frac{\partial^2 c}{\partial x^2} - v\frac{\partial c}{\partial x} - \alpha c = 0 \tag{34}$$

J.L. Fuchsman (Institute of Hydrobiology, Ukrainian SSR Academy of Sciences) identified this equation's parameters by I* and I-methods, using an ALGOL program for a BESM-6 digital computer. The program has been verified by the test data of Georgiyevskiy (1971); the accuracy of the solution of the hypothesis on the presence of v and α parameters by I-methods was verified. If the solutions of an equation, where α, or α and v were absent, are used, then with 10.6 these parameters become equal to zero. This program was used to interpret the ecological experiment described below.

Division of the Water Body into Segments

To solve the inverse problems using the results by hydrobiological and limnological observations, and to organize ecological observations on water bodies, categories which have uniform characteristics must be developed. A convenient method of grouping for inverse problems of water ecology has been developed at the Institute of Internal Waters Biology of the USSR Academy of Sciences by Butorin, Smirnov, Sklyarenko, and Malinin. This method offers: a) the initial classification of the system of criteria; b) transformation of the initial grouping into smaller associations using factor analysis and principle component analysis; c) the classification into approximately similar parts. This approach was developed by the authors using the data from 22 synchronous surveys during 1960-1964 at observation stations on the Rybinsk reservoir with a 110-dimensional vector of hydrological and biological indicators.

Method of Organizing Observations

This section reviews the methods of ecological observation by information interpreted by the inverse problem solution. The general conditions, satisfactory for natural ecological observations, are formulated by the identification and observation criteria (Li, 1966). This method of organizing ecological experiments established a system of data collection, number of points, number of measurements, and type of information measured for a simple determination of differential equation coefficients for ecosystem models.

Several features should be noted:

1. The minimum volume of experimental data is determined to interpolate to a minimum power by a second order polynomial. For example, to define one coefficient of the one-dimensional diffusion equation, it is necessary to have at least three points along the spatial coordinate and two measurements at each point. The two-dimensional diffusion equation requires six points along a plan not situated on a second order curve. When the number of parameters to be determined is larger, the minimum volume of experimental information must be increased so that a system of independent algebraic equations can be used.
2. The accuracy of parameter determination, when other conditions are equal, depends on the accuracy of integral calculation. Therefore, it may be expedient not to use the common equal-dimensional observation grid; instead, the measurement points and the data-collection features should be situated in nodes of orthogonal polynoms, if it is assumed that the quadrature and cubic capacity formulas of Gaussian and Tchebysheff's types are used.
3. To determine the ecological parameters that are independent coefficients in models, only the information on the equations' functions is necessary. The parameters will be found with the common character of the equations themselves. Determination of dependent coefficients necessarily requires some schematization and information on boundary condition values.

Field Experiment for Determining Ecological Equation Parameters

This method of identification serves as a basis for organizing a complex ecological experiment at the Kremenchug reservoir by the Institute of Hydrobiology, Ukrainian SSR Academy of Sciences, to determine the ecological model parameters based on an equation system (1.2). During 1972, five surveys were performed every three days in May, June, and October to measure hydrochemical, hydrobiological, and hydrophysical data. The species composition of phytoplankton, zooplankton, bacteria, and phytobenthos were measured; volatile organic substances, ammonia and nitrate nitrogen, iron, dissolved oxygen, pH, phosphorus, temperature, and volume at 5000 x 1000 x four meters at eight stations were also measured. The monitoring stations were organized along two lines at right angles--six stations along one axis and three stations along the other. Three depths were measured at each station.

D and α, β parameters are determined (μ is accepted to be 1; v is measured in the course of the experiment), which give a systemic characteristic of the biocenosis under observation. A dispersion analysis of the experimental data is made. The program was used in the treatment of the experimental data.

The research will determine the basis for constructing a hydrarchical series of models: stratification models, dynamic models, models of dimensionally heterogeneous biocenoses. Aspects of these models (such as the oxygen stratification model) conform well with field observations.

The inverse problems of ecology have been discussed with reference to several models of planktonic ecosystems. The models refer to some extremely complicated water ecosystems. Moreover, they are intended to study and forecast ecosystem behavior under anthropogenic influences. The problem of hydrobiological field data interpretation, given a description of ecosystems by differential or stochastic equations, is examined as a problem of identifying mathematical physics equations by stochastic processes. The identification is as follows: determination of ecosystem model parameters that contain a priori systemic characteristic of the ecosystem. A characteristic of the inverse problem in ecology is the necessity to reveal the structure of models on the basis of experimental data and to determine the parameters of models for which solution of direct problems is impossible (non-linear equations, stochastic boundary characteristic conditions, a priori unknown structure of models). The method of identification is based on integral transformations of ecological model equations. The structure or core of integral transformations of differential equations in general orthogonal curvilinear coordinates guarantees optimal algorithms in the sense of the amount of arithmetic operations. Parameters, which are algebraically both independent and dependent, constant, and variable coefficients of ecological model equations, have been identified.

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DISCUSSION

VAVILIN: Will the coefficients which are being determined depend on the type of interaction among the parameters in complex systems?

GEORGIYEVSKIY: We assume some kind of interaction, which actually does not exist. In the process of identification, it is necessary to propose a model by considering many parameters. After identification, it is possible to discard, according to coefficient value, those bonds which do not exist. The latter is more complex. If we propose a model which is simpler than reality, then the coefficient value can be random. The parameter value will not reflect a biological and physical meaning, but will be a certain characteristic more complex than the operator. What short cuts are there in ecology? In the first method, it is not difficult to propose all the interactions and find all the coefficients. The second method is considerably more difficult: When constructing ecological models, we generalize simplified groups.

OSTROMOGILSKIY: Can you give us an example of a real model which illustrates the basic premise of your report?

GEORGIYEVSKIY: In my report, I talked about real parameters which could not be determined by other methods. In the model of the Kremenchug Reservoir, reproduction and consumption coefficients were found which cannot be determined by any other methods. The reproduction and consumption coefficients for generalized groups were determined within the framework of the coefficients which characterize the predominant groups.

THOMANN: I have seen similar work in the U.S. and I consider this to be a promising approach. I would, however, like to make a few remarks. Inverse methods, in my opinion, preliminarily assume a certain ignorance of the ecosystem. For example, a large number of experiments show that there are determined phytoplankton growth limits. Over the last five years, scores of determinations of the assimilation of nutrients by phytoplankton have been made. There is also detailed information on zooplankton grazing. Experiments are extremely valuable when constructing models. A zooplankton would devour a phytoplankton, not knowing where it was located--in a bottle or in a lake. The inverse method can lead to a point where we will be able to say nothing definitive until we obtain more data. Experimental conditions are valuable. They must integrally be introduced into the model.

GEORGIYEVSKIY: The paper emphasizes that the two methods for studying ecosystem models (direct and inverse) must supplement one another. Let me give an elementary example. If we have a known predator/prey system, then prior to the inverse problem solution, we can state that the ratio of the coefficients of predator reproduction to prey extinction is greater than the ratio of coefficients of predator extinction to prey reproduction. It is necessary to expand the model by means of a large number of such relationships.

MATHEMATICAL MODELING OF THE HYDRODYNAMICS AND DISPERSION OF CONTAMINANTS IN THE NEARSHORE

Wilbert Lick

INTRODUCTION

Because of possible harmful effects of contaminants, it is necessary to be able to predict the dispersion and fate of contaminants in large lakes. The nearshore regions of these lakes are especially important since:

1. the nearshore regions are where contaminants are generally introduced and therefore their concentrations and effects are generally greater than in the offshore regions; and
2. the nearshore regions are of more particular interest to us for such uses as recreation, water supplies, and fishing.

Recent studies have demonstrated significant differences between nearshore and open-lake waters.

After a contaminant is introduced into a lake, it is dispersed by physical processes and transformed by physical, chemical, and biological processes. A large amount of work has been and is being done to understand the transformation processes of importance in lakes. At the same time, it has been recognized that the dispersion process must also be understood. In addition to field work, considerable theoretical work (much of it numerical) has been undertaken to predict the circulation in lakes. Until recently, most of this work has dealt with the overall circulation in lakes, not specifically with the nearshore. For example, most numerical models of the circulation in large lakes employ a numerical grid of two to six miles, a grid far too large to treat the nearshore in adequate detail.

In a part of research effort sponsored by the U.S. Environmental Protection Agency (EPA), we have developed numerical models which can realistically describe the currents and the dispersion of contaminants throughout large lakes and especially in the nearshore hydrodynamic and dispersion models, which necessarily include procedures for coupling the nearshore and offshore lake circulation models. Specific applications of these models have been made to predict the dispersion of contaminants under specific conditions and to understand the physical consequences of contaminant dispersion.

To illustrate our numerical models and some of their applications, these specific examples will be presented here. They are:

1. the wind-driven currents, both open-lake and nearshore, in Lake Erie under steady-state, constant density conditions;
2. the dispersion of a contaminant from a river into Lake Erie by wind-driven currents under present conditions and as modified by proposed large man-made islands such as a jetport in the lake; and

3. the dispersion of heat in the discharge from a power plant on Lake Michigan.

Several different numerical models of lake currents have been developed and are summarized by Lick (1975). Some models are relatively simple and give little detail. Others are more complex and detail the flow, but require relatively lengthy computer time. Two of our hydrodynamic models are used in the examples presented here. They are a constant-density, steady-state model and a variable-density, time-dependent model. Both are fully three-dimensional and give all three components of velocity at any point in the lake.

The basic equations used in the modeling of lake currents are the usual hydrodynamic equations for conservation of mass, momentum, and energy plus an equation of state. In sufficiently general form for most lake modeling, these equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\begin{aligned}\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} - fv &= -\frac{1}{\rho_r}\frac{\partial p}{\partial x} \\ &+ \frac{\partial}{\partial x}\left(A_H\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_H\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_v\frac{\partial u}{\partial z}\right)\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} + fu &= -\frac{1}{\rho_r}\frac{\partial p}{\partial y} \\ &+ \frac{\partial}{\partial x}\left(A_H\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_H\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(A_v\frac{\partial v}{\partial z}\right)\end{aligned} \tag{3}$$

$$\frac{\partial p}{\partial z} = -\rho g \tag{4}$$

$$\begin{aligned}\frac{\partial T}{\partial t} + \frac{\partial uT}{\partial x} + \frac{\partial vT}{\partial y} + \frac{\partial wT}{\partial z} &= \frac{\partial}{\partial x}\left(K_H\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_H\frac{\partial T}{\partial y}\right) \\ &+ \frac{\partial}{\partial z}\left(K_v\frac{\partial T}{\partial z}\right) + S\end{aligned} \tag{5}$$

$$\rho = \rho(T) \tag{6}$$

where u, v, and w are the fluid velocities in the x, y, and z directions respectively, t is the time, f is the Coriolis parameter which is assumed constant, p is the pressure, ρ is the density, ρ_r is the ambient or reference density, A_H is the horizontal eddy viscosity while A_v is the vertical eddy viscosity, K_H is the horizontal eddy conductivity while K_v is the vertical eddy conductivity, g is the acceleration due to gravity, T is the temperature, and S is the heat source term.

Several approximations are implicit in these equations. These are:

1. The pressure is assumed to vary hydrostatically;

2. The Boussinesq approximation, which assumes that density variations are small and can be neglected compared to other terms except in the hydrostatic equation, is valid; and

3. Eddy coefficients are used to account for turbulent mixing effects in both the momentum and energy equations.

The appropriate boundary conditions are dependent on the particular problem to be solved. At the free surface, $z = \zeta$, usual conditions are:

1. the specification of a stress due to the wind,

$$\rho A_V \frac{\partial u}{\partial z} = \tau_x, \quad \rho A_V \frac{\partial v}{\partial z} + \tau_y \tag{7}$$

where τ_x, τ_y are the specified wind stresses in the x and y directions, respectively;

2. a kinematic condition on the free surface,

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} - w = 0; \tag{8}$$

3. the pressure is continuous across the water-air interface and therefore the fluid pressure at the surface equals atmospheric pressure p_a,

$$p(x,y,\zeta,t) = p_a; \tag{9}$$

and 4. a specification of the heat flux at the surface,

$$q = -\rho K_V \frac{\partial T}{\partial z} = H(T-T_a) \tag{10}$$

where q is the energy flux, H is the surface heat transfer coefficient, and T_a is the air temperature.

At the bottom, the conditions are those of no fluid motion and a zero heat flux. Variations on these boundary conditions and other boundary conditions are discussed in the following sections as they are needed.

Wind-Driven Currents in Lake Erie

The Great Lakes, to which most of our models have been applied, are comparatively large, closed bodies of water. The Coriolis force is important but the variation in the Coriolis parameter is not. The currents in these lakes are primarily driven by the wind. Currents due to through-flows from incoming rivers to a single outflowing river are comparatively small except near the mouths of the rivers. In addition to two relatively continuous causes of currents, temperature and hence density, gradients cause currents and also modify existing currents. These thermally driven currents are important mostly in the summer months. For the rest of the year, when density is fairly constant throughout the lakes, a constant density assumption is valid. Time-dependent effects can of course be important. However, there are periods when currents in lakes (especially Lake Erie) are essentially steady or can be treated in a quasi-steady or time-averaged manner, and this makes a steady-state analysis appropriate and useful.

The problem considered here is a description of the steady-state, wind-driven currents in Lake Erie under constant density conditions (Gedney and Lick, 1972; Sheng and Lick, 1975). The numerical model used in the calculations is an extension of Welander's shallow lake model (Welander, 1957). In this model, in addition to the constant-density and steady-state assumptions, it is assumed that the nonlinear convection and horizontal diffusion terms can be neglected. One can show that the neglect of these terms is a valid approximation throughout the lake except in narrow regions or boundary layers near shore, regions which are less than 1 km wide (Sheng and Lick, 1975). These boundary layers are neglected in the present calculation.

The above assumptions, along with the assumption that the vertical eddy viscosity is constant or a simple function of depth, allows one to integrate the equations of motion analytically. This procedure reduces the governing equations to two dimensions. Introduction of integrated velocities defined by $U = \int_{-h}^{0} u dz$ and $V = \int_{-h}^{0} v dz$, where $h = h(x,y)$ is the local depth, and an integrated stream function ψ defined by $U = \partial\psi/\partial y$ and $V = -\partial\psi/\partial x$ reduces the equations to the single equation (Gedney and Lick, 1972):

$$\nabla^2\psi + \gamma_1 \frac{\partial\psi}{\partial x} + \gamma_2 \frac{\partial\psi}{\partial y} = \gamma_3 \qquad (11)$$

where γ_1, γ_2, and γ_3 are functions of the local depth and bottom slopes, and γ_3 is also a function of the applied wind stress. The appropriate boundary condition is that of no normal flux at the shore, or ψ is constant along shore. Once this equation has been solved for ψ, all three components of the velocity can be calculated as continuous functions of depth as well as the surface elevation.

The topography of Lake Erie is shown in Figure 1. Some of its more important features are as follows. It is approximately 386 km long and 80 km wide near the mid-point of the long axis. Topographically, it can be separated into three basins: 1) a shallow western basin with an average depth of 7 m, 2) a large, relatively flat central basin with an average depth of 18 m, and 3) a deeper, cone-shaped eastern basin with an average depth of 24 m. The western and central basins are separated by a rocky chain of islands, which creates multiple connections on the surface of Lake Erie.

The value of the stream function on the mainland shore is determined by the river inflows and outflows. The values of the stream function on the island boundaries are not known a priori. These values are determined by the condition that the surface elevation be continuous around each island, i.e., $\oint(\partial\zeta/\partial s)da = 0$, where the integration path is around each island. In the calculations presented here, three islands were incorporated.

The above equation for the stream function was solved by finite difference methods. At all grid points, a five-point central difference equation was used. For purposes of the computation, Lake Erie was divided into two regions. One was a region surrounding the islands approximately 80 km by 64 km in which a 0.805 km (0.5 mile) square-grid was used. The second region was

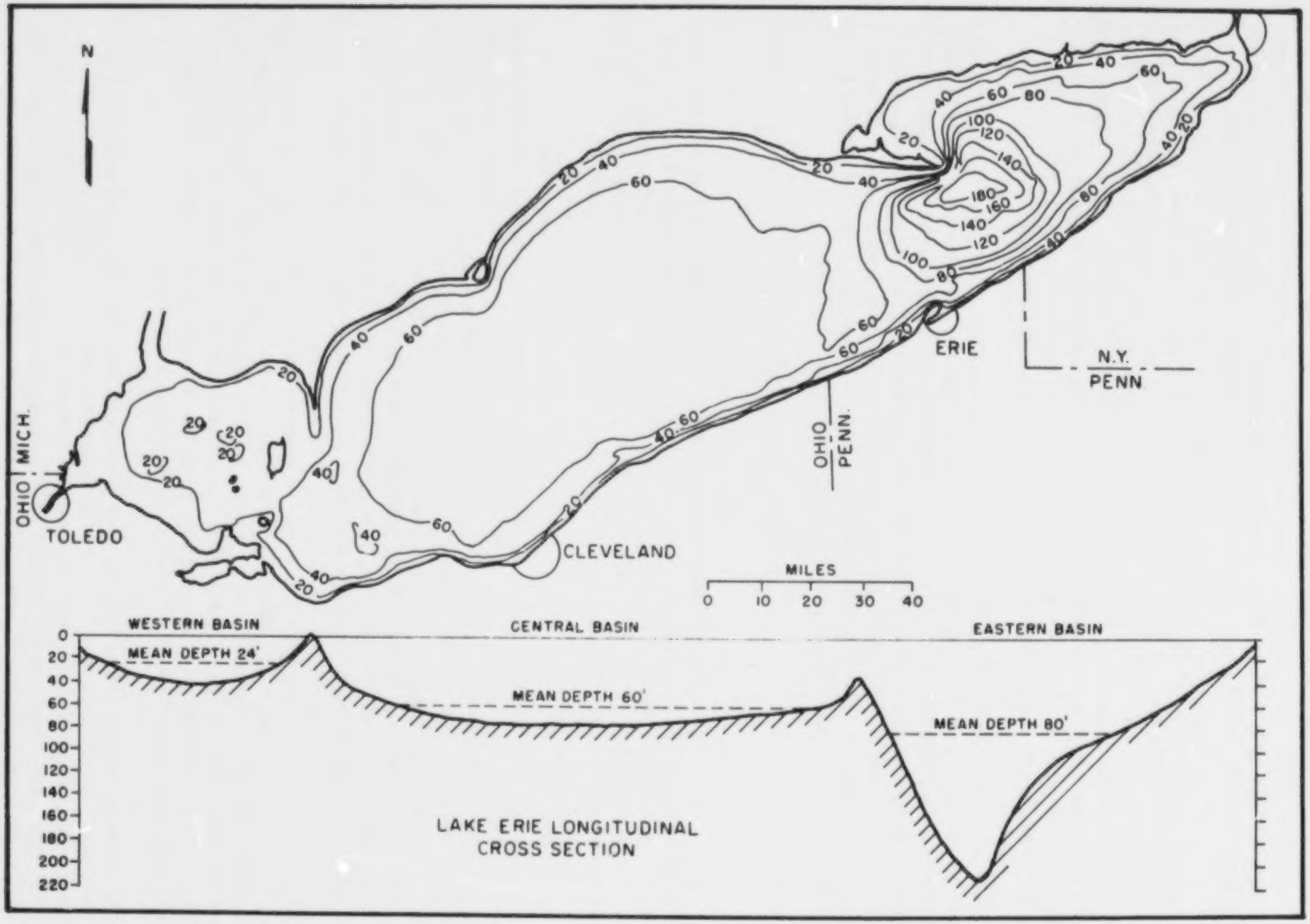

N
OHIO
MICH.
TOLEDO
CLEVELAND
ERIE
OHIO
PENN.
N.Y.
PENN
MILES
0 10 20 30 40
20
40
60
80
100
120
140
160
180
WESTERN BASIN
CENTRAL BASIN
EASTERN BASIN
MEAN DEPTH 24'
MEAN DEPTH 60'
MEAN DEPTH 80'
0
20
40
60
80
100
120
140
160
180
200
220
LAKE ERIE LONGITUDINAL
CROSS SECTION

Figure 1. Lake Erie bottom topography (Monin and Yaglom, 1973).

composed of the remainder of Lake Erie where a 3.22 km (2 mile) square-grid was used. A total 5050 grid-points were used. The 0.805 km grid-size was found necessary to obtain consistent and accurate ζ line integrals around each island and also to accurately represent the island boundaries.

A combination of successive over-relaxation by points and lines was used to solve the system of finite-difference equations. A complete iteration was first performed in one region. From these values, interim interface values between the two regions were formed for the second region. A complete interaction was then performed in the second region. From these values in the second region, interface values were in turn formed for the first region. The process was repeated until the maximum relative error between successive iterations at any grid point was less than 10^{-5}.

For a constant eddy viscosity, numerical solutions for the stream function and velocities in Lake Erie were obtained for a variety of wind directions and magnitudes. For the calculation presented here, the wind was assumed to be uniform over the entire lake surface with a magnitude of 10.1 m/sec and direction W50°S. This uniform wind condition was found to be a valid approximation for the period for which the calculations and field observations were compared. A friction depth d ($d=\sqrt{2A_V/f}$) of 27.4 m was used because it provided the best agreement between the numerical results and current-meter measurements. This value of d corresponds to an eddy viscosity A_V of 38.0 cm^2/sec. From the results of many different calculations, it was found, for best agreement between the calculations and observations, that A_V must be taken to increase as the wind magnitude increases with either a linear or quadratic relationship (Simons, 1974, for a similar conclusion). The results presented here include a Detroit River inflow of 5380 m^3/sec and an equal outflow via the Niagara River.

Plots of the horizontal velocities are shown in Figures 2 and 3. In these figures, the beginning of the arrow represents the actual location of the current represented by the arrow. The magnitude of the velocity can be determined from the velocity scale indicated on the figure. Note that the velocity scale is different for each figure.

The solution for the velocities represents the sum of the Ekman drift currents and the gradient current. Figure 2 shows that a top surface mass flux is being transported toward the eastern boundaries, primarily in the direction of the wind but deflected to the right by the Coriolis force. As shown in Figure 3, a subsurface current driven by the pressure gradient returns the surface mass flux in the opposite direction. In the central and eastern basins, surface currents are in general smaller in the center of the lake than near the shore. This effect is essentially due to the relatively large subsurface return current down the center of the lake which is opposite in direction to the surface current and subtracts from it.

EPA established a system of automatic current-metering stations in Lake Erie in May 1964. The EPA data consisted of velocity readings taken every 20 minutes. To compare the water current measurements with our calculations, the 20-minute readings were vectorially summed over a 24-hour period to form a resultant current. The resultant magnitude is equal to the vector sum

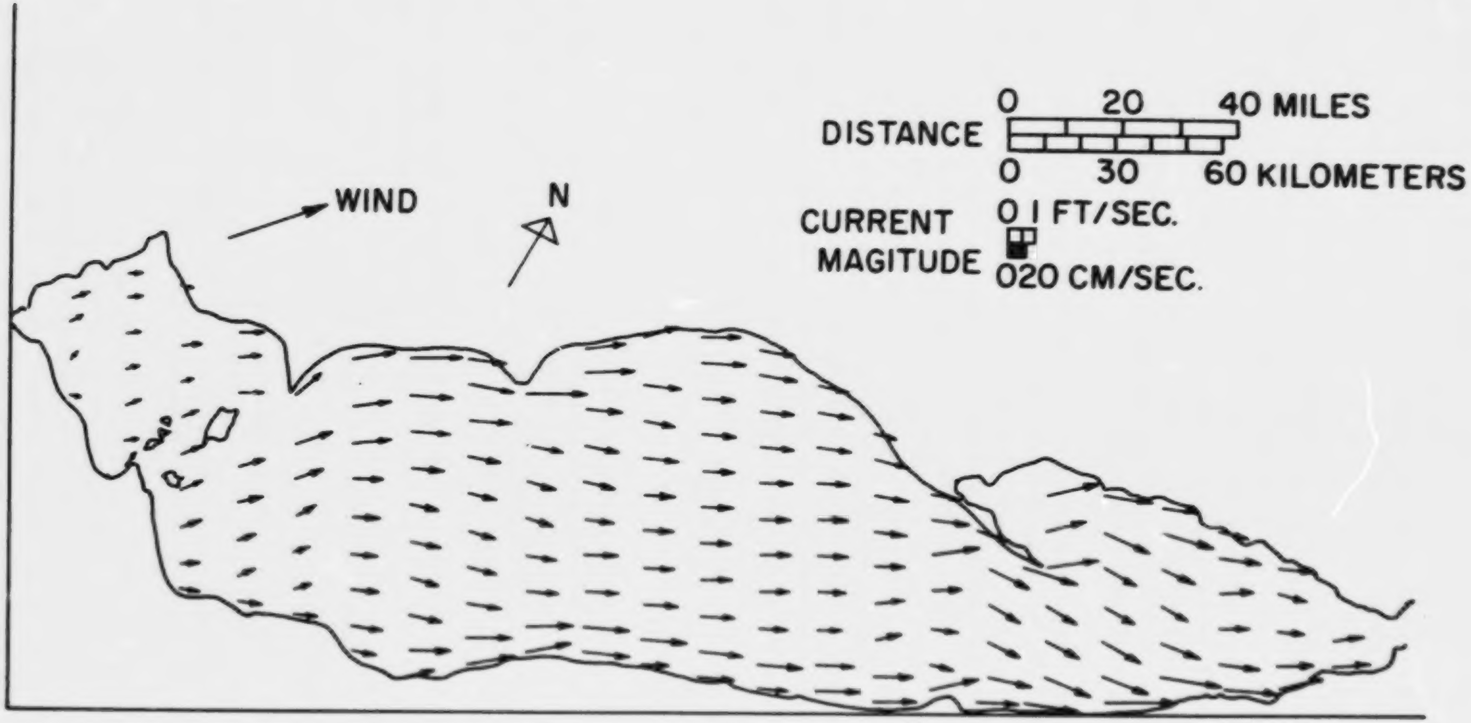
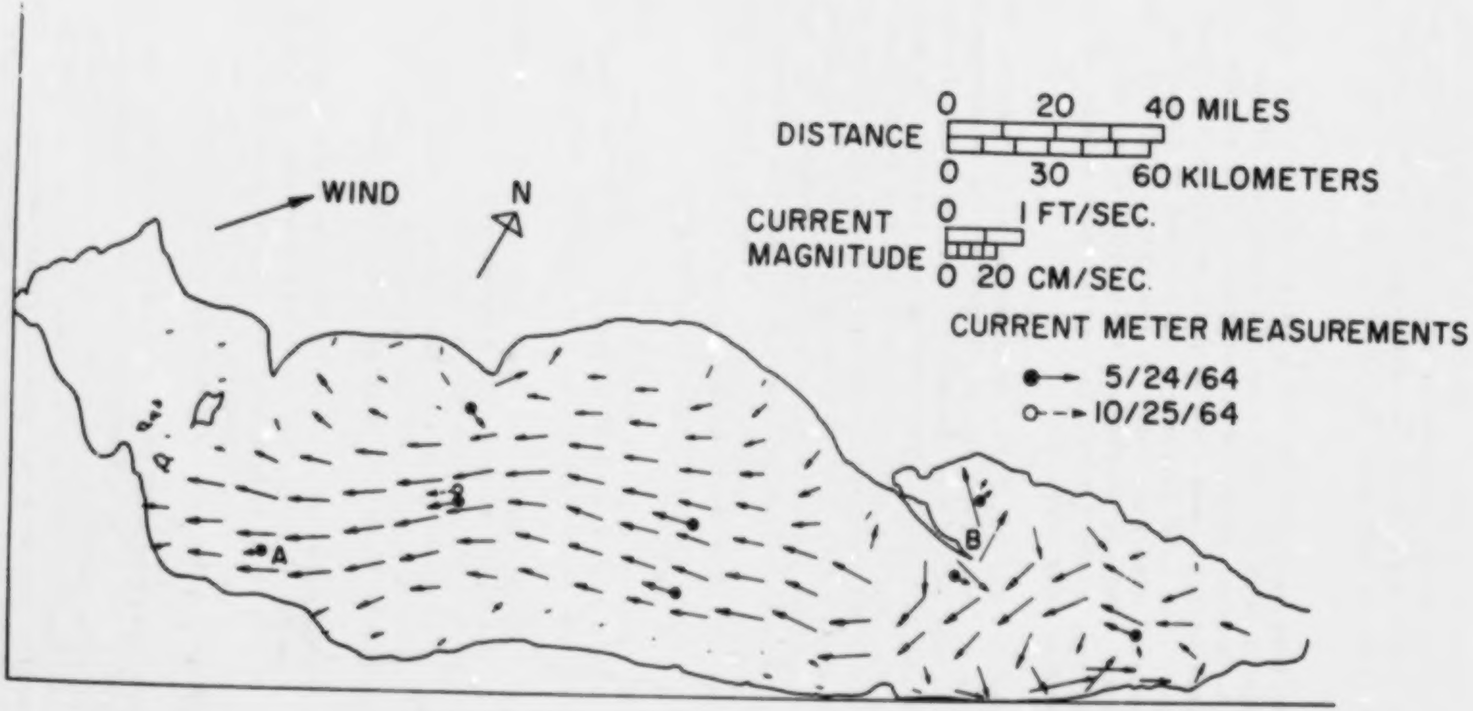

WIND
N
DISTANCE
0 20 40 MILES
0 30 60 KILOMETERS
CURRENT
MAGITUDE
0 1 FT/SEC.
0 20 CM/SEC.

Figure 2. Horizontal velocities at a constant 0.4 m (1.5 ft) from surface. Wind direction, W50S; wind magnitude, 10.1 meters per second (22.7 mph); friction depth, 27.4 meters (90.0 ft); rivers: Detroit, Niagara.



DISTANCE
0 20 40 MILES
0 30 60 KILOMETERS
CURRENT MAGNITUDE
0 1 FT/SEC.
0 20 CM/SEC.
CURRENT METER MEASUREMENTS
5/24/64
10/25/64
WIND
N
A
B

Figure 3. Horizontal velocities at a constant 9.9 m (32.8 ft) from surface. Wind direction, W50S; wind magnitude, 10.1 meters per second (22.7 mph); friction depth, 27.4 meters (90.0 ft); rivers: Detroit, Niagara.

magnitude divided by the number of readings. During the time periods used for comparison, wind was fairly steady for two or more days so that any seiche currents were small. The vector summing of the data over a 24-hour period should remove part, if not all, of any small seiche current since the largest seiche period is 14 hr.

Wind used in the numerical calculations for a particular day was determined by taking the average of both the direction and magnitude of the 24-hour resultant wind at each U.S. Weather Bureau shore station. This average wind, with its magnitude increased by 1.48, was then used to determine the shear stress at the water surface. The 1.48 factor was determined by comparing shore data with "over-the-lake" wind data taken by the EPA. On May 24, 1964, the resultant "over-the-lake" wind was determined to be 10.1 m/sec with direction W50°S. The resultant winds for two days prior were within 20° of this direction and at somewhat less magnitude.

The current-meter data for May 24, 1964, as measured at 10 m below the surface, is shown in Figure 3. Note that the positions of the measurements are different from those of the calculated currents. The agreement is markedly good in both magnitude and direction. The discrepancy between the magnitudes of the measurements and calculations at point A at 10 m is believed to be a measurement error since this measurement became erratic at a later date. The magnitudes of the measurements in the region of point B at first appear to be considerably different from the calculated values. However, the agreement is believed to be satisfactory when one considers that the currents are changing rapidly with distance in the point B area. Also in Figure 3 the meter measurements are plotted for a W43°S wind at a velocity of 8.6 m/sec taken on October 25, 1964. Again the agreement is quite good.

Figure 4 shows the vertical velocities at mid-depth in the lake. Arrows pointing toward the top of the plot represent vertical velocities toward the lake surface. Large upwellings and downwellings are apparent near shore.

The same basic model described above has also been applied to a nearshore region of Lake Erie (Sheng and Lick, 1975). The purpose was to analyze in more detail the currents in the offshore Cleveland area (Figure 1) in the absence of, and including, a proposed jetport. Advocates of this airport have proposed that it be situated on a large man-made island in the lake approximately eight kilometers offshore in waters approximately 15 m deep.

The steady-state currents were calculated for various jetport configurations and for different wind velocities. In these calculations, a 0.4 km grid was used nearshore and a 3.2 km grid offshore. Two jetport configurations were studied in detail: an island approximately 4.8 km by 3.2 km, and this same island extended with an 0.4 km wide causeway to shore (Figure 5).

An interesting result obtained from the calculations was that the island configuration did not modify the flow appreciably but the island-with-extension did. The reason for this follows: calculations determined that in the absence of the jetport, although currents are strong near the surface at any horizontal location, opposing currents are generally also strong near the bottom. The result is that the vertically integrated mass-flux in most loca-

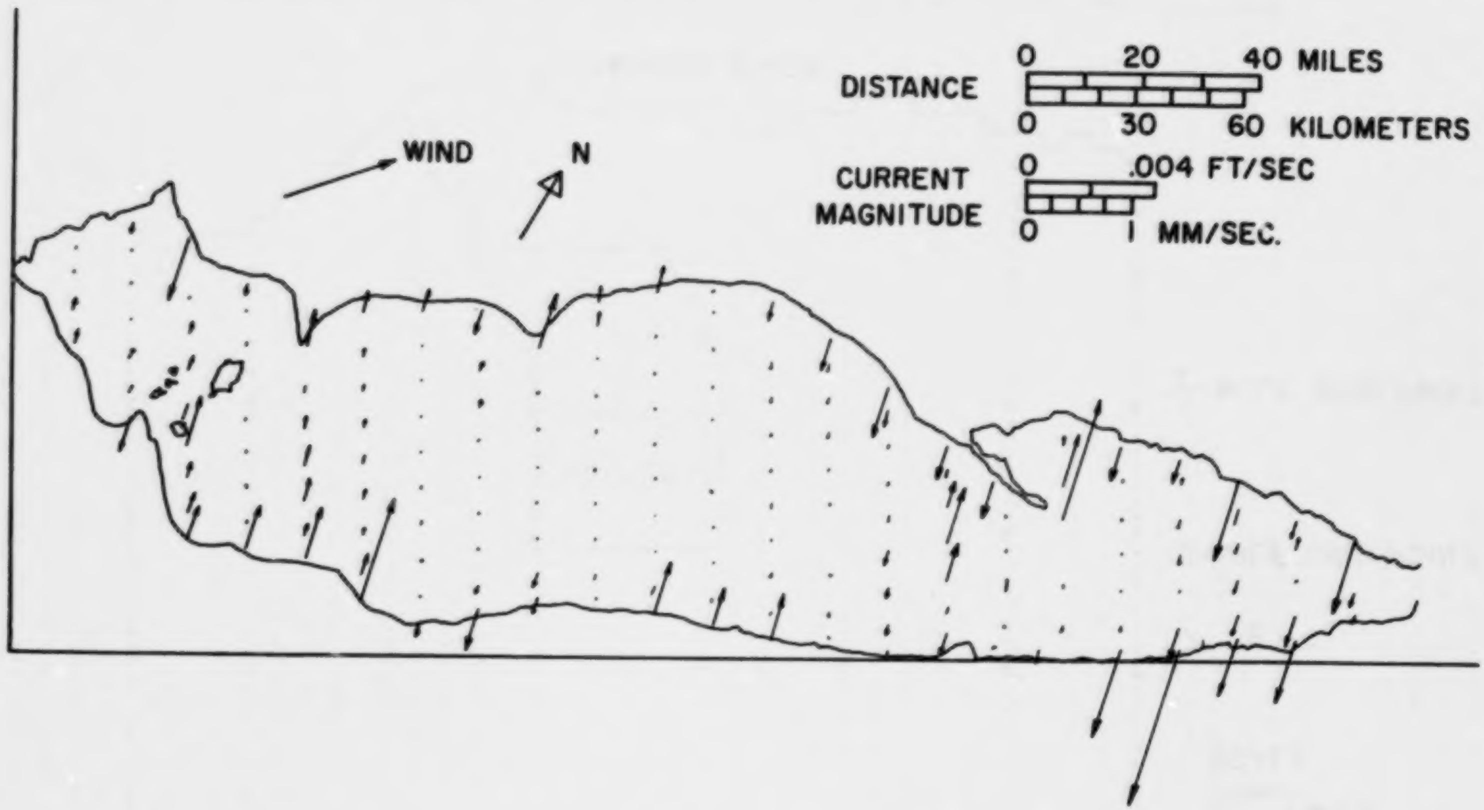

WIND
N
DISTANCE
0 20 40 MILES
0 30 60 KILOMETERS
CURRENT MAGNITUDE
0 .004 FT/SEC
0 1 MM/SEC.

Figure 4. Vertical velocities at mid-depth. Wind direction, W50S; wind magnitude, 10.1 meters per second (22.7 mph); friction depth, 27.4 meters (90.0 ft); rivers: Detroit, Niagara.

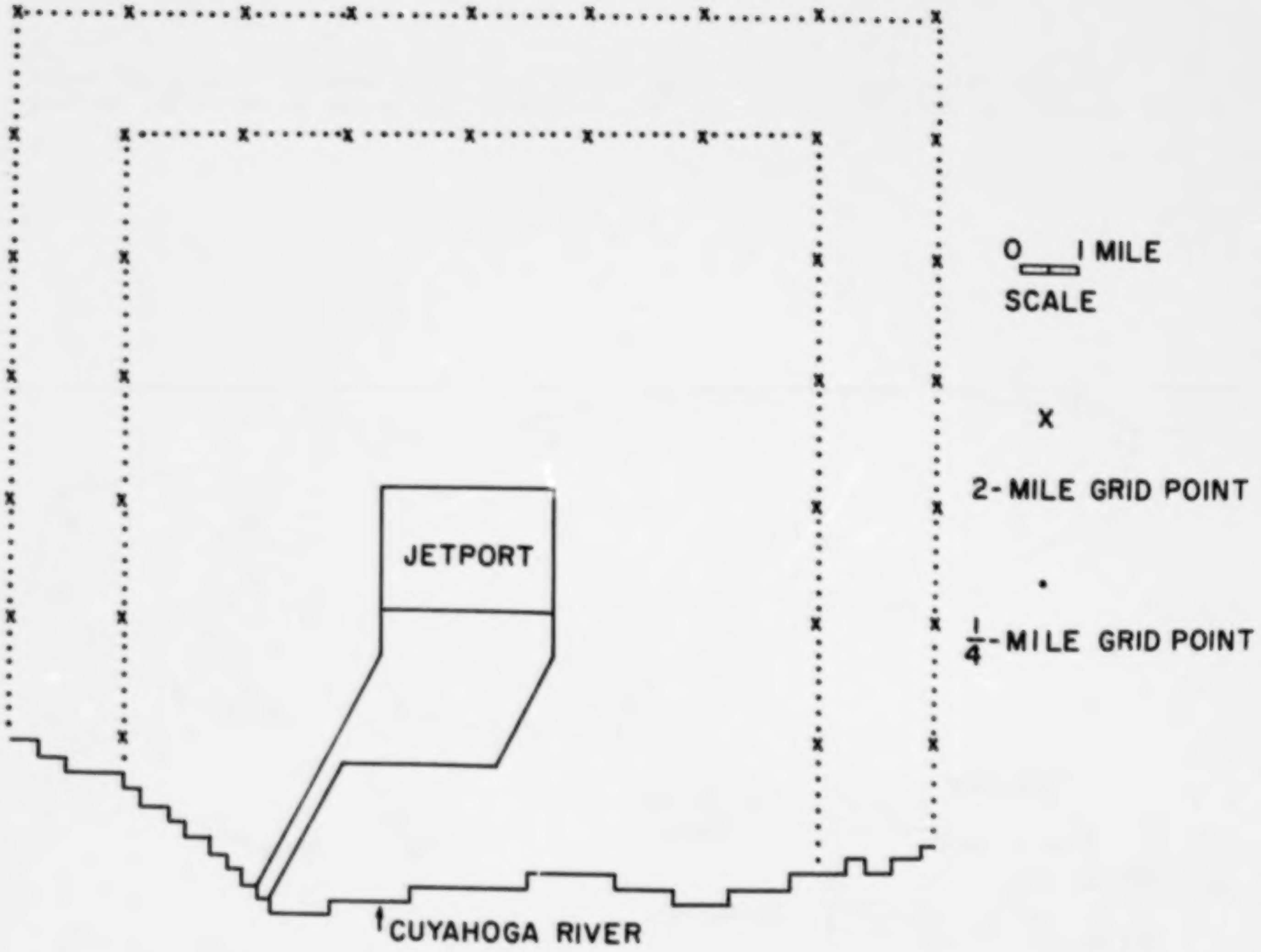

JETPORT
CUYAHOGA RIVER
0 1 MILE
SCALE
X
2-MILE GRID POINT
•
1/4-MILE GRID POINT

Figure 5. Topography of jetport and grid structure in the shore region near Cleveland.

tions near the island and offshore is approximately zero. This is not true near shore where the vertically integrated mass-flux is moderately large and directed towards the east. Therefore, the island does not appreciably block the flow; however, a jetport which extends to shore from the island does block the flow and extensively modifies the nearshore flow field.

This blockage can be seen in Figures 6 and 7 which show the horizontal velocities at the surface and at a 10-m depth for an advanced concept of the jetport. The currents are strongly deflected near the jetport. It can also be shown that strong upwellings and downwellings occur near the jetport.

In general, the coupling between the nearshore and farshore grids was treated by an iteration process as described above. A simpler procedure, although approximate, is to first calculate the overall circulation in the lake, and then calculate the flow quantities in the nearshore region assuming that conditions at the boundary of the nearshore region remain fixed. No further iteration is required if the nearshore region is large enough to offset the effect of the jetport outside the nearshore region. In the above calculations the nearshore region was taken to be approximately 25.6 km by 22.4 km. The dimension was sufficiently large so that the island configuration could be modeled without iteration (this was shown by comparison of solutions with and without iteration). For the island with an extension to shore, however, iteration was necessary.

Dispersion by Wind-Driven Currents

The problem considered here is the dispersion of contaminants by wind-driven currents after the contaminants have entered Lake Erie from the Cuyahoga River (Sheng and Lick, 1975). The effect of a jetport on this dispersion is also investigated. The river itself is treated as a point source of pollution and the effect of the river on the flow in the lake is neglected. The contaminant is assumed to be discharged from the river at a constant rate starting at some initial time. It is also assumed that the concentrations of the contaminant are small enough that the effect of the contaminant on the flow can be neglected.

The basic equation describing the dispersion of a substance is

$$\frac{\partial C}{\partial t} + \frac{\partial Cu}{\partial x} + \frac{\partial Cv}{\partial y} + \frac{\partial Cw}{\partial z} = \frac{\partial}{\partial x}\left(D_H\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_H\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(D_v\frac{\partial C}{\partial z}\right) + S \qquad (12)$$

where C is the concentration, u and v are particle (or fluid) velocities in the (horizontal) x and y directions respectively, w is the particle velocity (the sum of the fluid velocity and the settling velocity w_s of the contaminant relative to the fluid) in the z direction, D_H is the horizontal eddy diffusivity, D_v is the vertical eddy diffusivity, and S is a source term.

A boundary condition sufficiently general to describe practically all situations encountered in the dispersion of a physical substance can be written as (Monin and Yaglom, 1971):

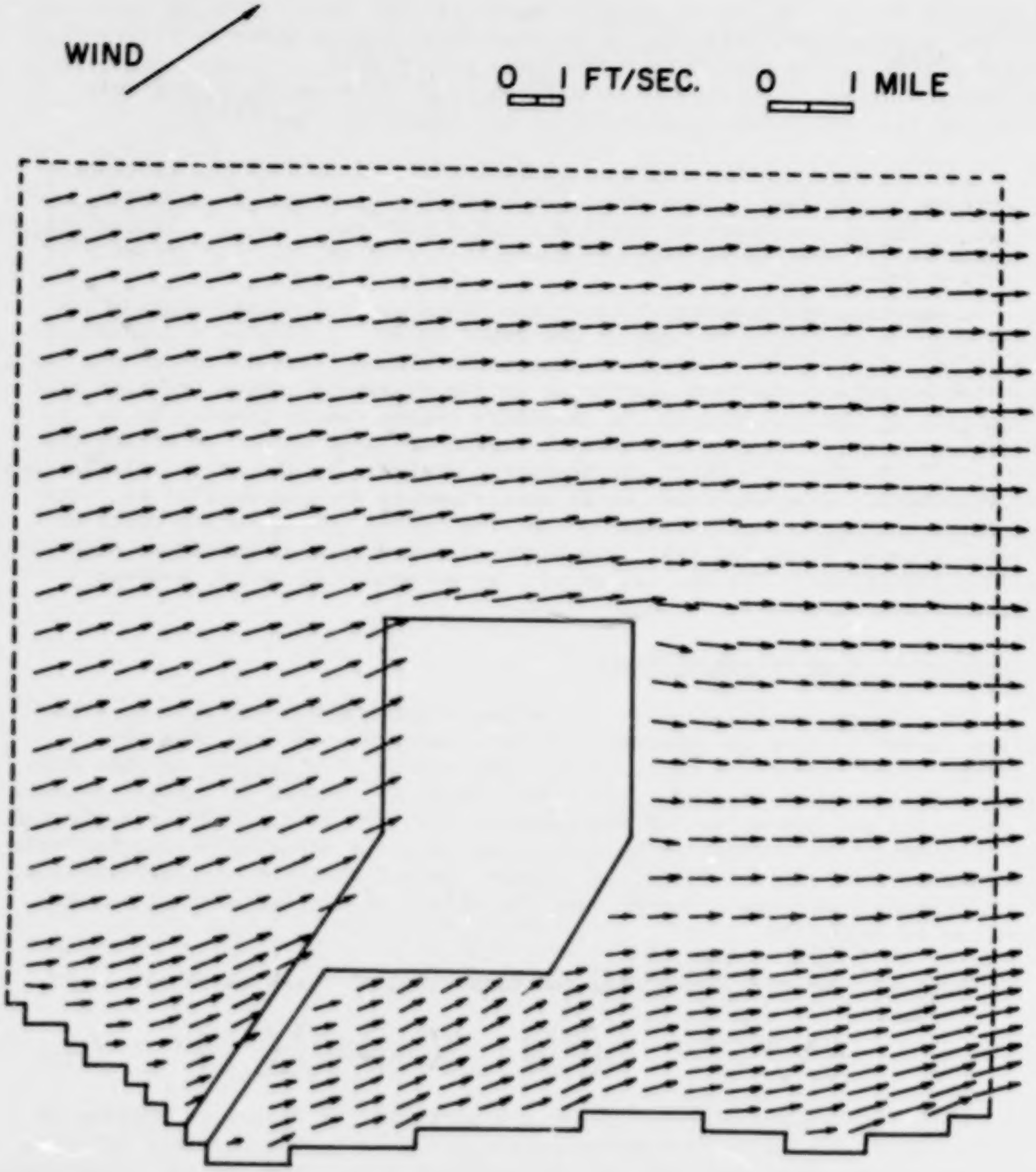


Figure 6. Steady-state nearshore currents in the presence of a jetport island with extension to the shore with (τ_x, τ_y) = (1.55 dyne/cm^2, 1.1 dyne/cm^2). Horizontal velocities at the surface.

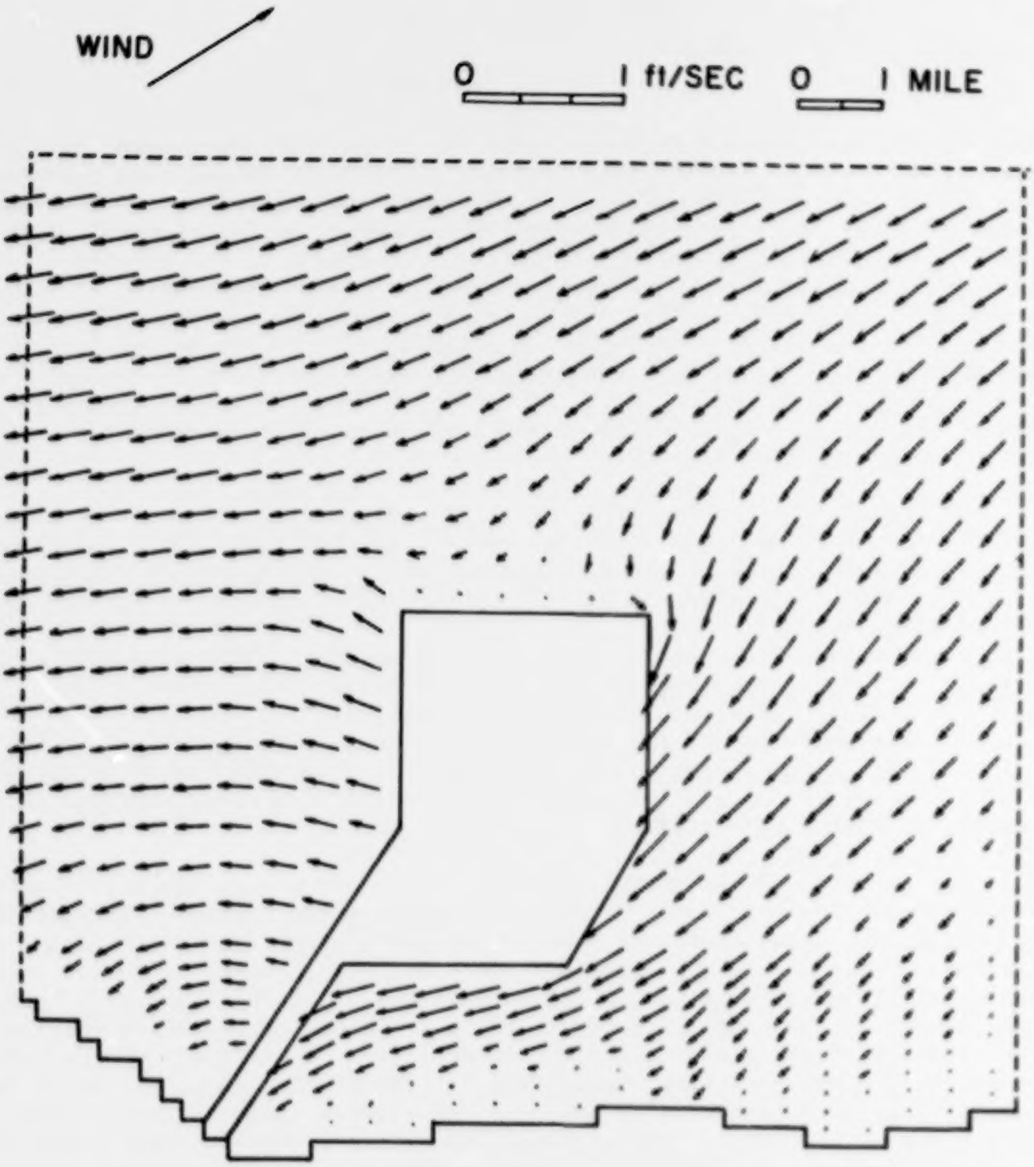


Figure 7. Horizontal velocities at a constant depth of 30 ft. from the surface.

$$w_s C - D_v \frac{\partial C}{\partial z} = -\beta C + E$$

$$= -\beta(C - C_{eq}) \qquad (13)$$

On the left-hand-side of this equation, the first term represents the flux to the boundary due to gravitational settling, while the second term represents the flux to the boundary due to vertical turbulent diffusion. On the right-hand-side, the first term depends on the "porosity" or "stickiness" of the wall. The case $\beta=0$ corresponds to perfect reflection of the substance from the boundary while the case $\beta\to\infty$ corresponds to perfect absorption at the boundary. For $0<\beta<\infty$, partial reflection and absorption occurs. The second term on the right-hand-side is due to entrainment. In the second form of the equation, C_{eq} represents an equilibrium concentration which in general depends on the shear stress and sediment composition. When the concentration is greater than C_{eq}, net deposition results while when the concentration is less than C_{eq}, net erosion results.

In the example considered here, it is assumed that the boundary condition at both the upper and lower surfaces is that of zero flux. This corresponds to setting the right-hand-side of Equation 13 to zero, i.e., no entrainment and a perfectly reflecting boundary. Other conditions assumed for the calculation are the following:

1. a steady-state velocity field is caused by a W32°S wind with a velocity of 5.2 m/sec and a wind stress of 0.9 dynes/cm^2;
2. variable density effects are negligible;
3. the mass flux of the Cuyahoga River is 27 m^3/sec;
4. the concentration of the contaminant in the river is 10 units; and
5. the vertical eddy viscosity and diffusivities are 17 cm^2/sec while horizontal mixing is neglected.

For these conditions, the velocities u, v, and w were first calculated using the hydrodynamic model described in the previous section. The surface velocities are shown in Figure 8 while the near-bottom (1/6 of the depth from the bottom) velocities are shown in Figure 9. Strong currents in the along-shore direction are apparent at the surface and it can be shown that they extend almost to the bottom. However, the near-bottom currents, especially in the vicinity of Cleveland, are directed away from shore. From the calculations, it can be shown that downwelling is present throughout the water column over the entire nearshore area shown in the figures. Due to the sharper bottom-slopes near shore, the vertical velocities in regions near shore are larger than those in regions far from the shore.

Once the velocities are known, the dispersion of a contaminant can be calculated by means of Equation 12. In all cases, a strictly conservative numerical scheme was used. In addition, a stretching of the vertical coordinate proportional to the local depth was used. With this transformation, the same number of vertical grid points are present in shallow as in deeper parts of the basin. Thus, in the shallow areas, there is no loss of accuracy in the computations due to lack of vertical resolution, a significant factor in nearshore calculations.

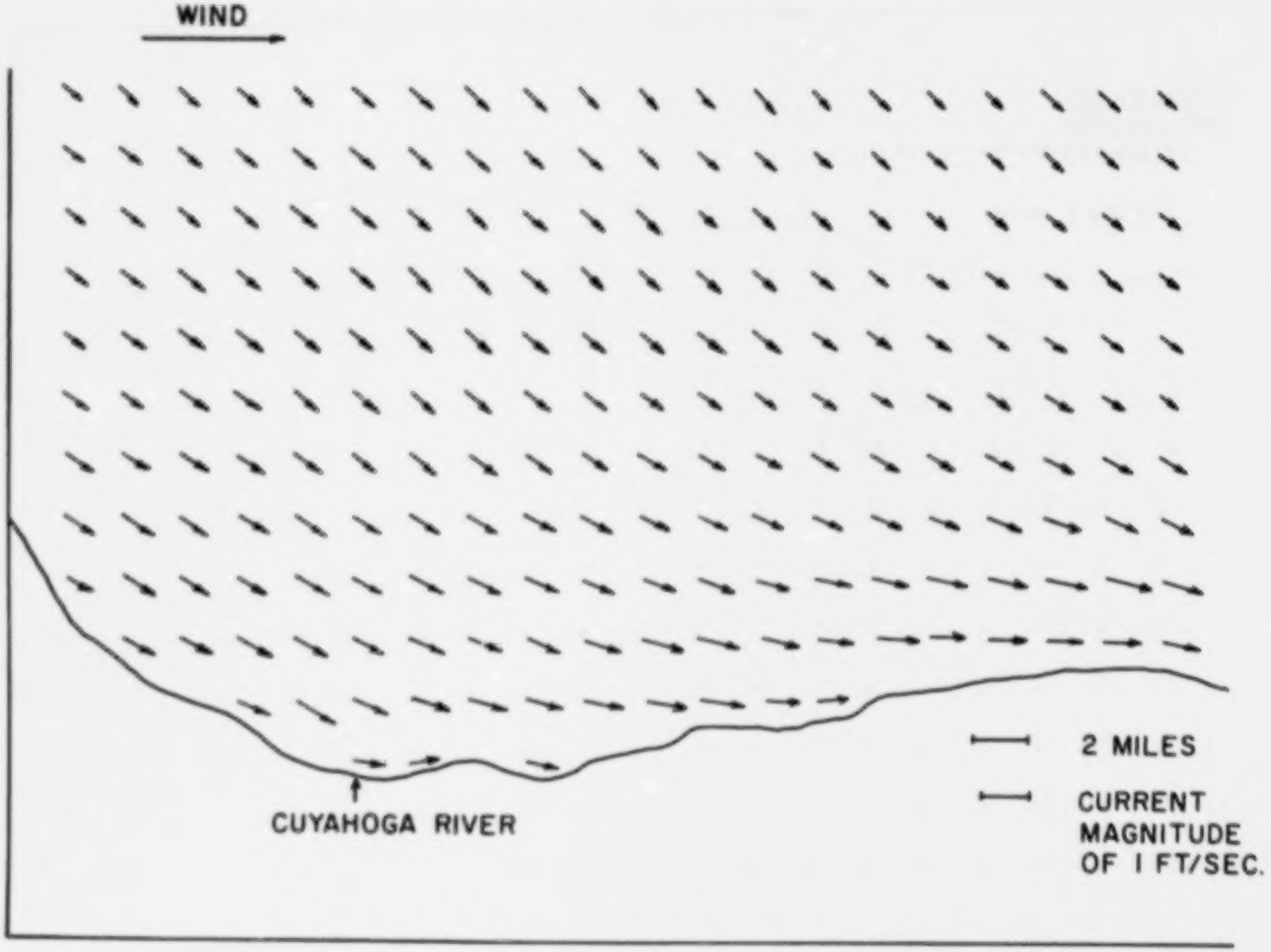

WIND
CUYAHOGA RIVER
2 MILES
CURRENT
MAGNITUDE
OF 1 FT/SEC.

Figure 8. Horizontal velocities at surface in the nearshore with $\tau_x = 0.9$ dyne/cm^2.

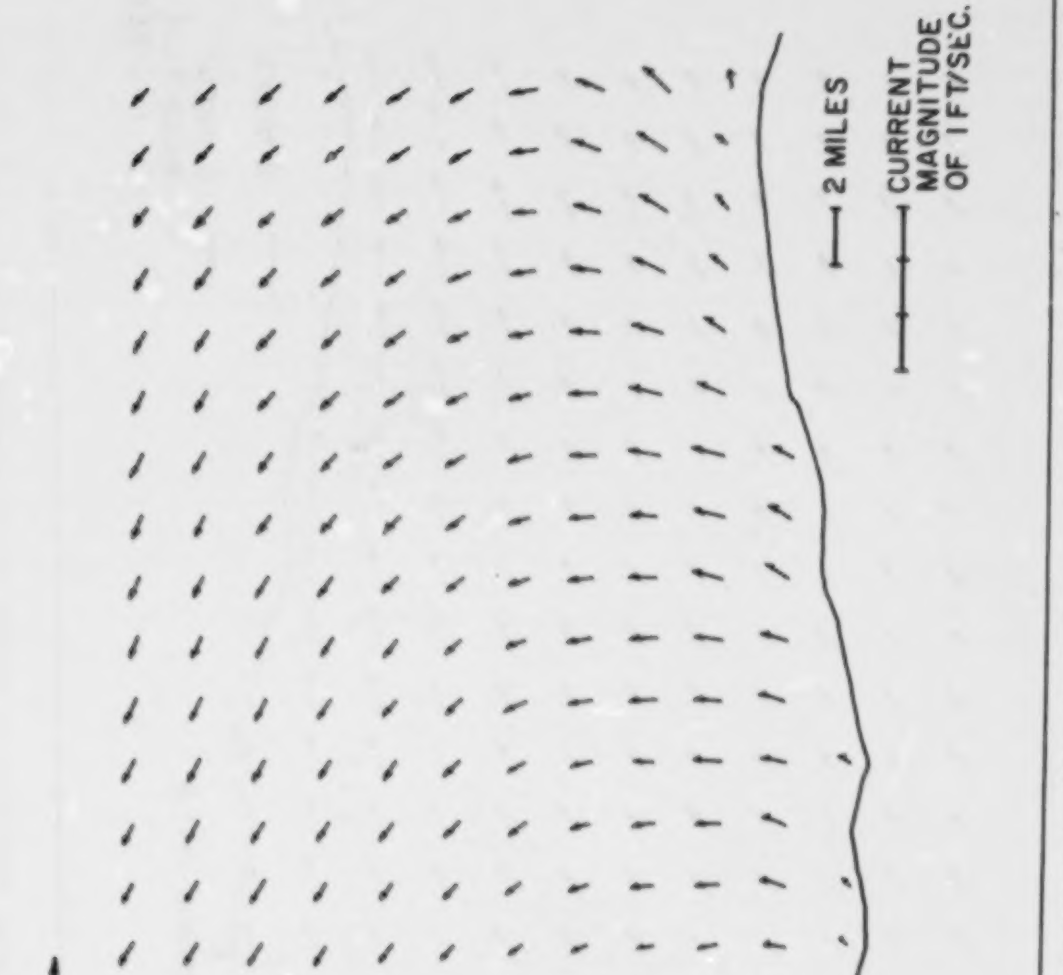


Figure 9. Horizontal velocities at 5/6 - depth in the nearshore with τ_x = 0.9 dyne/cm^2.

In the present calculation, the dispersion of a conservative, dissolved substance was considered. Relative concentrations at the lake surface at 128 hours are shown in Figure 10. It can be seen that the contaminant is transported along shore towards the east but with some transport in the offshore direction. This offshore transport is mainly due to the bottom currents in the offshore direction followed by vertical mixing. A horizontal diffusivity of $2x10^5$ cm^2/sec made little difference in the results.

To show qualitatively the effects of an offshore jetport on the currents and the dispersion of a contaminant, two additional cases were calculated under the same conditions as above but with two different proposed configurations of a jetport. These configurations were:
1. an island 6.4 km square approximately 9.6 km offshore of Cleveland, and 2. a landfill extension of this island to shore.
The results of the calculations for the surface concentrations at 128 hours are shown in Figures 11 and 12. The island configuration, as seen in Figure 11, does not affect the contaminant transport appreciably, except in the immediate vicinity of the island. This is due to the fact that the flow over most of the nearshore region is not appreciably modified by the presence of the island. In the case of the island with an extension-to-shore, the contaminant is transported much further offshore than in the previous two cases. The extension blocks the flow in the long-shore direction. Very little contaminant is found at the downwind side of the jetport. It can be shown for this case that there are strong sub-surface, offshore currents in regions near the river mouth and upwind of the jetport.

Calculations were also made for the same conditions as above except that it was assumed that the contaminant had an average settling velocity of -0.5 mm/sec relative to the surrounding fluid. Without the presence of a jetport, the concentration distribution after 128 hours at the surface and near the bottom is shown in Figures 13 and 14. By comparing the surface concentration distribution with that of the neutrally buoyant contaminant (Figure 10), it can be seen that the along-shore surface transport is reduced appreciably. Near the bottom, the contaminant was transported much farther in the offshore direction than in the previous case. This was due to the settling and the offshore currents present near the bottom under these wind conditions.

Thermal Plumes from a Power Plant

A time-dependent, variable-density model (Paul and Lick, 1973, 1974; Lick, 1975) has been used to calculate the flow-field and temperature distribution in the discharge from the Point Beach power plant on Lake Michigan. The calculations include realistic geometry, buoyancy effects, wind stresses, and crossflows in the lake. The basic numerical procedure of solution and details of the calculations are presented in the references mentioned above and so, except for a brief statement of the assumptions used in the model, the model will not be discussed in detail here. However, as examples of the dispersion of heat, the results of two calculations and their comparison with field observations will be presented.

In addition to the usual assumptions stated in the introduction, the present hydrodynamic model assumes that the rigid-lid approximation is valid,

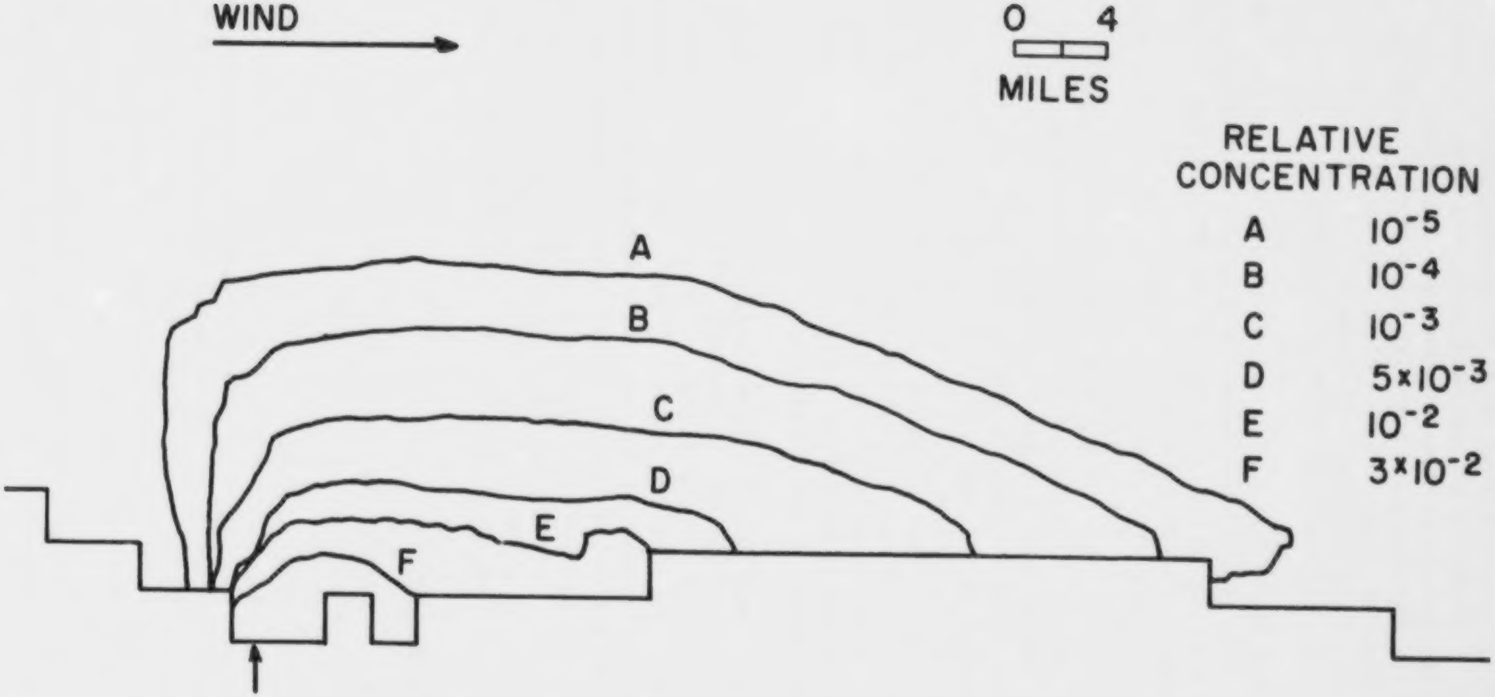

WIND
0 4
MILES
RELATIVE CONCENTRATION
A 10⁻⁵
B 10⁻⁴
C 10⁻³
D 5×10⁻³
E 10⁻²
F 3×10⁻²
A
B
C
D
E
F

Figure 10. Surface concentration contours in the nearshore at 128 hours.

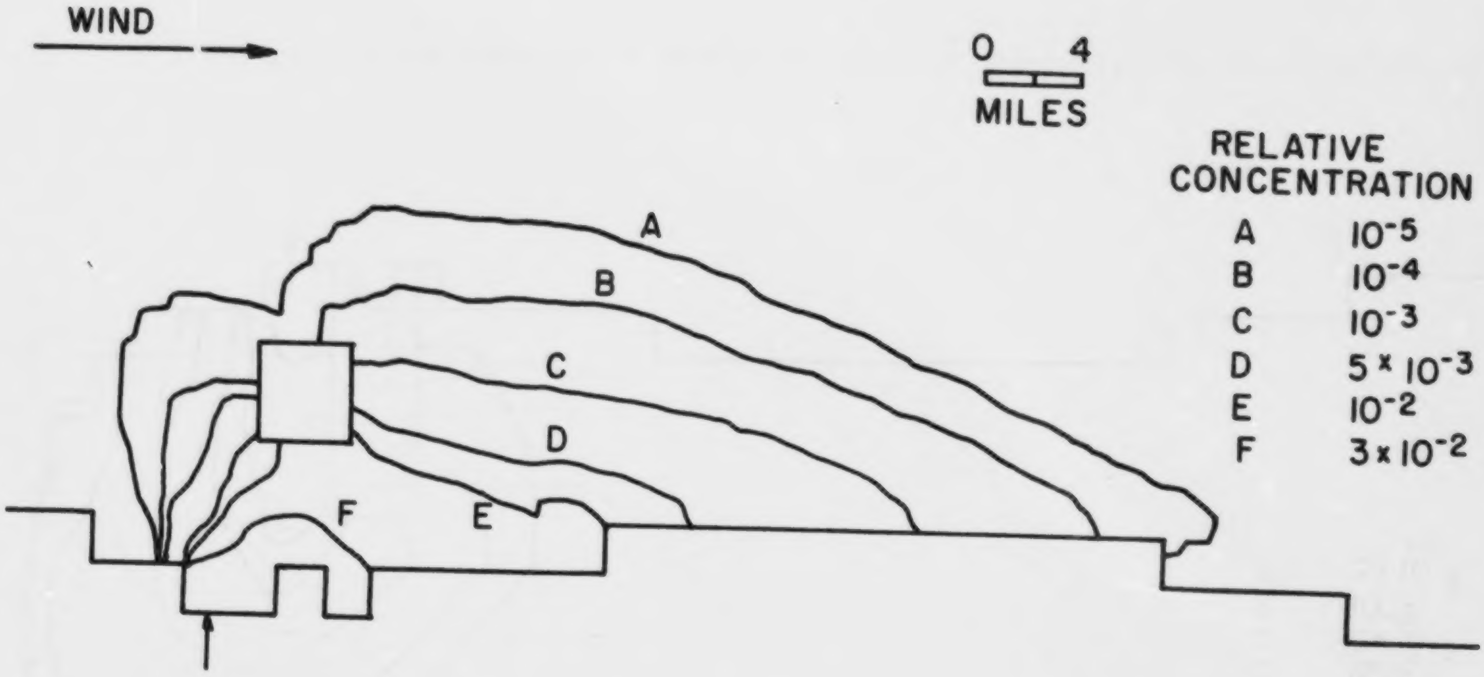

WIND
0 4
MILES
RELATIVE CONCENTRATION
A 10^{-5}
B 10^{-4}
C 10^{-3}
D 5×10^{-3}
E 10^{-2}
F 3×10^{-2}
A
B
C
D
E
F

Figure 11. Surface concentration contours at 128 hours, island jetport.

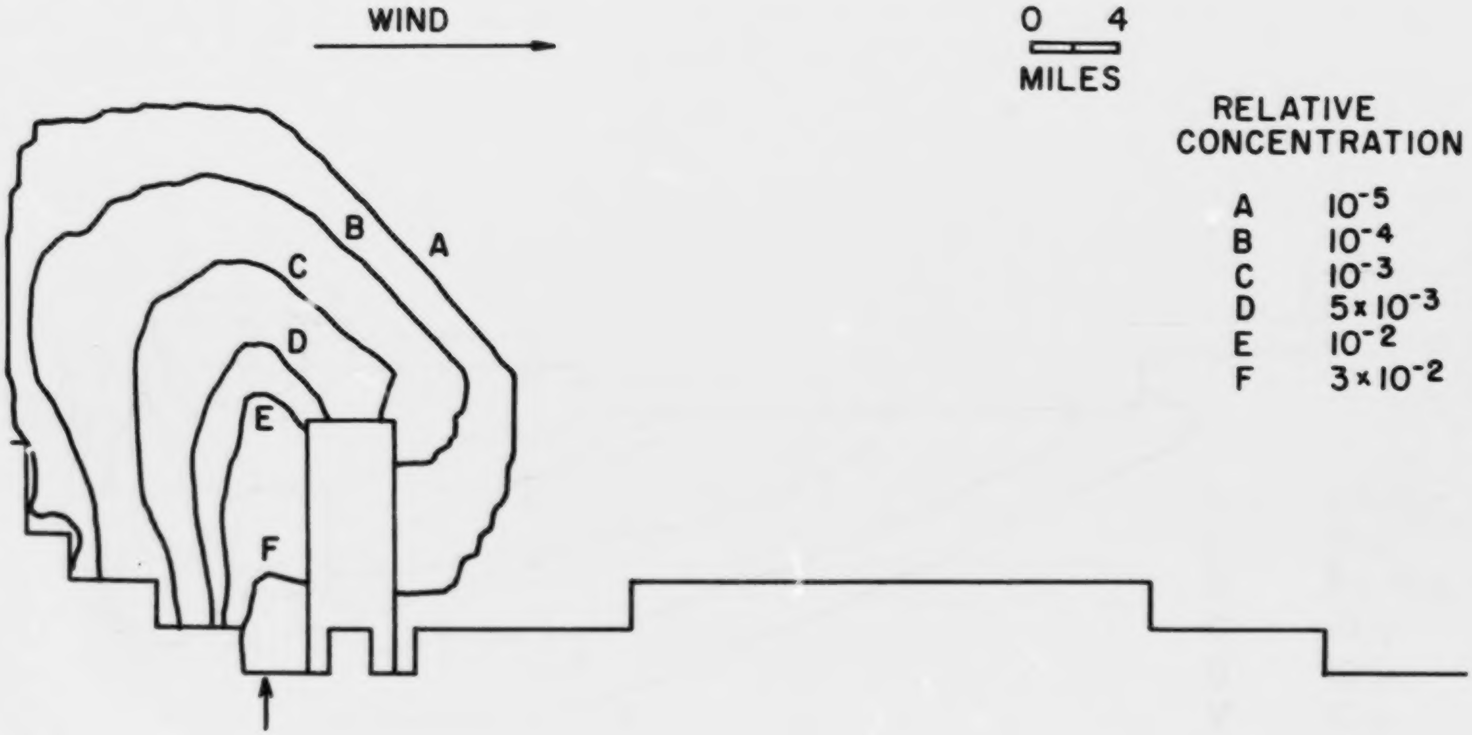


Figure 12. Surface concentration contours at 128 hours, island with extension to shore.

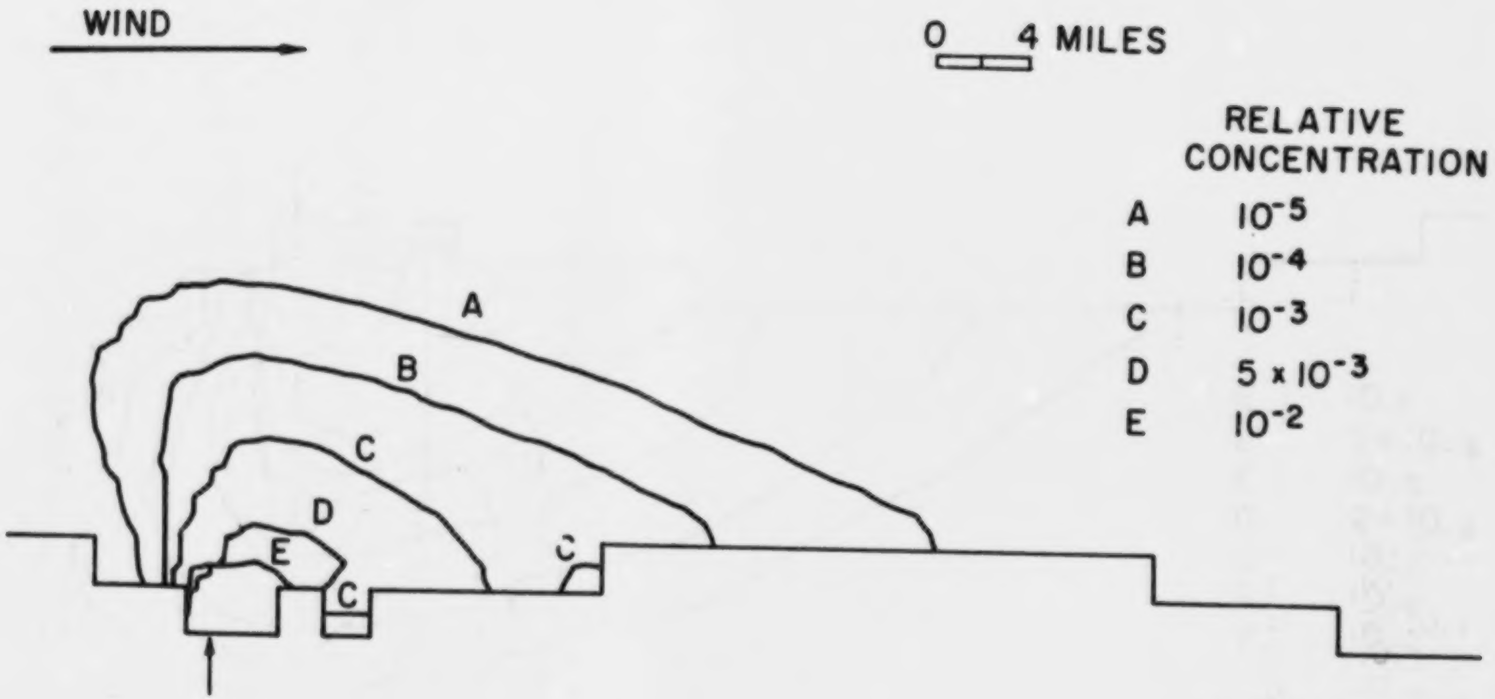

WIND
0 4 MILES
RELATIVE CONCENTRATION
A 10^{-5}
B 10^{-4}
C 10^{-3}
D 5×10^{-3}
E 10^{-2}
A
B
C
D
E
C
C

Figure 13. Surface concentration contours in the nearshore at 128 hours. Settling velocity = -0.5 mm/sec.

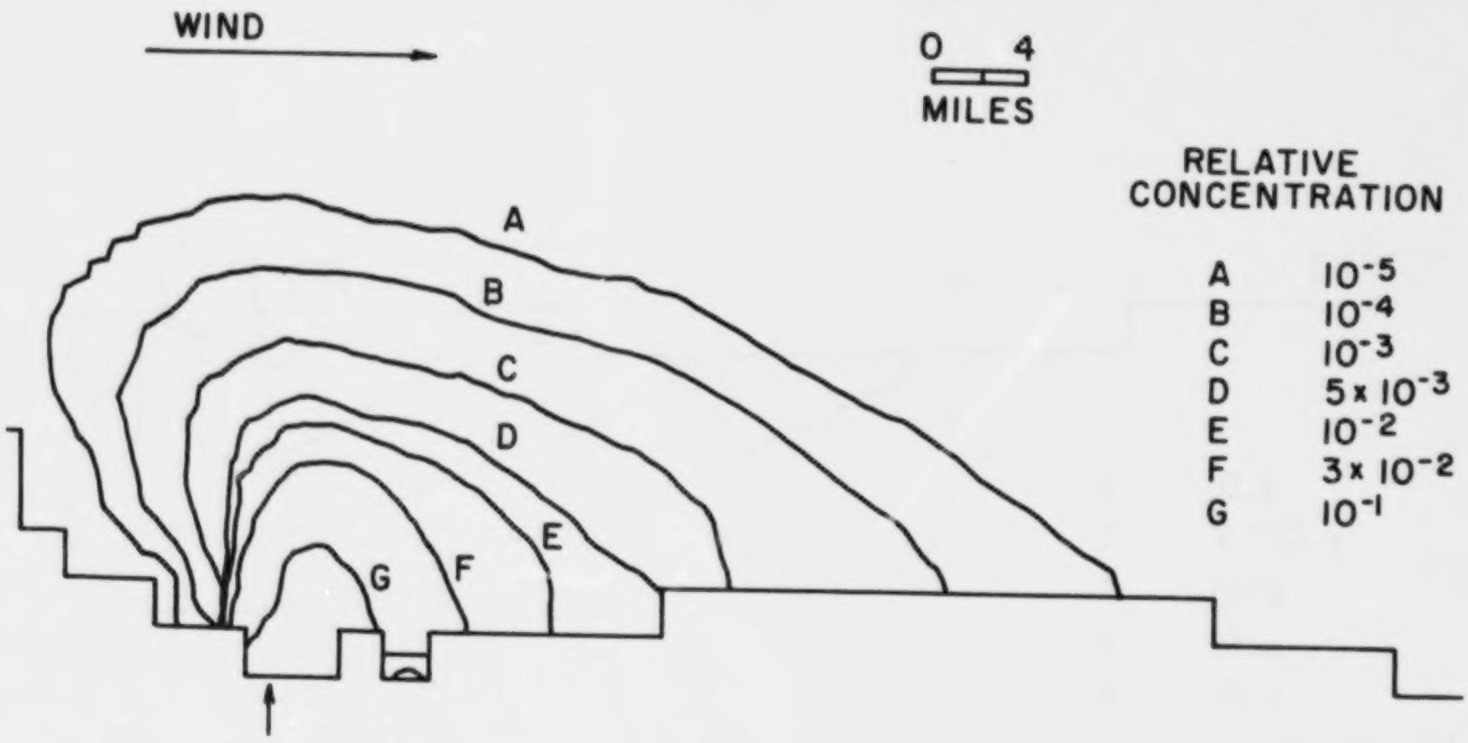

WIND
0 4
MILES
RELATIVE CONCENTRATION
A 10^{-5}
B 10^{-4}
C 10^{-3}
D 5×10^{-3}
E 10^{-2}
F 3×10^{-2}
G 10^{-1}
A
B
C
D
E
F
G

Figure 14. Near-bottom concentration contours in the nearshore at 128 hours. Settling velocity = -0.5 mm/sec.

i.e., $w(z=0) = 0$. This approximation is used to eliminate surface gravity waves and the small time scales associated with them, greatly increasing the maximum time-step possible in the numerical computations. In this approximation, only the high frequency surface variations associated with gravity waves are neglected while the steady-state results, with which we are solely concerned here, are calculated correctly and are the same as for the free-surface case. As in the previous calculation of contaminant dispersion, we used strictly conservative numerical schemes and stretched the vertical coordinate proportional to the local depth.

Calculations have been made for two cases of the discharge from the Point Beach power plant. These discharges flow into a quiescient lake and into a lake with a crossflow and a surface wind. The relevant parameters, based on field measurements taken at the site of the discharge (Frigo et al., 1974), are listed in Table I. The bottom topography is shown in Figure 15. The outfall extends into the lake and the discharge forms a 60° angle with the shore.

TABLE I. PARAMETERS OF DISCHARGE FROM POINT BEACH PLANT.

Flow rate	$24.7\ m^3/sec$
Outfall width	10.8 m
Outfall depth	4.2 m
Ambient lake temperature	9.5°C
Vertical eddy coefficient	$(50 - 200 \frac{\partial T}{\partial z})\ cm^2/sec$
Horizontal eddy coefficient	$1000\ cm^2/sec$
Surface heat transfer coefficient	$30\ watt/m^2 - {}^oC$
Temperature variation	8.5°C
Maximum velocity	0.9 m/sec
Densimetric Froude number	4.2
Equation of state	$\Delta\rho = -1.25 \times 10^{-3} \Delta T$

The boundary conditions are as follows: the inlet velocity profile was specified as a smoothed average of that measured at the outfall. The inlet temperature was taken as the constant value measured. A surface heat transfer proportional to the difference in temperature between the surface water and the air was assumed with the heat transfer coefficient determined from the work of Edinger and Geyer (1965). The stress acting on the water surface due to the wind (measured) is calculated by the formulae developed by Wilson (1960). The outer x and y boundaries for the numerical calculations must be taken at some finite distance. Roache (1972) discusses outer boundary conditions and concludes that the actual boundary conditions are not that important as long as the boundary conditions do not severely restrict the flow. The boundary conditions used here are that the normal derivatives of the velocities and the temperatures are zero.

The vertical eddy coefficient is taken as dependent on the local vertical temperature gradient and is assumed to be given by:

$$A_V = \alpha - \beta \frac{\partial T}{\partial z} \qquad (14)$$

where α and β are constants depending on local conditions. The constant α is equal to the vertical eddy diffusivity under vertically stable conditions.

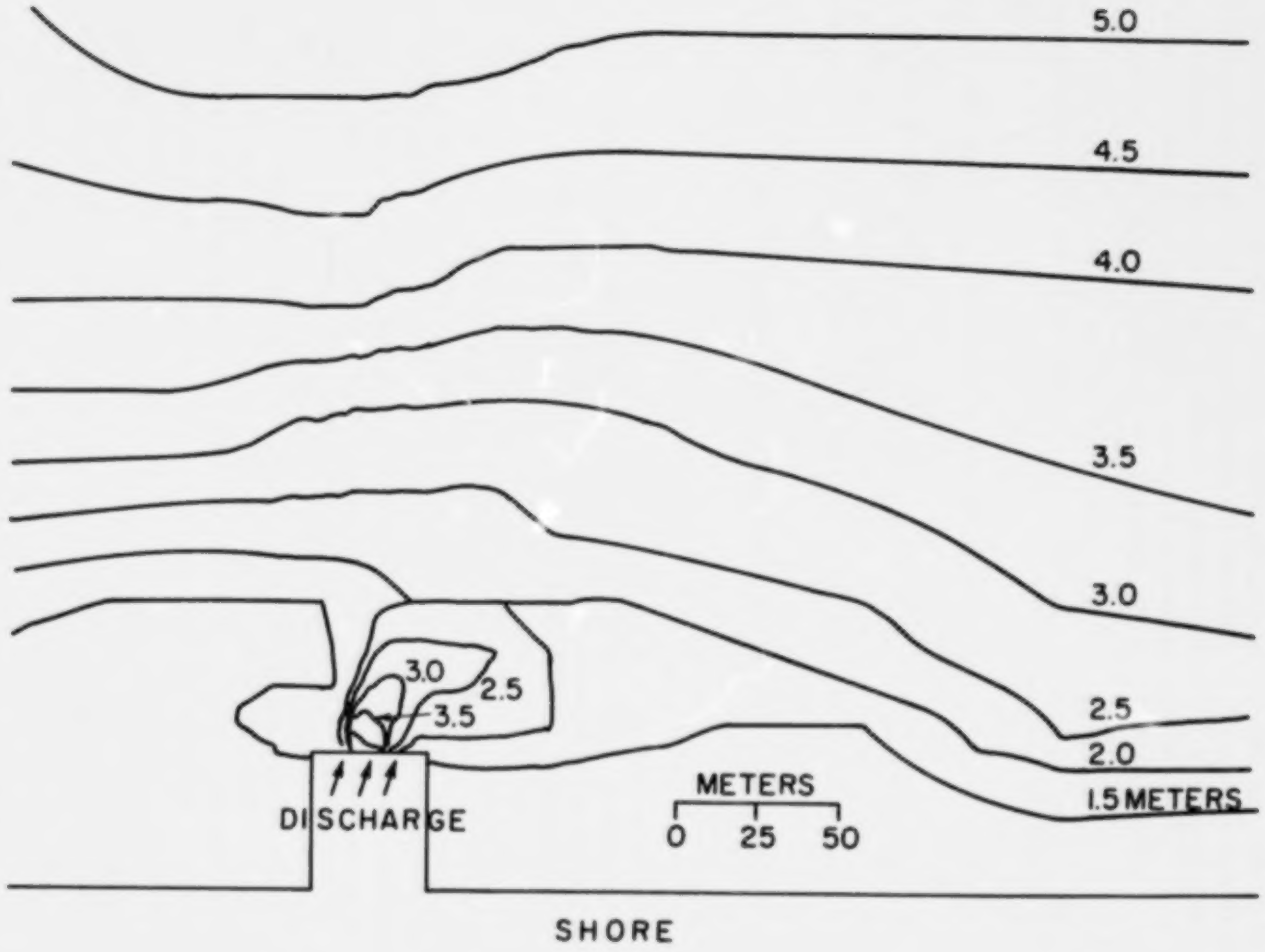

5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5 METERS
3.0
2.5
3.5
DISCHARGE
METERS
0
25
50
SHORE

Figure 15. Bottom topography for the Point Beach power plant.

For the above boundary conditions and parameters, time-dependent calculations started with an initial guess of the flow and proceeded until a steady-state had been reached. The first case was that of the discharge into a quiescent lake. No winds or crossflows in the basin were present. From the calculations, one can show that as the flow is discharged from the outfall, it is forced towards the surface by the decreasing depth (see Figure 15) as well as by buoyancy. The large densimetric Froude number of 4.2 indicates that buoyancy effects are not initially dominant. After about 75 m from the outfall, the depth begins to increase. After this point, the discharge tends to remain very near the surface. The resulting surface temperature field is shown in Figure 16.

A second calculation was made for the case when a crossflow (current of 9.1 cm/sec at an angle of 125°) and a wind (approximately 5 m/sec at an angle of 270°) were present. In this case, the discharge is physically swept in the direction of the crossflow. The temperatures (Figure 17) are displaced in the direction of the crossflow and towards the shore.

For this second case, a comparison of the calculated results with field observations (Frigo et al., 1974) is shown in Figures 18 and 19. Figure 18 shows the temperature decay along the centerline while Figure 19 shows the isotherm areas for various temperatures. It can be seen that there is good agreement between the predicted results and field observations. Similar agreement was obtained between predicted results and field observations for the first case above. The calculations are presently being extended to include more of the flow field.

Additional field data are available (Frigo et al., 1974) from which one can determine more details of the flow field. However, the flow, due to its turbulent nature (Csanady, 1973 for a general description of the turbulent diffusion and nature of plumes), is highly variable both in space and time. Continuous field measurements must be made over a sufficiently long period of time to average out these variations before more general comparisons between our calculations and observations can be made. This has not yet been done. However, the above calculated results, although limited, seem more than adequate at this point and establish confidence in the numerical model.

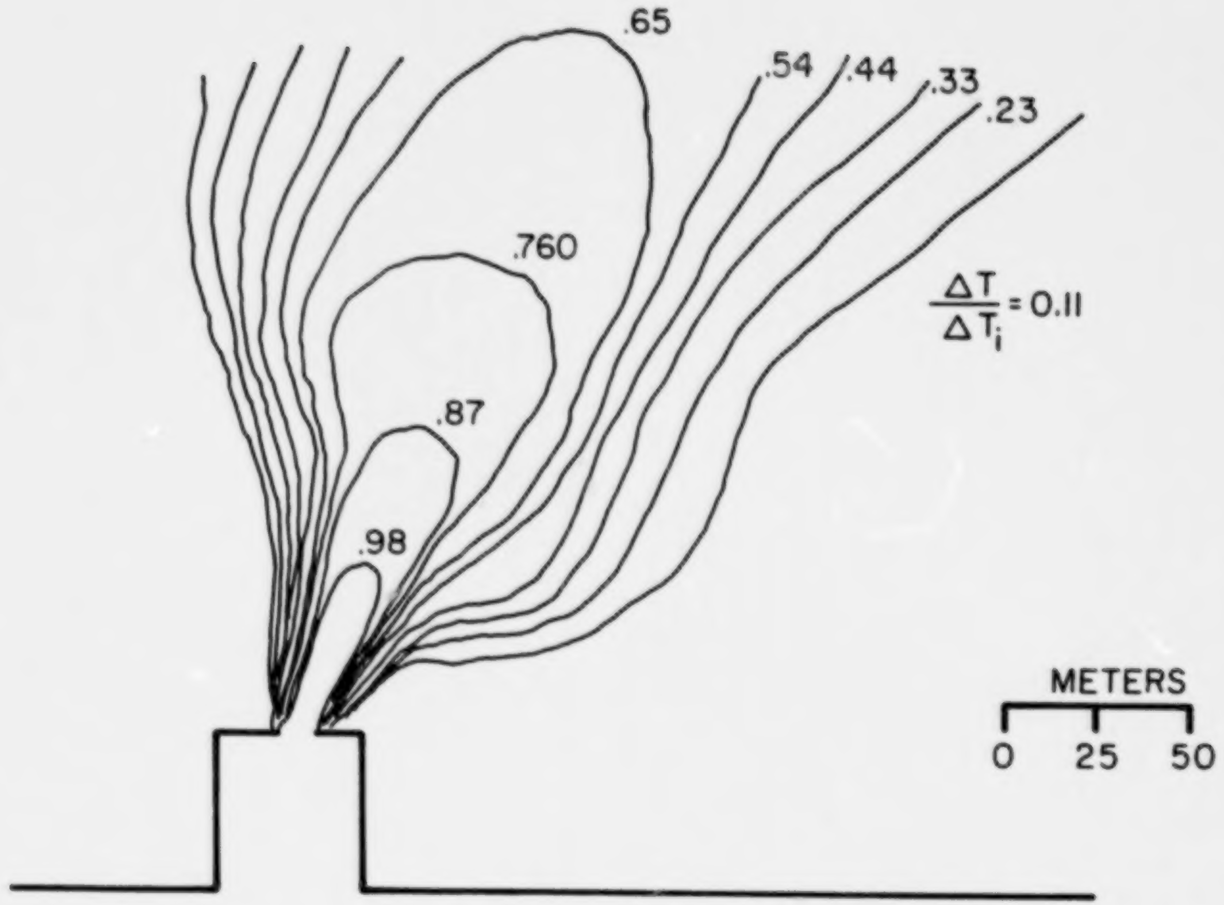


Figure 16. Surface temperature distribution, no wind, no cross-flow.

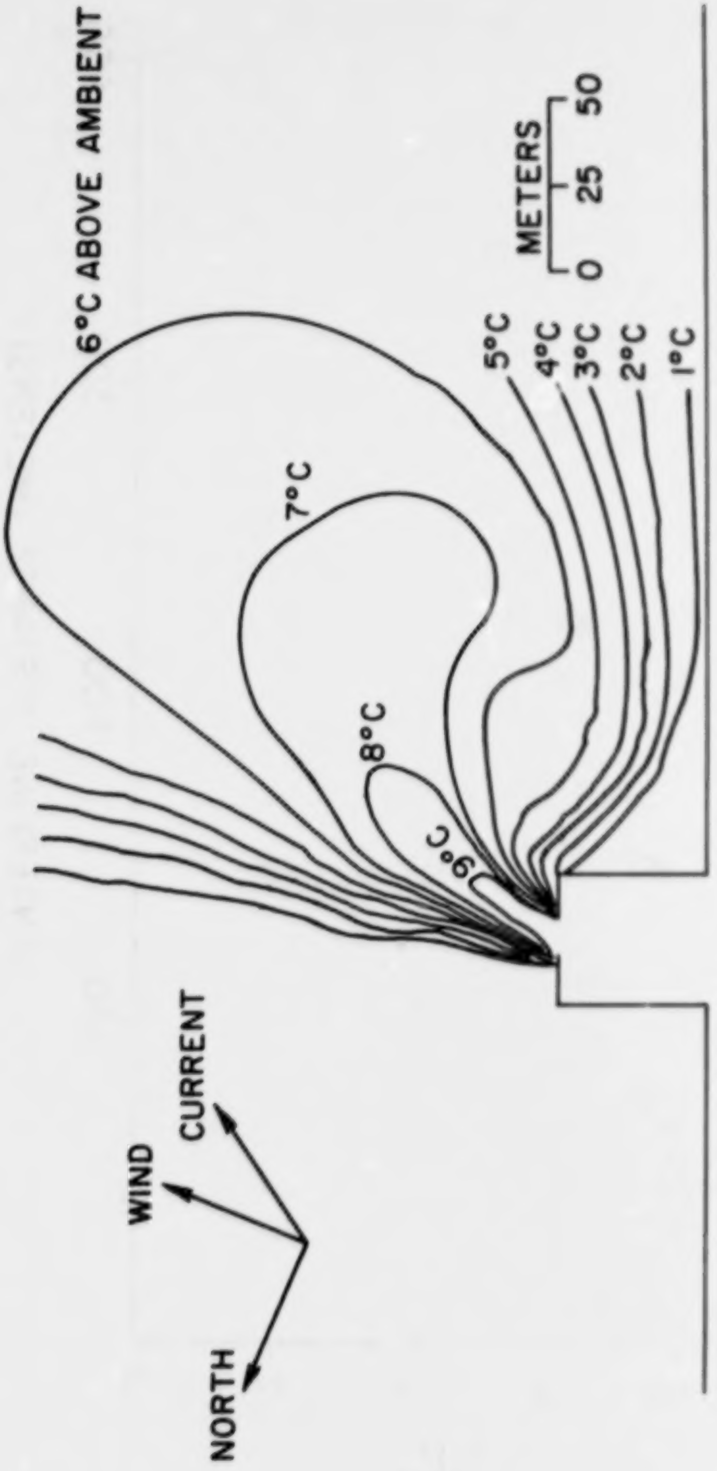


Figure 17. Surface temperature distribution with wind and cross-flow.

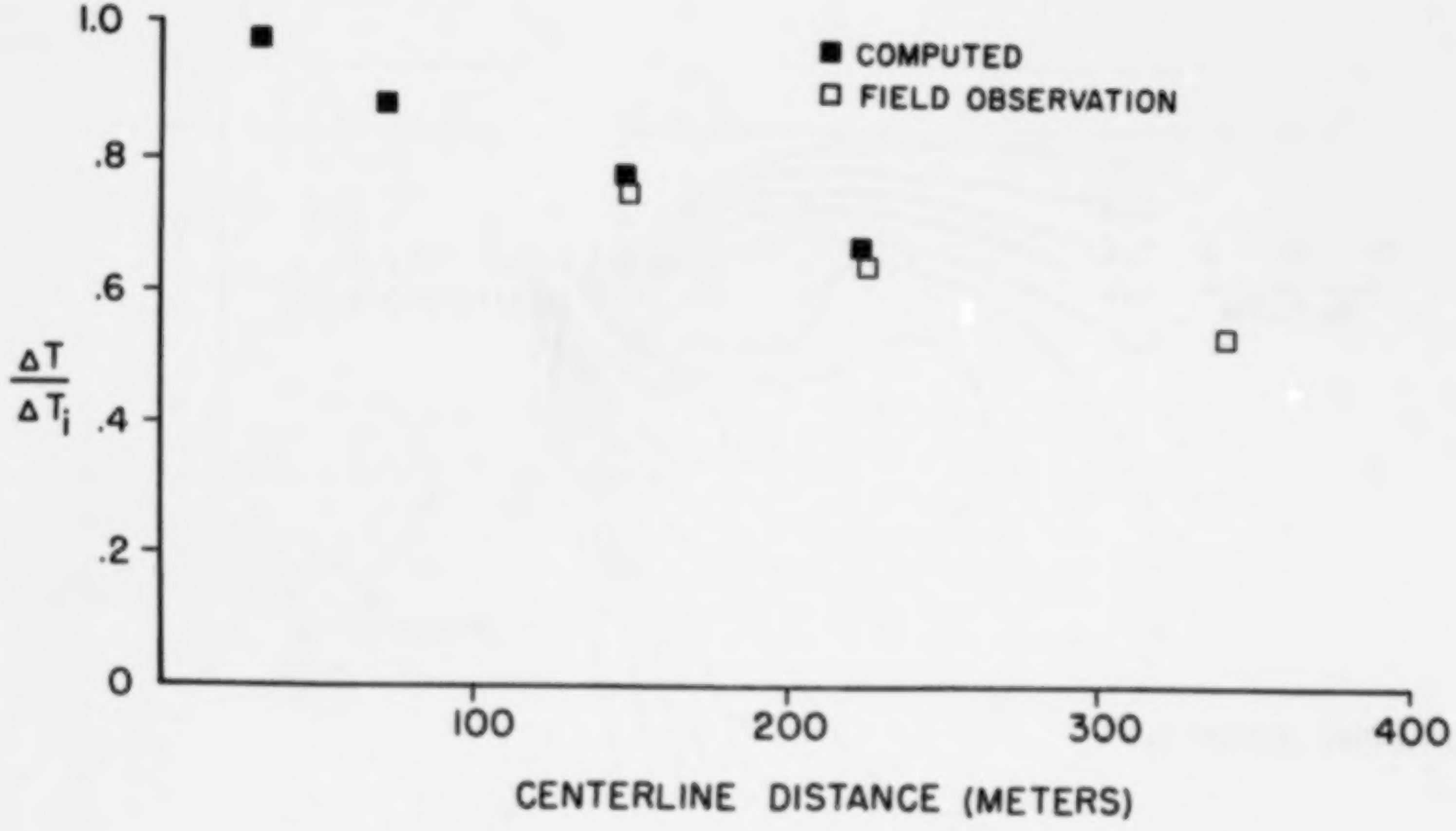


Figure 18. Surface temperature decay along centerline with wind and cross-flow.

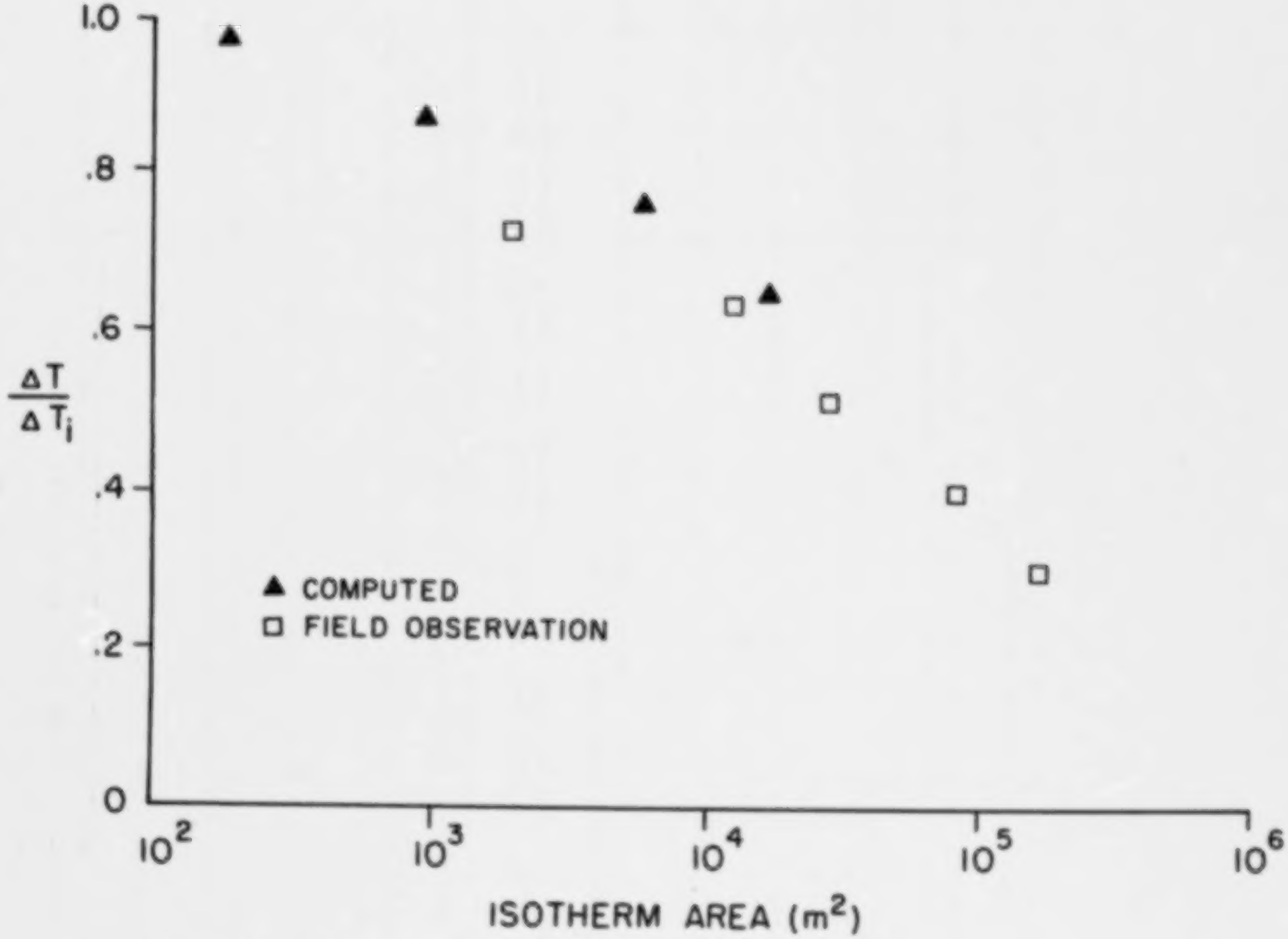


Figure 19. Surface isotherm areas with wind and cross-flow.

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DISCUSSION

BARANNIK: My question concerns modeling thermal streams. Several models have accepted that the eddy viscosity coefficient depends on the Richardson's number. Dr. Lick uses a linear dependency between the eddy viscosity coefficient and the temperature coefficient. I would like you to comment on this in more detail.

LICK: We had only two field observations, and we determined the parameters on the basis of field observations. Therefore, we did not use very complicated formulas.

LOZANSKIY: Why does the range of parameters within which field research work was carried out not correspond to the range of parameters within which calculations were carried out?

LICK: This is due to the insufficient quantity of field data. The calculations gave those fluctuations which are not far from the field observations.

DAVIES: The majority of hydrodynamic specialists criticize ecosystem models because of inadequate verification. However several circulation models predict flows which, in actuality, are not observed. How do you reconcile these inconsistencies?

LICK: We assume that we have working numericals models. Further efforts must be directed to verifying these models.

VASILYEV: In your discussions, there is a discrepancy between thermal viscosity and conditions where there is no bottom friction. How do you use information on wind? How many monitoring stations do you have to obtain information on the lake's surface?

LICK: We applied non-bottom friction conditions because Lake Erie is shallow. In this case, it is better to apply conditions of non-bottom friction than conditions of bottom friction. If we use a boundary condition, then for stationary flow the general results differ. If the pressure is proportional to the velocity, then it is necessary to determine two parameters: bottom pressure and viscosity. And we think that it is not reasonable with a limited amount of field data to use a lot of parameters.

The wind was measured at different points over the lake, and a steady-state situation was assumed. The average wind velocity value for two days was taken. The wind was measured at six reference points on the shore, and the wind velocity value at any point on the lake was interpolated between these points. We accept the wind-driven stress as proportional to the velocity square. multiplied by the coefficient, which was determined by field measurements. We introduced the correction coefficient in order to increase the wind velocity over the lake. It is interesting to note that the latter is always larger over the lake than over dry land. Experiments prove that it is 1.5 times larger.

MORTIMER: Dr. Lick described one of the principle difficulties in a model of lake hydrodynamics, that of weak use of meteorological data, not only the wind velocity value over dry land and over the lake, but also its direction. It is also interesting to note that wind velocity and direction are changed when there is a transition from a smooth to rough surface. It is necessary for us to study wind-driven motion. Flows are measured using instruments. We need LaGrangian measurements with floating instruments, transmitted through earth satellites. Reception stations can be located in the mountains. For this, Baikal is an ideal model.

BARANNIK: The model compares vertical turbulent exchange coefficient values which were selected using a comparison of field observation data with calculation results. In similar models for modeling flows in large water bodies, the dependency between the vertical turbulent exchange coefficient and the wind velocity was based on field data. This evaluation showed that the vertical turbulent exchange coefficient which Dr. Lick used is close to our dependencies. It is possible to conclude that Dr. Lick's model is close to those models which are used in the USSR for calculating flow in large lakes.

YEREMENKO: Dr. Lick's method is a fully acceptable method for solving complicated problems. We have developed a similar full flow method. I would like to briefly discuss the research which we conducted using the Azov Sea as an example. The problem was solved where there were conditions of bottom adhesion and steady viscosity. No essential deviation in circulation was observed. When there were adhesion and steady viscosity conditions, we essentially overestimated the bottom resistance. Divergence with the Chezy conditions took place 5 to 7 times. In this case, I think that Chezy's law for wind-driven flow is also an assumption. Researching the turbulent viscosity distribution, given wind-driven flow, we were successful in solving a problem with bottom friction, but with turbulent viscosity data obtained for wind-driven flows. It turned out that in this case, the divergence between the adhesion conditions when there is steady viscosity was decreased in comparison with the case when Chezy's law was used.

ZNAMENSKIY: It is possible to distinguish steady state processes from field data. In this case, the larger the averaging level, the better it is. On Lake Baikal, the basic air currents are stable. The division line for two multi-directional air flows over a period of several years varies insignificantly. We used this principle when constructing a hydraulic model of Lake Baikal. Using the established steady state systems, we modeled drift and compensation flows. We obtained results which are close to the field observations. Dr. Lick's model allows us to formulate a series of delicate questions on flow structure in water bodies. Using this model, we obtained an evaluation of the rise and fall of the water masses in the nearshore regions. I think that the models presented are sufficiently well evaluated, not only from a practical aspect, but also from an aspect of knowledge of natural processes.

ROZHKOV: If you examine flow in large lakes, then it is necessary to turn your attention to their probability structure. I will describe the flow spectrum in a stationary approximation using data from field observations in Lake Ladoga. The observations were taken over several months. The diagrams clearly show the maxima of synoptic changes with the cycle of operations lasting three days, the inertia fluctuations over a period of 14 hours, and the inertia fluctuation overtone. If you examine a non-stationary probability model, then it is possible to note that the intensity of all these fluctuations is changed in time. The frequency spectral density will be the most complete characteristic of the two. The synoptic fluctuations are changed with a half-month cycle of operations. These are the basic characteristics of the flow structure in large lakes as a non-stationary non-uniform vector process.

LICK: For deeper lakes, we used a three-dimensional model and developed several models for steady-state conditions. We computed several examples using these models. Additional difficulties take place when modeling in nearshore zones, when the size of the grid disguises different flows, and when we apply valid limited conditions--coefficients of fluctuation and entrainment. Little is known about the entrainment and adhesion coefficients. Still, the numerical models are useful for reconstructing real processes for understanding situations which arise when water bodies are stratified.

MODELING AND VERIFICATION; THE TWO INDISPENSABLE LEGS FOR PROGRESS TOWARD UNDERSTANDING AND MANAGEMENT OF AQUATIC SYSTEMS AS DEMONSTRATED BY A STUDY OF COASTAL CURRENTS IN LARGE LAKES

C.H. Mortimer

INTRODUCTION

Today, computerized mathematical modeling of aquatic systems, fed by some successes, partial successes, and more promises, has become fashionable with environmental managers in governmental agencies and industry. This development, although welcome in many ways, contains two serious risks: 1) that limited funds for research will not be spent in an optimum, interactive balance between model development and field experiment; and 2) that, when it eventually becomes apparent that the hoped-for or promised predictive capability is all too often a mirage, adverse reaction will starve the development of better models. This paper is addressed to the avoidance of the first risk. If this risk can be avoided, as I believe it can, the second risk becomes smaller.

Considering lake examples we must not, on one hand, create a state of affairs in which limnological modelers and managers prefer to put questions to their models rather than to lakes. Nor must we, on the other hand, waste effort in unplanned or random collection of data. Our best hope of progress toward deeper knowledge and eventual predictive capability lies in a proper balance between questions directed both to models and to lakes, either simultaneously or, more often, in an iterative two-legged sequence in which models assist the planning of optimized schemes of data acquisition. Models can then be tested and improved. "Classical limnology" has not lacked for models, concepts, and hypotheses to guide (and misguide) it; but the prime lesson to be learned from a century of lake research--from Forel to modern times--is that soundly conceived and adequately based "hard" sets of data, from experiments and lake surveys, are the only grindstones on which models can be sharpened to the point at which their full power can be brought to bear.

As you see, I am discussing models used as scientific tools to define the most revealing questions we can ask of lakes. While eventual predictive capability is the ultimate goal for environmental management, even incomplete and invalid models can generate *new ideas* and better *define the questions*. But, above all, I would urge careful and thorough *listening* to the answers that will put us more quickly on the right road to knowledge than modeling alone. We need two legs to walk that road--modeling *and* verification; theory *and* experiment. I could give many examples--from hydrobiology, hydrochemistry, and hydrophysics--of how listening to signals transmitted by the lakes themselves can shortcut a great deal of unproductive work and put us much further forward on the road to productive modeling and understanding. But for brevity, and because the results may turn out to be very relevant to Lake Baikal, I have chosen an example from the physics of large lakes, namely the responses of the stratified basin to wind impulses.

Models of Thermocline Dynamics

Summer wind storms, the driving force on large stratified lakes and reservoirs, produce displacements of the thermocline level which, in turn, generate internal waves. In the largest basins, the Coriolis force of the earth's rotation combines with the wind to force the thermocline to the surface, causing extensive upwelling in coastal waters in a nearshore band some 20 km wide along the shoreline to the left of the wind direction. A Lake Michigan example, produced by strong wind from the north, is shown in Figure 1. Similar upwelling from strong wind was noted on Lake Baikal by Skabitschevsky (1929); and many other examples have been described, for example, the detailed monograph of Rossolimo (1957).

With upwelling as the "hammer blow" and the thermocline as the "gong," let us listen for the vibrations which follow. This we can do, at little cost, by plotting (Figure 2) the temperature readings at city water intakes on opposite sides of the basin. When intake pipes are near thermocline depth, temperature records from water intakes on opposite sides of the lake basin--three pairs in Figure 2--show a flip-flop motion of the thermocline (down one side, up the other) after strong wind. If a period of weak winds follows, a slow-moving internal wave is generated and progresses counterclockwise around the shore until destroyed by the next storm.

The simplest analytical model for this wave response is a combination of two internal Kelvin waves (Figure 3) in a channel of uniform depth. I wish to stress here that it is not always necessary, at the outset, to construct an elaborate numerical model for each particular lake. Much physical insight can initially be gained from simple generalized analytical models.

The model in Figure 3 tells us that the Kelvin wave amplitude is greatest at the shoreline and decreases exponentially away from the shore, at a rate which depends on latitude and on the density difference across the thermocline. The Kelvin wave currents are everywhere shore-parallel; but beyond a certain distance from shore, the Kelvin wave currents become negligibly small. In Lake Michigan, with a typical internal Kelvin wave speed of about 1.6 km/hr, the associated current amplitude decreases with increasing distance from shore, falling to 5% at about 14 km from shore. Because of the higher latitude and smaller density-difference across the Lake Baikal thermocline, the internal Kelvin wave speed is about 1 km/hr in late summer*, and the offshore distance of the 5% amplitude level is about 7 km. Internal Kelvin waves are therefore trapped near the shore. They are set in motion by winds strong enough to bring about upwellings on one side of the basin and downwelling on the other. In the northern hemisphere such internal Kelvin waves travel counter-clockwise around the basin, but they complete the circuit

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* Assuming a thermocline at 20 m, and upper and lower layer temperature at 11° and 4°C, respectively.

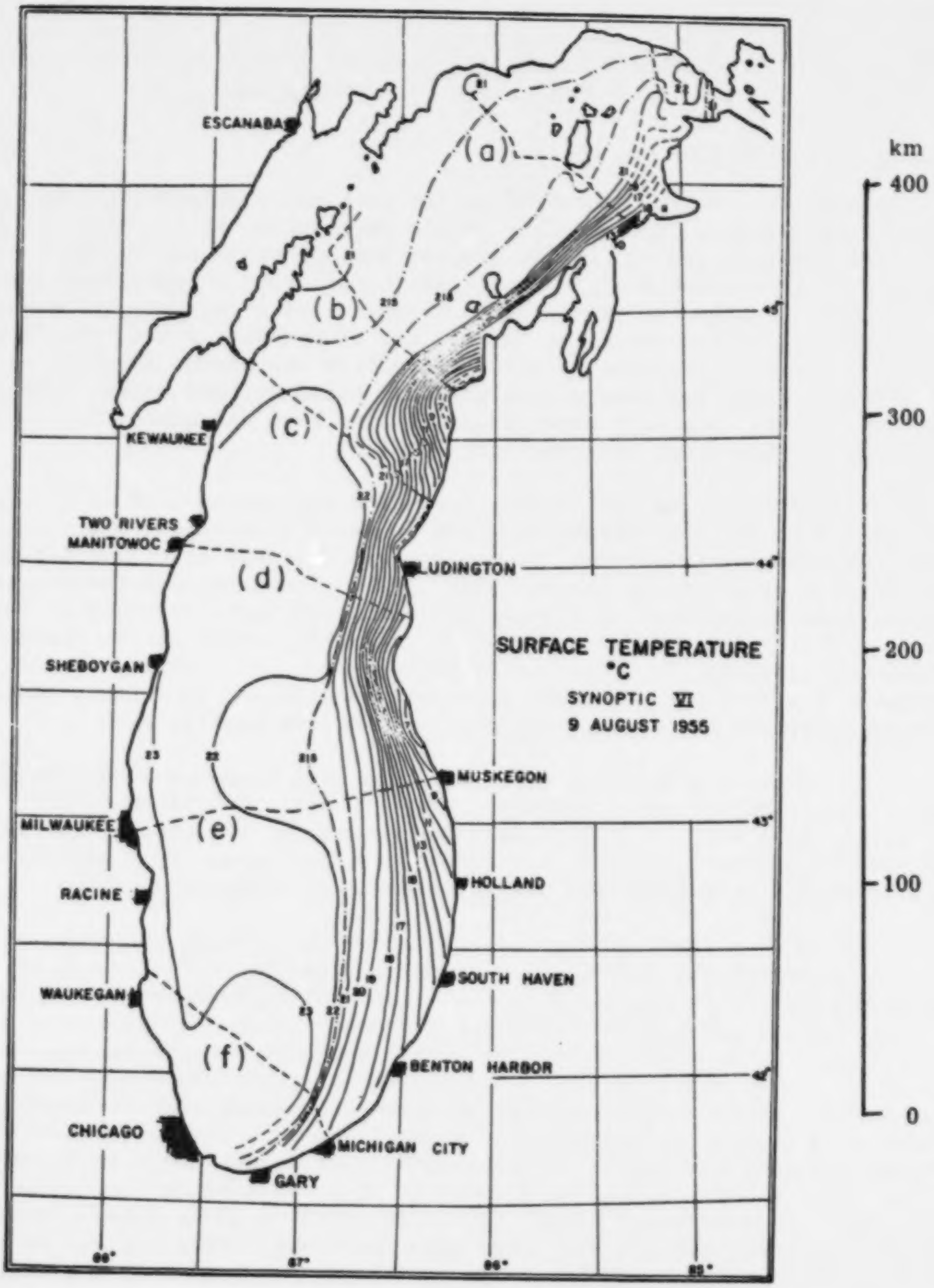

km
400
300
200
100
0
ESCANABA
(a)
(b)
(c)
KEWAUNEE
TWO RIVERS
MANITOWOC
(d)
LUDINGTON
SURFACE TEMPERATURE
°C
SYNOPTIC VI
9 AUGUST 1955
SHEBOYGAN
MUSKEGON
MILWAUKEE
(e)
RACINE
HOLLAND
WAUKEGAN
SOUTH HAVEN
(f)
BENTON HARBOR
CHICAGO
MICHIGAN CITY
GARY
45°
44°
43°
42°
88°
87°
86°
85°

Figure 1. Lake Michigan, 9 August 1955. Distribution of surface temperature, °C, derived from seven "synoptic" temperature surveys by Ayers et al. (1958). Vessel tracks are shown as broken lines.

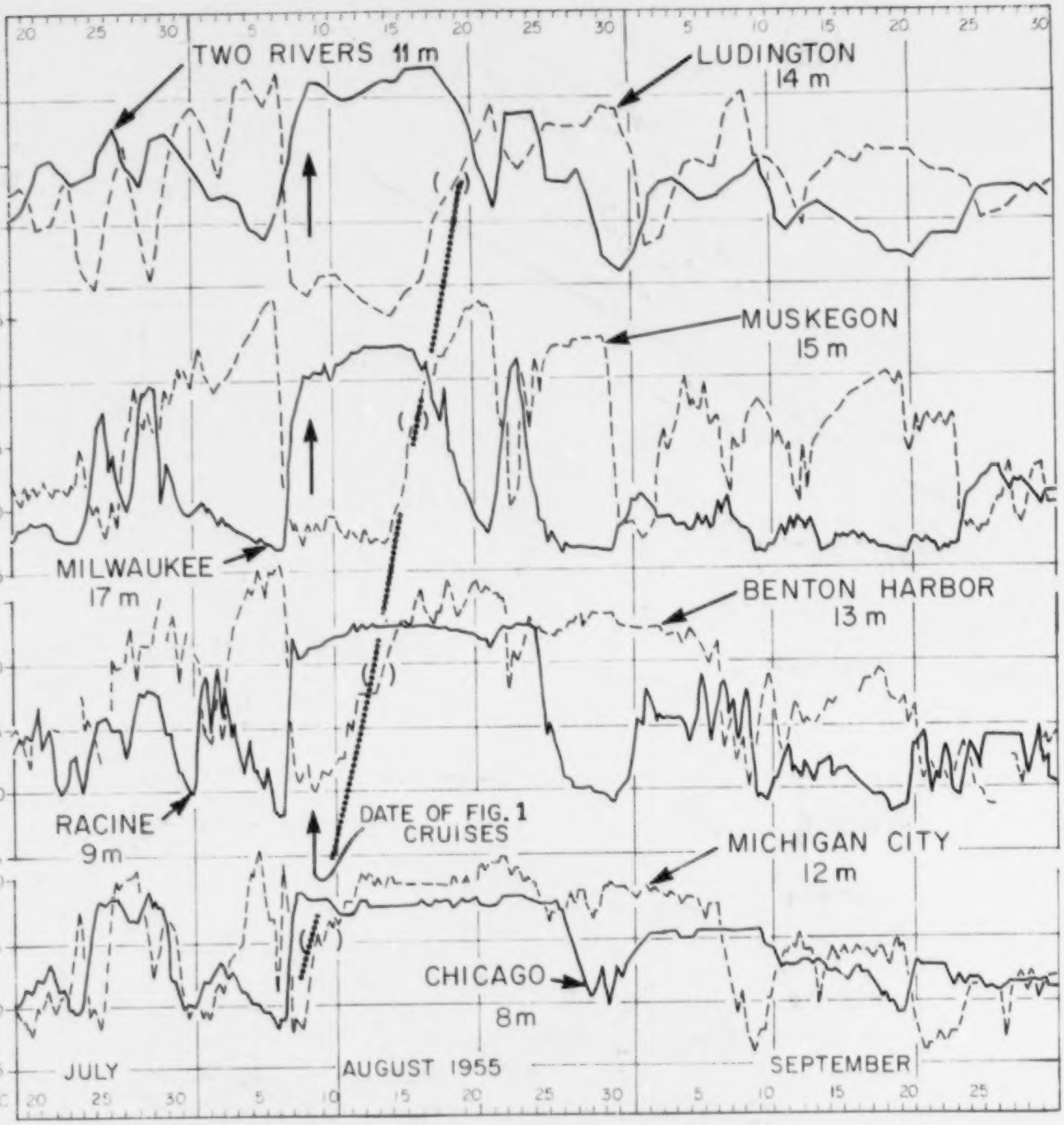


Figure 2. Daily or twice-daily temperature readings at the water intakes of cities around Lake Michigan, 1955 (Mortimer, 1971). The intake pipes extend about 1000 m into the Lake; the depths are shown on the diagram. The temperature records for the intakes on the western shore (unbroken line) are paired with the corresponding record (broken line) for the opposite intake on the eastern shore (see Figure 1). The vertical arrows at 9 August denote the time of the Figure 1 cruises. The thick, sloping dotted line follows the progress northward along the eastern shore of a presumed internal Kelvin wave (see text) generated by the Figure 1 upwelling.

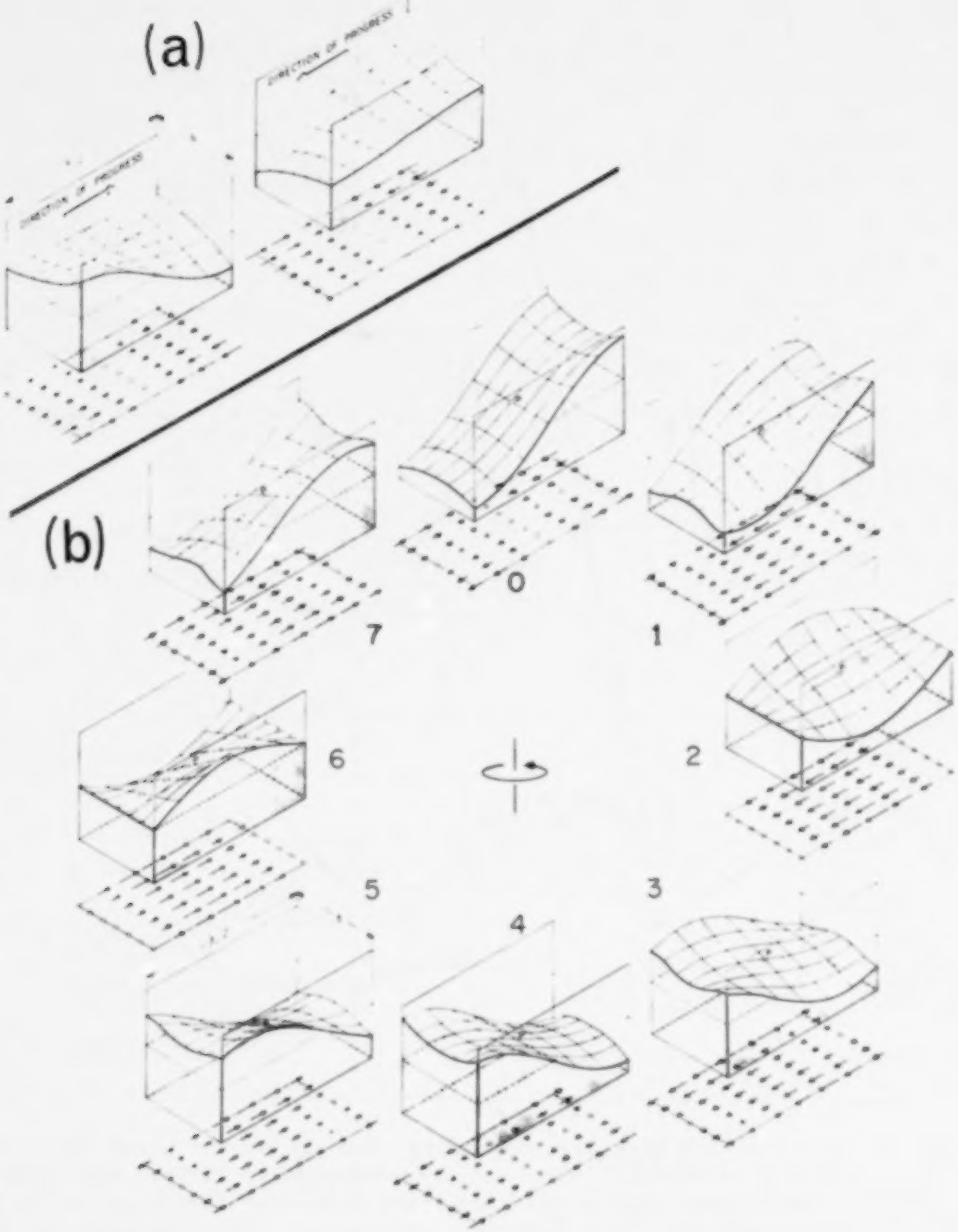

(a)
DIRECTION OF PROGRESS
DIRECTION OF PROGRESS
(b)
0
1
2
3
4
5
6
7

Figure 3. (a) Two identical Kelvin waves traveling in opposite directions. These combine to form the amphidromic standing wave and associated currents, successive (1/8 cycle) phases of which are illustrated in (b) assembled from Mortimer (1975).

only if not disturbed by another storm. Completion of the circuit in Lake Michigan would require a month, and the Lake is unlikely to remain undisturbed for so long. In Lake Baikal, I calculate the period for one circuit to be two months. Therefore the clue provided by Schostakovitsch (1926) in Figure 4 is of interest. During the months of July to September 1914, the 4°C isotherm depths at opposite ends of the Baikal basin rose and fell in opposition and with apparent periodicity of about 40 days. The data are not complete enough to decide whether an internal wave or an upwelling interpretation is appropriate (see Rossolimo, 1957, p. 548).

If we increase the frequency-resolution of our lake-signal receiver, as in the Figures 5 and 6 recordings of waterworks intake temperatures, snatches of higher frequency signals come through, when the thermocline stays for several days near intake depth. The thermocline "gong" is heard to be vibrating at a set of frequencies (free modes) which are closely tuned to the local inertial frequency, but which never exceed it. The equivalent periods are 14 to 17 hours for Lake Michigan, which has an inertial period of 17.4 hours, i.e. 12 hours divided by the sine of the latitude. For Lake Baikal (inertial period 15 hours) the preferred frequencies are expected to lie between 12 and 15 hours. The simple model, proposed in 1963 for this near-inertial wave response and later verified (Mortimer, 1963, 1971, 1974), is that of an internal Poincaré wave in a channel of uniform depth (Figure 7).

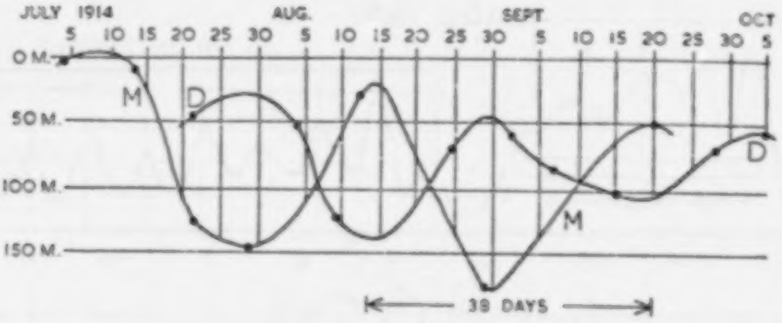


Figure 4. Lake Baikal 1914. Variation in depth of the 4°C isotherm, July to October, at Marituj (M) and Douschkatschan (D), i.e. at opposite ends of the lake, redrawn from Schostakovitsch (1926), with a 38-day time interval added.

In contrast to the slow-moving internal Kelvin waves confined to a nearshore strip about 15 km wide and with wave currents always shore-parallel, the Poincaré wave is a cross-channel internal seiche which vibrates the whole thermocline in a cellular pattern with clockwise-rotating currents. As with seiches in small lakes, several modes can be forced into oscillation by a storm, but in very large lakes (Baikal and Michigan, for example) all observed modes possess periods close to (but never greater than) the local inertial period.

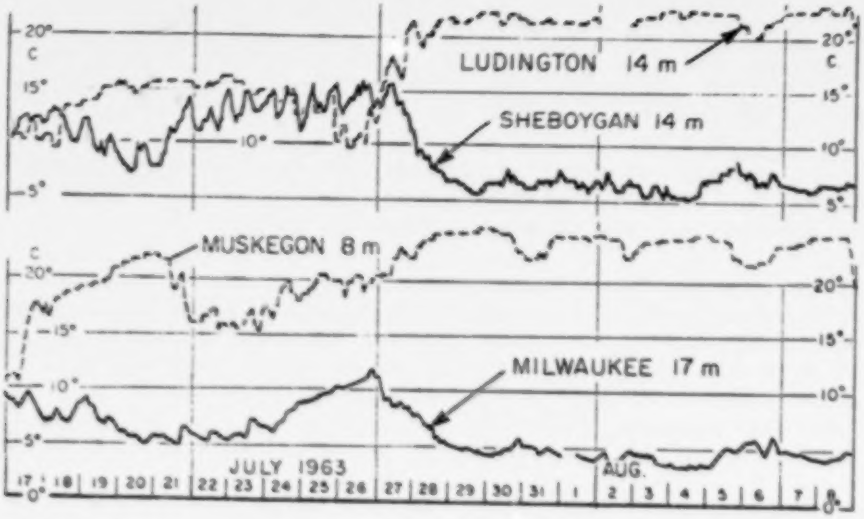


Figure 5. Hourly temperatures at Lake Michigan water intakes, 1963 (Mortimer, 1971).

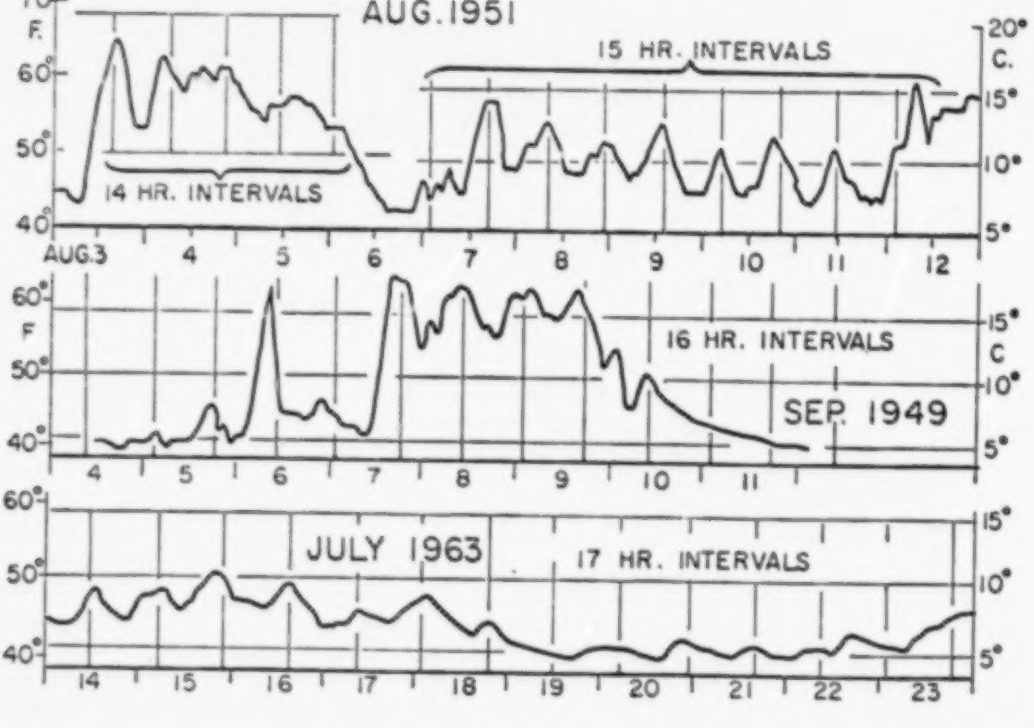


Figure 6. Hourly mean temperatures, during various episodes, at the Milwaukee Wisconsin water intake (Mortimer, 1971).

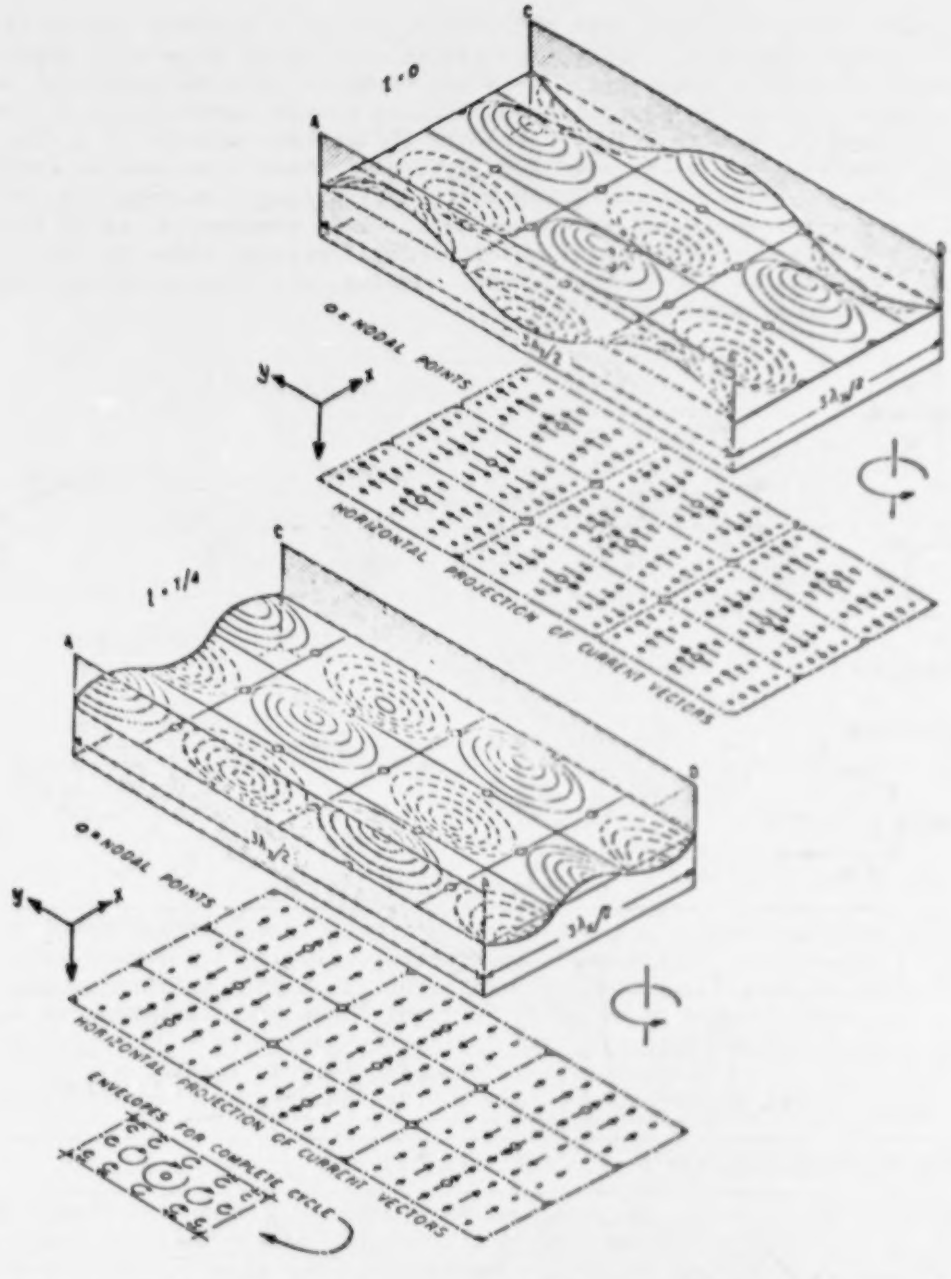

C
t = 0
A
D
O = NODAL POINTS
3λ/2
y
x
HORIZONTAL PROJECTION OF CURRENT VECTORS
C
t = T/4
A
D
O = NODAL POINTS
3λ/2
y
x
HORIZONTAL PROJECTION OF CURRENT VECTORS
ENVELOPES FOR COMPLETE CYCLE

Figure 7. A standing Poincaré wave in a wide, rotating channel of uniform depth. Separated by 1/4 cycle, two phases of the oscillation are shown for the cross-channel trinodal (3rd mode) case, with a ratio of a long-channel to cross-channel wavelength of 2/1. The clockwise rotation and cellular distribution of the current vectors are illustrated on the horizontal planes below the channel (Mortimer, 1974).

We must therefore infer from the lake's responses and from our simple models of those responses, that summer storms over large lakes will generate three types of current responses: 1) shore-parallel patterns within 15 km of the shore, produced by upwelling/downwelling events (geostrophic currents) generating internal Kelvin waves; 2) rotary (Poincaré) currents at offshore distances greater than 15 km; and 3) a mixture of these responses at intermediate distances. The various forms which this mixture can take are modeled by simple combinations of rotary and unidirectional components, as in Figure 8. With increasing proportions of unidirectional current added to the rotary (Poincaré) current, we obtain meandering, cusping, and looping current tracks, illustrated in later examples.

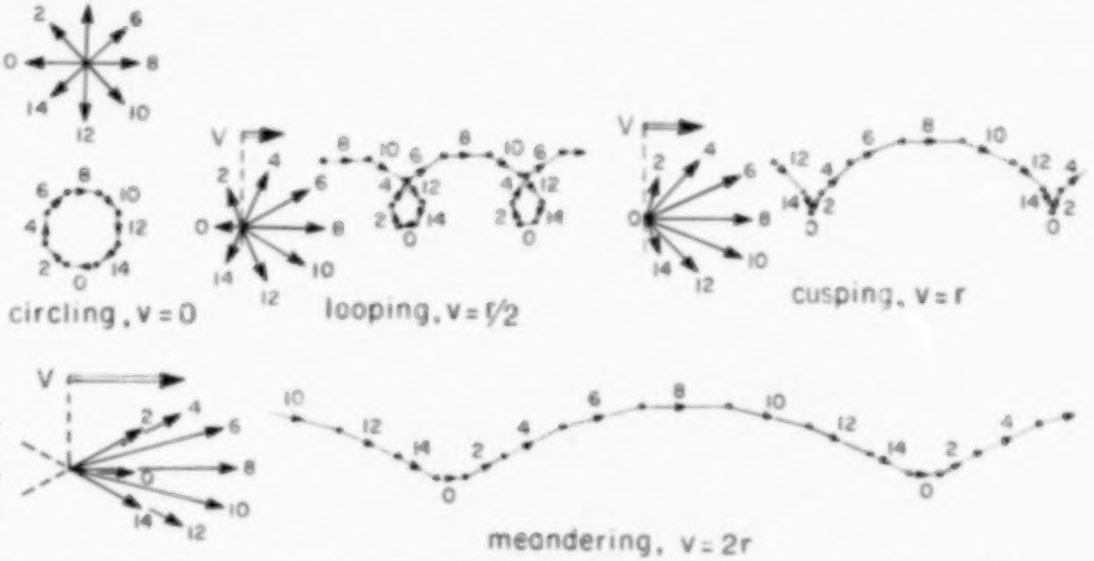
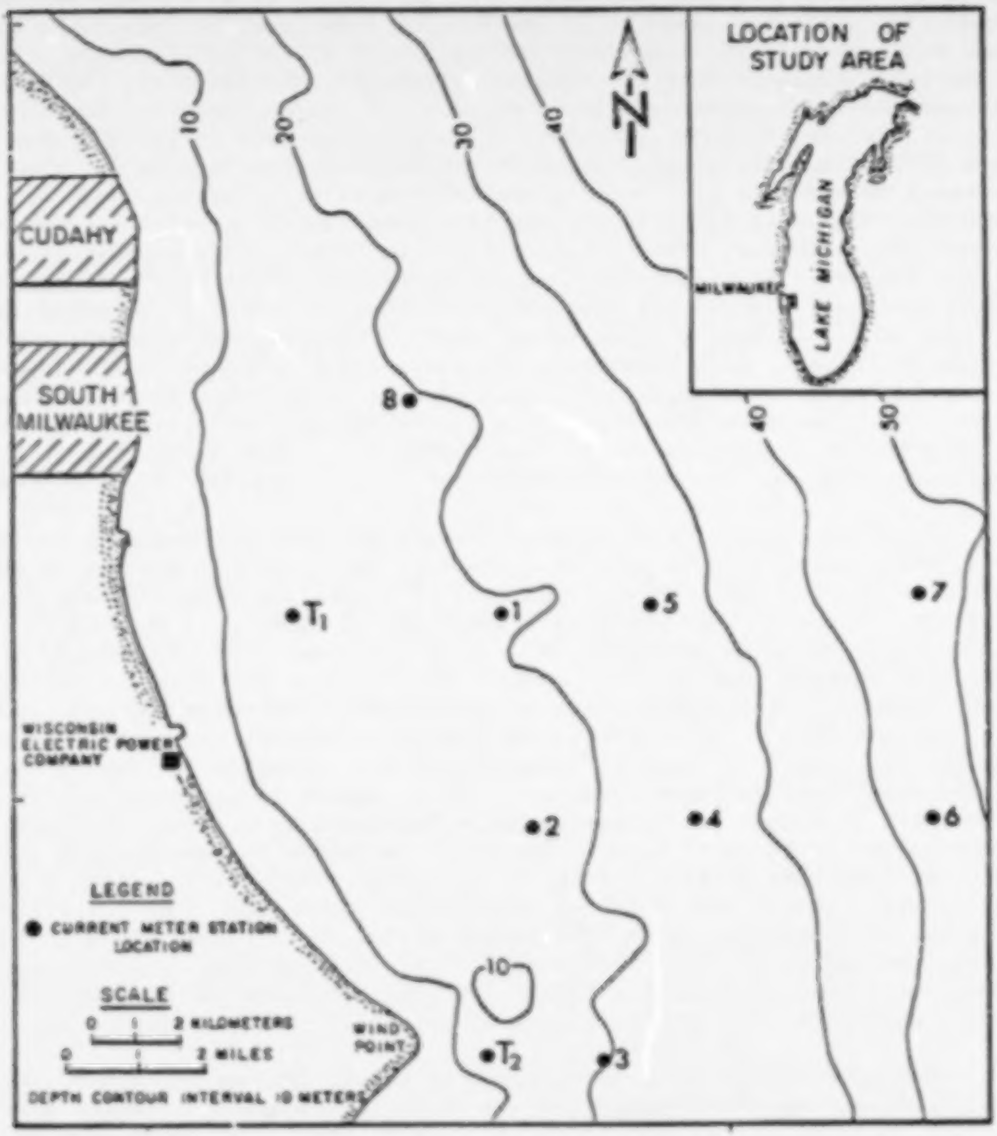


Figure 8. Combinations of a rotating current vector of constant speed r (shown at 2 hr intervals and corresponding to a Poincaré wave of 16 hr period) with increasing proportions of a unidirectional current vector of speed v. Current "roses" and progressive vector diagrams are shown for the following models: v = 0 (motion in the inertial circle); v = 1/2 r (looping current track); v = r (cusping current track); and v = 2r (meandering current track).

Signals from Large-Basin Coastal Currents

With the above models to guide us, my student G. K. Sato and I were in a position to "listen" to the motions in the coastal zone. Although complex, these motions are of great practical importance. For this is the zone into which most man-made wastes are discharged and dispersed, from which water is extracted for industrial purposes, and in which most recreational uses develop.

Our listening post for signals from coastal currents occupied a nearshore area of Lake Michigan about 15 km square (Figure 9), with ten current meters moored in depths ranging from 10 to 26 m (Sato and Mortimer, 1975).



CURRENT METER STATION LOCATIONS

LOCATION OF STUDY AREA
MILWAUKEE
LAKE MICHIGAN
N
CUDAHY
SOUTH MILWAUKEE
WISCONSIN ELECTRIC POWER COMPANY
WIND POINT
10
20
30
40
40
30
10
8
T1
1
5
7
2
4
6
T2
3
LEGEND
CURRENT METER STATION LOCATION
SCALE
0 1 2 KILOMETERS
0 1 2 MILES
DEPTH CONTOUR INTERVAL 10 METERS

Figure 9. Location of instruments to measure current speed, current direction, and temperature off Oak Creek, Wisconsin, western Lake Michigan (Sato and Mortimer, 1975).

The following diagrams present extracts of continuous records of water flow (speed and direction) past the fixed instruments moored at various depths at the station positions shown in Figure 9. The water current speeds and directions are compared, in these diagrams, with wind speed and direction at a nearby land station. When the coastal waters were unstratified, the currents followed the local winds relatively closely and reproduced the episodic character of the wind force (Figure 10). This was also true in early spring (June 1972, Figure 11), when, although the shallow nearshore waters were beginning to stratify and form the thermal bar first described in detail by Tikhomirov (1963) in Lake Ladoga, the main lake basin was still isothermal without a thermocline. The episodic character of the wind-responsive currents is well displayed in Figures 10 and 11. When the wind direction changed and picked up speed, the current also changed direction, after a few hours lag, and also picked up speed, often very rapidly. Changes from northgoing to southgoing currents were accompanied by sharp rises in temperature (downwelling motions) while northgoing winds brought lower temperatures. This suggests that the development and fate of the thermal bar will be determined, not only by the sequence of heat input during the spring months and by the convergence model of Tikhomirov (1963), but also by episodic wind influences.

After whole-basin stratification became well-established with a distinct "permanent" thermocline by mid- or late-June, the current responses in the nearshore/offshore intermediate region (5 to 10 km from shore) became modified by internal Poincaré wave dynamics (Figure 12, 18 June to 17 July 1972, Station 6, 23 m, 10 km offshore), i.e. they were less directly dependent on local wind forcing (see 24 to 27 June). Depending on the relative proportions of the unidirectional (wind-driven or geostrophic) component and the rotary Poincaré component of the current, the resultant current was either "meandering," "cusping," or "looping" (see Figure 8). Examples of looping currents are shown for Lake Michigan (Figure 13, 12 m, August to September 11, km offshore) with a current track showing loops (produced by a rotary Poincaré wave component) every 16 to 17 hours. The only continuous current records available to me from Lake Baikal (Figure 14, September 1960, two stations, 6 and 21 km offshore) clearly show Poincaré wave motion, with about four rotations in 2.5 days, corresponding to an approximate period of 15 hours, close to the inertial period at 54° North latitude. I predict that this type of rotary current will be conspicuous in Lake Baikal, particularly as the width of the coastal zone of Kelvin wave influence is relatively narrow, about 7 km.

Much nearer shore in Lake Michigan in August (Figure 15, 3 km), the rotary current component disappears and the currents are more nearly shore-parallel, related not so much to the wind as to flows associated with upwelling and downwelling motions (see temperature record in that figure).

A summary of what we have learned about coastal currents is contained in spectra of the kinetic energy associated with the currents at two stations, respectively 6 km and 11 km from shore in Lake Ontario (Figure 16, Boyce, 1974, unpublished observations by J.O. Blanton). There is a distinct gap in both spectra between low-frequency motions with periods of 3 days or more and Poincaré-wave motions with periods close to the inertial period, 17.5 hr. At the 11 km station, the near-inertial energy component is greater and the low-frequency energy component is less than at the 6 km station.

1 to 25 November 1972: wind (at Mitchell Field) compared with current at Station T_2, 10 m depth.

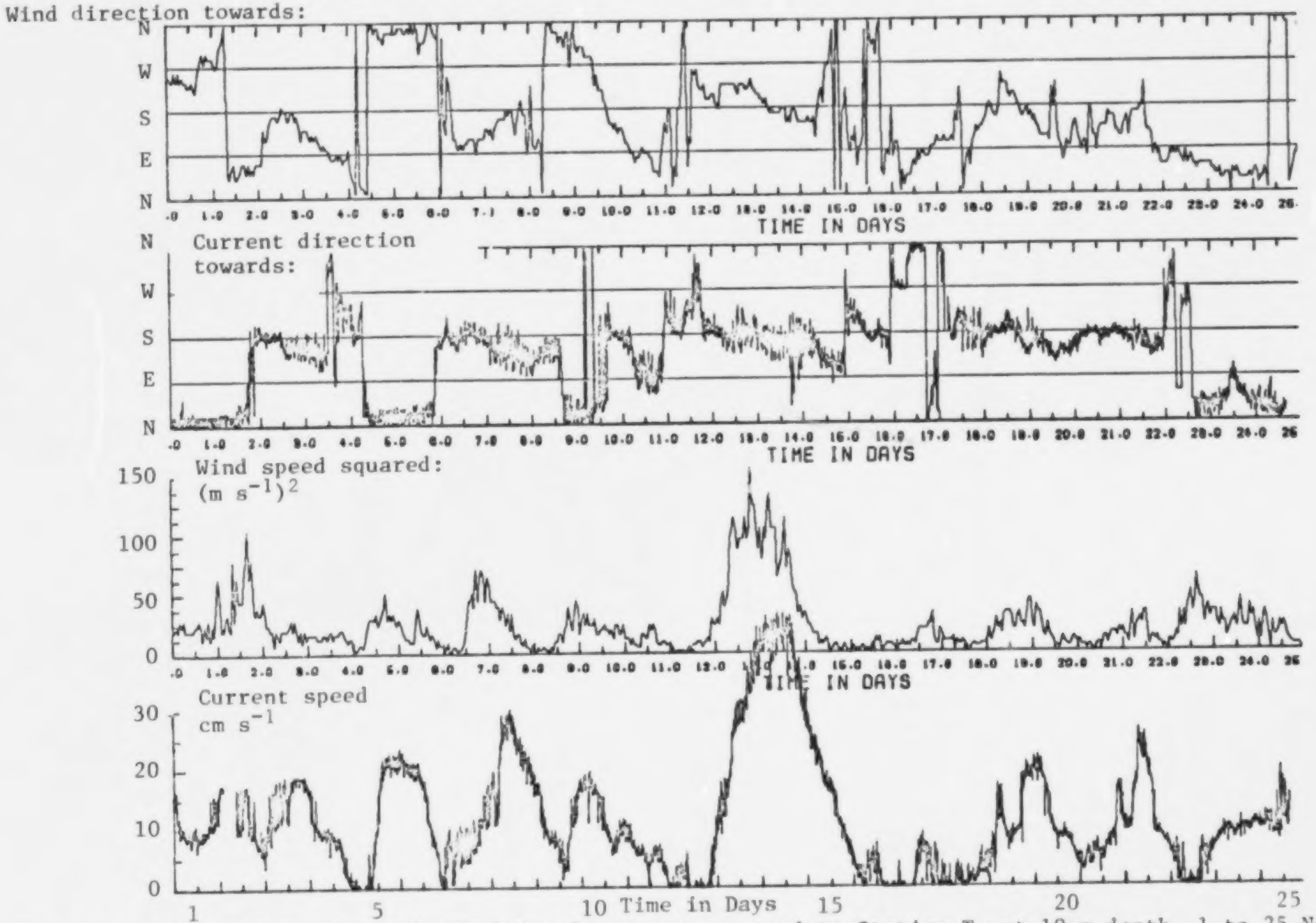


Figure 10. Direction and speed of wind and current compared to Station T_2 at 10 m depth, 1 to 25 Nov., 1972. The directions shown are those toward which both the current and wind are moving.

1 to 26 June 1972: Wind (at Mitchell Field) compared with current and temperature at Station 3, 16 m depth.

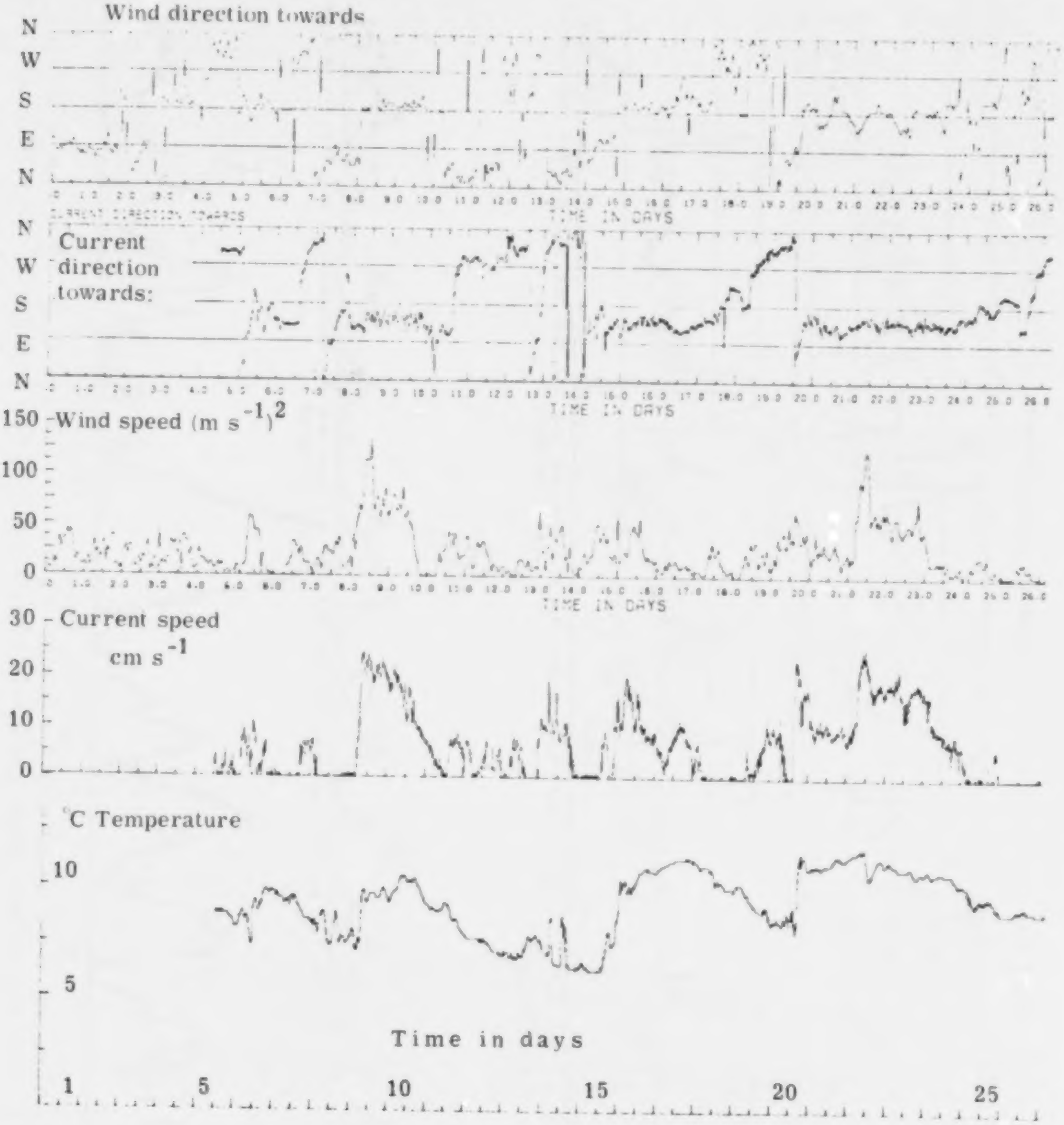


Figure 11. Direction and speed of wind and current compared (plus temperature), at Station 3 (see Figure 9), 16 m depth, 1 to 26 June 1972 (Sato and Mortimer, 1975).

21 June to 16 July 1972: Wind (at Mitchell Field) compared with current and temperature at Station 6, 23 m depth.

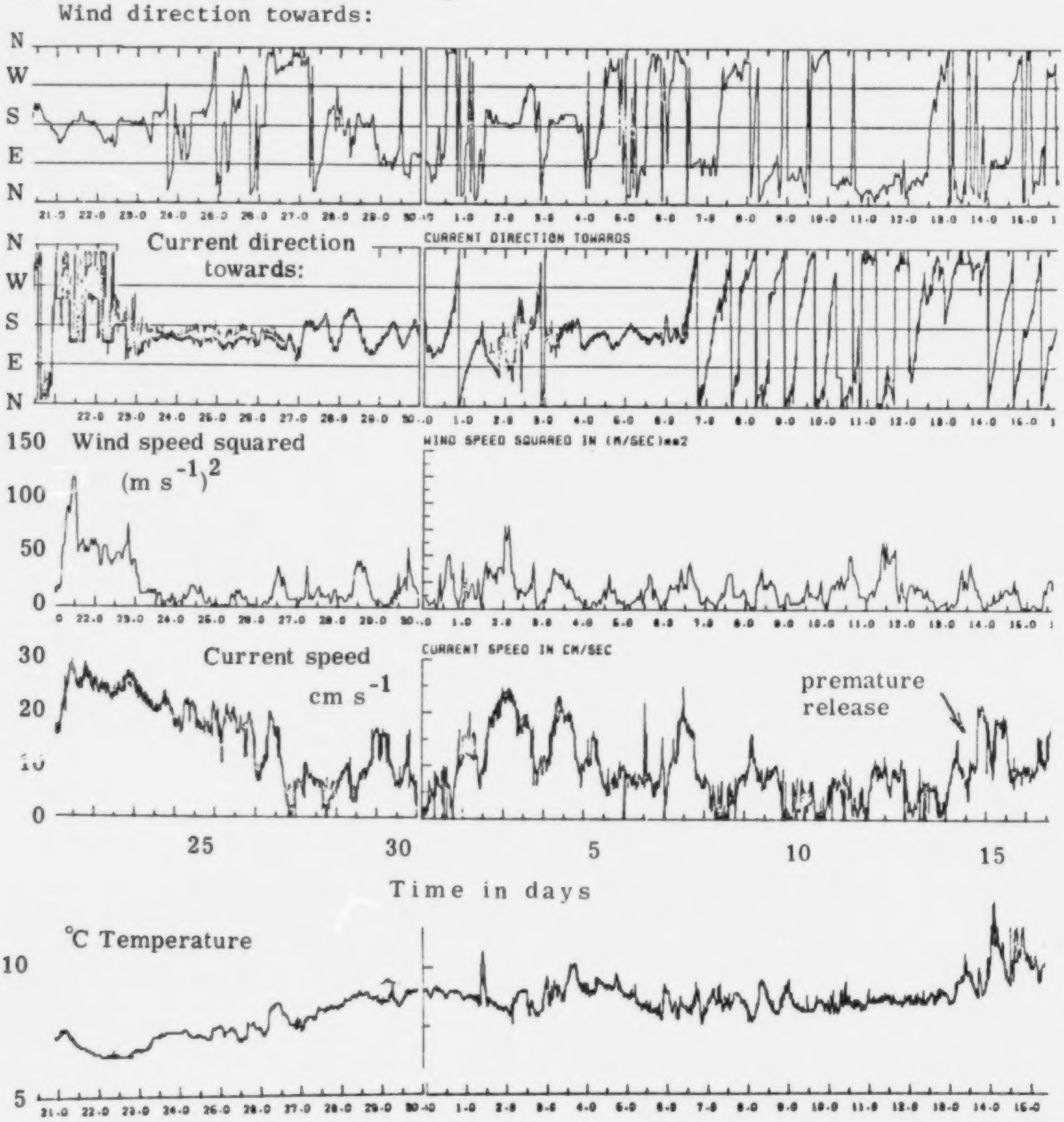


Figure 12. Direction and speed of wind and current compared (plus temperature) at Station 6 (see Figure 9) 23 m depth, 21 June to 16 July 1972 (Sato and Mortimer, 1975).

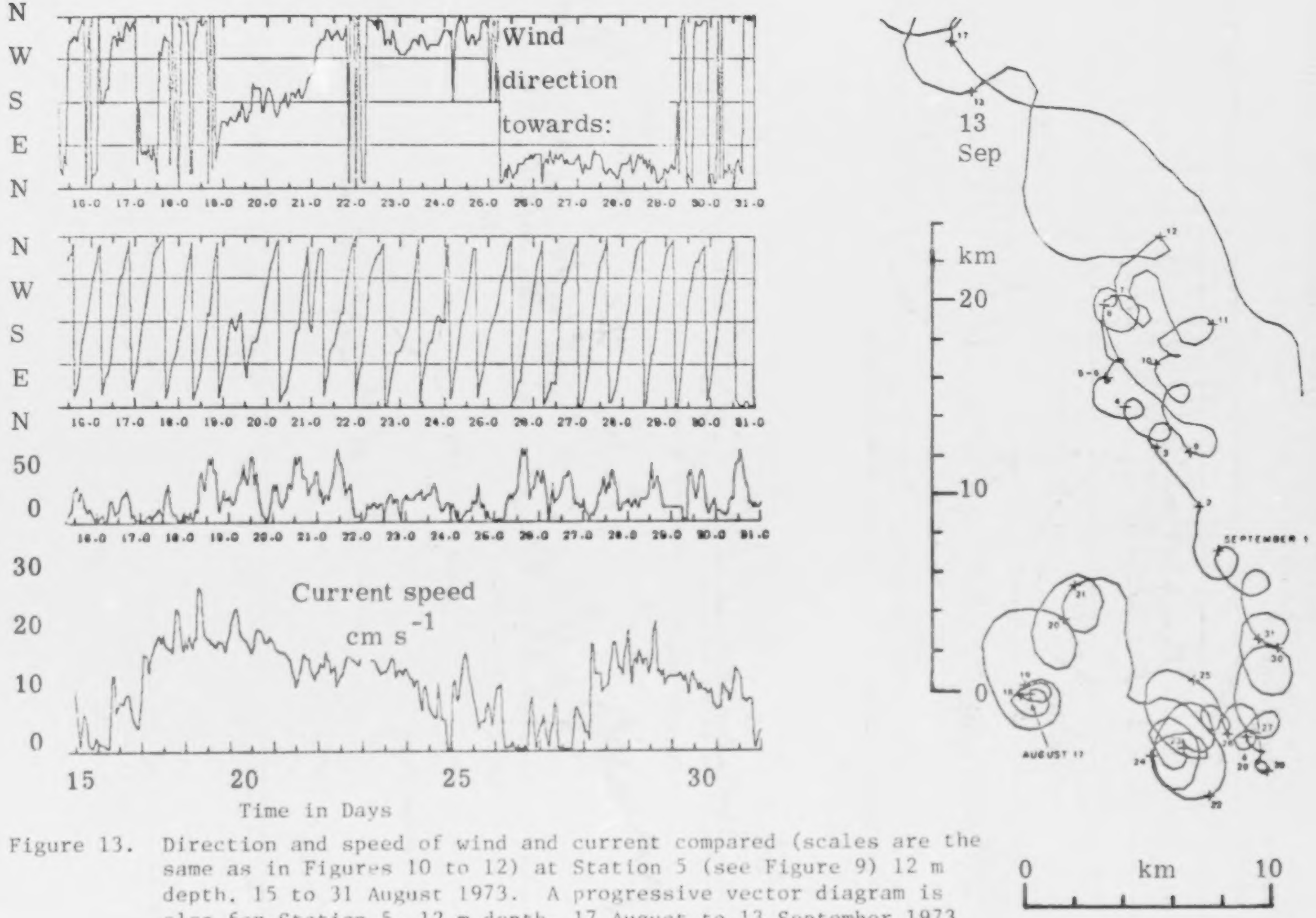
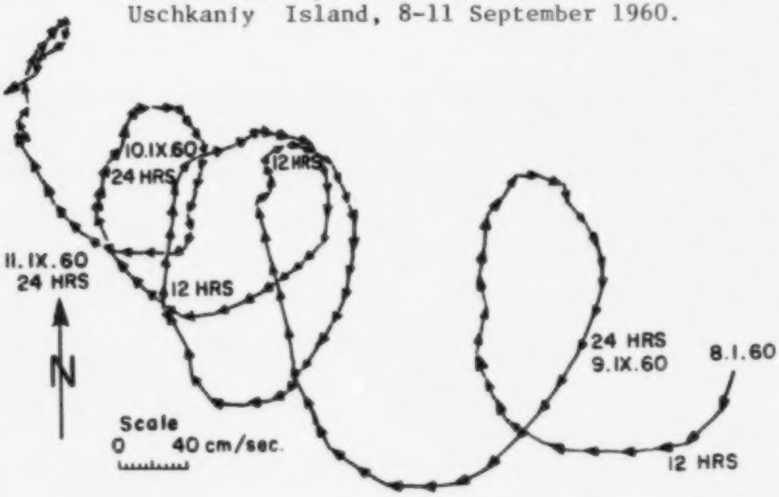


Figure 13. Direction and speed of wind and current compared (scales are the same as in Figures 10 to 12) at Station 5 (see Figure 9) 12 m depth, 15 to 31 August 1973. A progressive vector diagram is also for Station 5, 12 m depth, 17 August to 13 September 1973 (Sato and Mortimer 1975).

Trekhsutochnaya Station, 6 km southwest of Bol. Uschkaniy Island, 8-11 September 1960.



Peninsula anchor station, 21 km north of Choboi Point (Olchon Island), 16-18 September 1960.

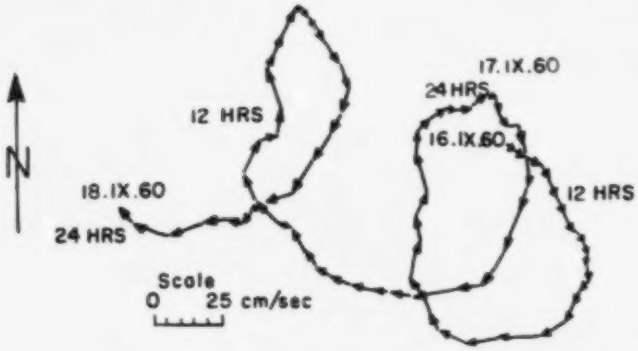


Figure 14. Progressive vector diagrams of currents at 10 m depth near the "Academic Ridge," Lake Baikal (from "Atlas of Baikal," Limnological Laboratory, Acad. Sci. USSR, Siberian Section, 1969).

1 to 23 August 1972: Wind (at Mitchell Field) compared with current and temperature at Station T_1, 11 m depth.

Wind direction towards:

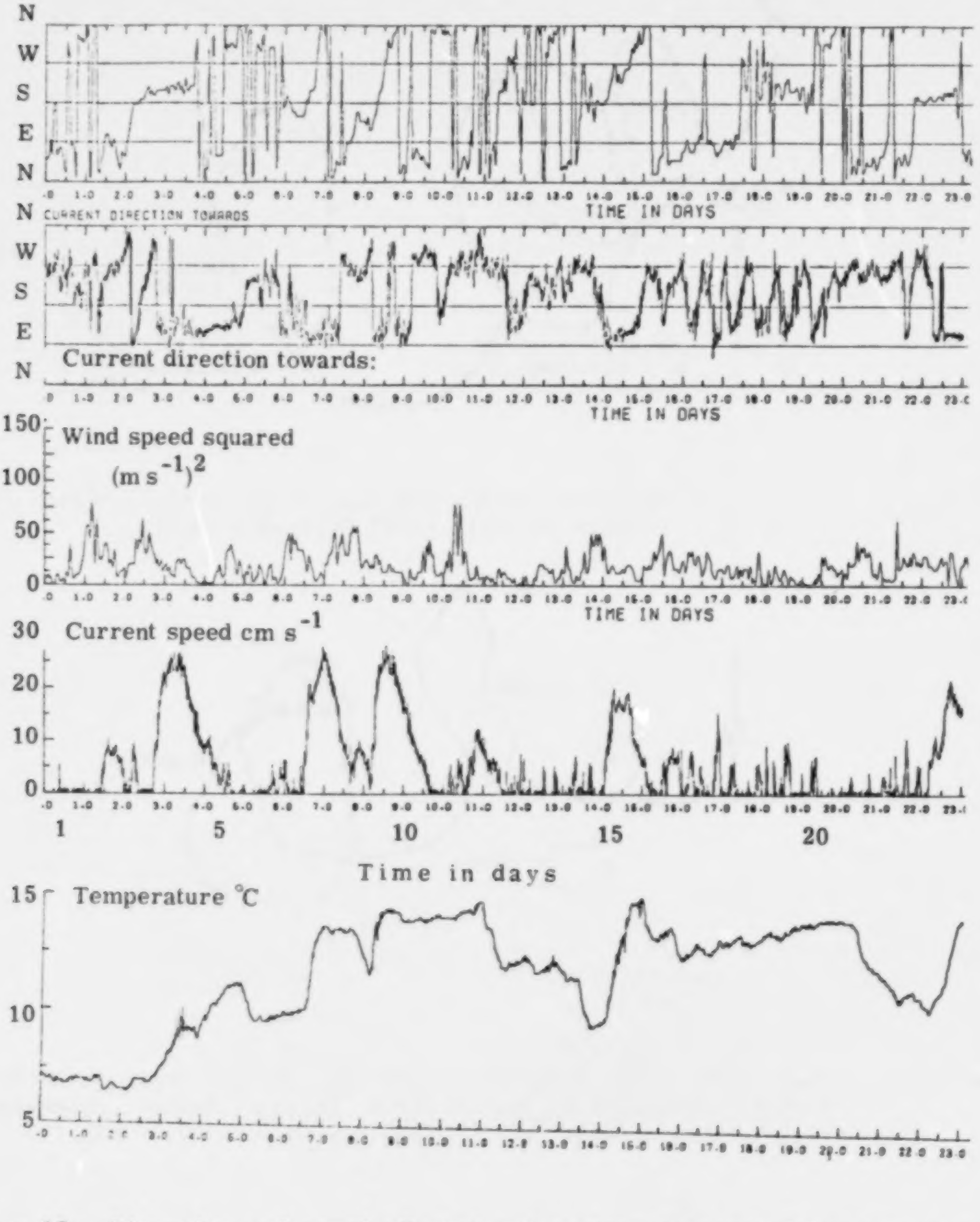


Figure 15. Direction and speed of wind and current compared (plus temperature), at Station T_1 (see Figure 9) 11 m depth, 1-23 August 1972 (Sato and Mortimer, 1975).

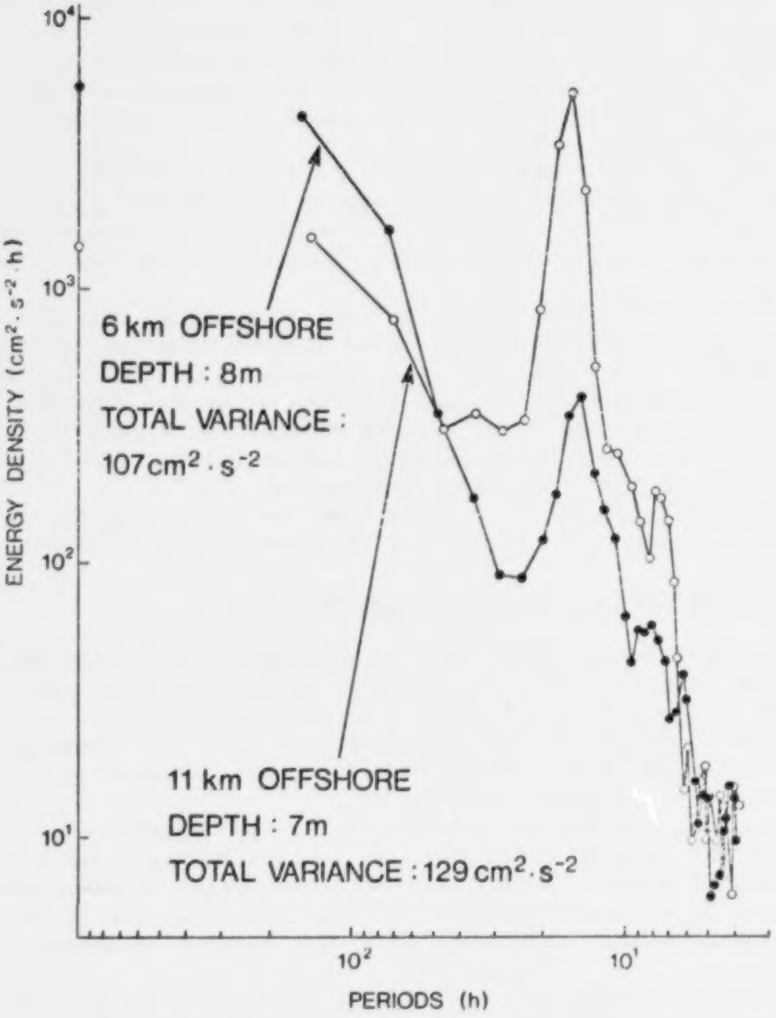


Figure 16. Frequency spectra of kinetic energy in the currents at two stations in Lake Ontario, 3-17 July 1970 (Boyce, 1974).

Lessons Learned from Listening to Lakes

Listening to the lake itself has put us a useful step forward on the road toward physical understanding, upon which realistic three-dimensional models of motions in particular basins can be soundly based. For our next state of modeling--i.e. for the next step in our two-legged progress--the lake's signals have told us that:

1. During seasons of no stratification, nearshore currents are episodic, shore-parallel, and directly responsive to wind, with characteristic steep rises and slower falls in speed (Figure 11). The rate of speed decrease provides information on coastal friction.

2. During stratification, nearshore currents are still partially shore-parallel, episodic and wind-responsive, but also respond (as shore-parallel geostrophic currents) to upwelling/downwelling events and to the low-frequency internal Kelvin waves generated by the upwelling/downwelling motions.

3. Beyond 15 km from shore, Poincaré wave dynamics, with nearshore inertial clockwise rotation of currents, are dominant. In the range 3 km to 20 km offshore, combinations of shore-parallel and rotating current patterns are encountered.

4. The simple, linear Kelvin and Poincaré wave models do not fully explain the observed nonlinear features of the internal waves arising because the (linear) assumptions, a) of uniform depth and b) of small wave amplitude relative to total depth, do not hold. Nonlinear models must therefore be developed.

5. The final lesson is that we must continue to maintain our listening posts--and develop new ones--to receive signals from the system we are trying to understand. We now perceive some features in outline--and we can recognize some of the impediments to clearer vision--but we are reminded by lake messages from new listening posts (namely satellites, Figure 17) that there is still a great deal to be learned about large-scale motions, particularly mechanisms and scales of horizontal dispersal.
It is clear that, for a long time to come, we shall need the two legs of theory and experiment, i.e. modeling verified by signals from nature.

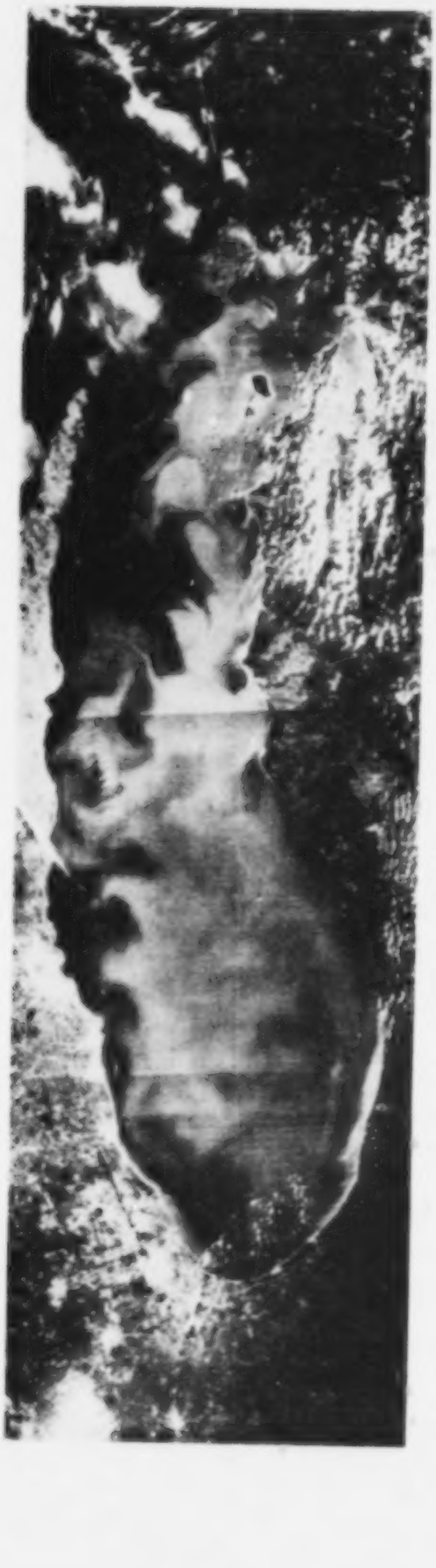
Some General Remarks on Lake Ecosystem Modeling

While the examples given here of interactive iteration between listening and modeling have been physical, it will be profitable to adopt a similar two-legged approach to modeling of chemical and biological events and processes. Models applied to aquatic ecosystems fall into three general classes:

1. hypotheses or mathematical models, not necessarily valid, which suggest new ideas or paths to explore;
2. tunable mathematical models, which can be adjusted to the conditions in a particular lake or river, but are limited in application elsewhere; and
3. models based on more fundamental knowledge of the controlled mechanisms in a wide range of real systems. Such models will be eventually capable of providing reliable predictions in a wide range of examples.

Of class 1. little need be said; it is part of the general method of advancement of scientific knowledge. The "curve-fitting" models of class 2. often are useful in environmental management, but independent verification of them is rarely possible; and, in using them, it should not be forgotten that the environmental manager's most pressing need is to be able to predict the effects of large perturbations of the system. In lakes at least, large perturbations are characterized by sweeping changes in species composition. The stage is swept clear, to be invaded by a different set of hungry actors, to which the earlier tunable model can no longer be made to fit.

Class 3. models of lakes, capable of universal, independent verification, are still in embryonic or component stages, with hydrodynamic modeling the most advanced component. It is clear that the most rapid advance will come, not through separate élites of "classical limnologists" and modern systems analysts, but through an intimate, interactive partnership between the two.



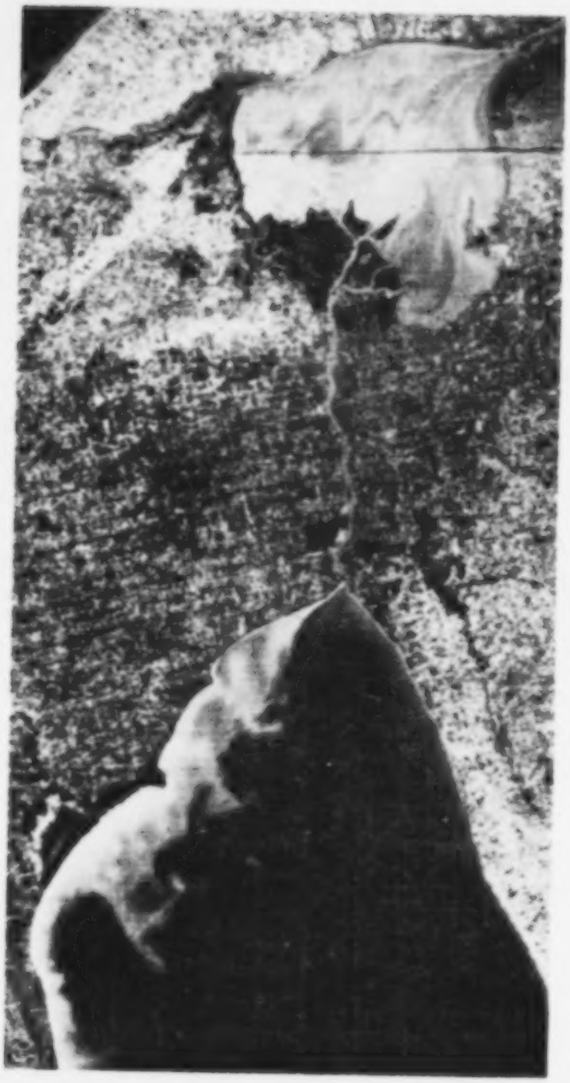


Figure 17. LANDSAT (ERTS satellite) images* of: (above) lower Lake Huron and Lake St. Clair, 27 March 1973; and (left) Lake Michigan, 21 August 1973. The "white water" in Lake Michigan has been attributed to calcium carbonate precipitation. The turbidity in Lake Huron was produced by wave erosion of the shoreline.

*from negatives supplied by the U.S. National Aeronautic and Space Administration.

The question is not which partner is superior or more up-to-date, but which partner is lagging behind the other. Models can be powerful, interpretative tools, but the discoveries and the verifications are still to be sought and listened for in the lakes themselves. The availability of modern tools does not diminish the usefulness of signposts to limnological progress erected by the pioneers.

> "Un lac se rapproche par ses proportions d'un laboratoire òu le naturaliste peut répéter a volunté ses recherches, instituer des expériences, interroger la nature au lieu de se borner à en écouter les lecons." Forel (1892, Vol. I, p. vii).

which, freely translated, reads:

> "In its dimensions a lake may be compared to a laboratory, in which the naturalist can repeat his researches at will, set up his experiments, and interrogate nature, rather than confine himself to listening to lectures."

This seems to be a fitting note on which to end this lecture!

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DISCUSSION

ROZHKOV: First, if the reaction in the lake is determined and the influence force is random, will a model based on stochastic hydrodynamics levels be suitable? Second, in the report, flow spectra obtained by field data are presented. Do you have any internal fluctuation spectra? You did not illustrate them in the report. How do you relate them?

MORTIMER: Due to insufficient time, I did not speak about quasi-stationary conditions and geostrophic flows near the shore. The impulse of any natural system can be two-fold: quasi-impulse and fluctuating. I spoke about one determined group of lake frequencies; the second group of frequencies is surface fluctuations.

The second question relates to spectral data on flow. The model used is simple and based on influx theories. We had this type of model 50 years ago. If you know the basin dimensions and the density differences in the thermocline, it is possible to calculate the frequency of the discharge motion fluctuation. When the basin is large, then the fluctuation frequencies are close to inertia frequencies. Therefore, you see such a large peak on the spectrum. In small water bodies, there is a better division of the model according to frequency. The spectrum shows us a series of peaks.

GAVRILOV: I would like to mention the results obtained from research on the Mozhayskiy water reservoir. Unfortunately, I cannot show you any graphic materials, but analysis shows that automatic measurement of parameters as pH, temperature, dissolved oxygen, light penetration, and so forth, is subordinated to the same spectrum and carries impressions of the same phenomenon which has been examined from the hydrodynamics point of view. One main question is what sampling method should be used for hydrodynamic analysis? Data obtained from 1974 to 1975 show that during the spring algal bloom, fluctuations in the parameters examined above have quasi-sinusoidal character.

ROZHKOV: Stochastic hydrodynamic equations should be the basis for models of natural processes (flows, internal waves). Such models already exist. The principles of internal fluctuations, about which Dr. Mortimer spoke, are valid not only for lakes, but also for oceans.

MORTIMER: Large lakes have their own characteristics, which differ from those of the ocean, but they are often presented as models for oceans. I do not think this is valid.

ZNAMENSKIY: I would like to emphasize that it is important that mathematical modeling and field research go hand in hand. When everyone understands the danger of pollution, questions of environmental protection will be extremely important.

CONSTRUCTION OF A MODEL OF LAKE BAIKAL ON PRINCIPLES OF SELF-ORGANIZATION

A.G. Ivakhnenko

INTRODUCTION

Maintenance of water purity in Lake Baikal, which contains 80% of the freshwater reserve in the USSR and 20% of the world source, is a task of vital urgency. The Limnological Institute of the Siberian branch of the USSR Academy of Sciences, was organized to monitor and study the processes in Lake Baikal. During the years of its activity it has undertaken a considerable amount of work which is published in many scientific publications (see Bibliography). Galkin has published methods of modeling the lake self-purification processes using differential equations and utilizing Monte-Carlo statistical methods to determine average characteristics of the Lake Baikal pollution zone. Galazi (1960-1964) also published the results of lake pollution studies on the basis of the research conducted in Baikal. Shimarayev (1971) showed that the forecasting of the hydrobiological processes in the lake should be divided into two parts: the forecast of a small number of low-frequency factors and the forecast of a large number of high-frequency factors.

The Biological Faculty of Irkutsk University, the Institute of Cybernetics of Ukr. SSR Academy of Sciences, and the Limnological Institute have joined in the effort to study the application of the self-organization method to forecast the Melosira biomass yield in Lake Baikal (Kozhova et al. 1973).

Irkutsk University and the Institute of Cybernetics of Ukr. SSR Academy of Sciences have also carried out joint research aimed at a better understanding of correlation links between phytoplankton and zooplankton in Lake Baikal. The results of this research are shown in Appendix I.

The main part of this report reflects the results of a joint investigation carried out by the Novocherkassk Hydrochemical Institute and the Institute of Cybernetics of Ukr. SSR Academy of Science (1973-1975). It is concerned with the construction of three mathematical models of Lake Baikal mixing zones on the basis of self-organization principles.

Construction of Models of Lake Baikal's Self-Purification Process by Self-Organization Algorithms*

For a number of years, the Novocherkassk Hydrochemical Institute has been conducting observations and investigations of Lake Baikal mixing zones in that region of the lake which receives the effluents of the Baikal pulp plant (BPP) and the paper factory (on the Selenga River)--the main "polluters" of the lake (Limnological Institute Report, 1968).

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*The authors are: Zenin, A.A., A.A. Matveyev, V.L. Pavelko, A.G. Ivakhnenko, and P.I. Koval'chuk.

The results of these observations have been used to construct mathematical models of Lake Baikal's mixing zones by the following three hydrochemical parameters: (1) dissolved solids; (2) suspended solids; (3) chemical oxygen demand (COD).

It was assumed that these parameters do not interact with one another. (To refine the models these interactions will be taken into consideration.)

The models obtained are intended to:

1. develop methods to forecast changes in the concentration of hydrochemical parameters;
2. develop methods to identify the mixing zones and to extrapolate them beyond the region of available measured data;
3. determine the maximum permissible concentrations that do not induce lake eutrophication ("critical quantities problem").

These results were derived from the latest measured data, least subject to the influence of wind-induced currents. In addition, the effect of only one point source of pollution was considered. The problems involving several sources of pollution, when there is an inflow of freshwater, are now in their definition stage (problems of Lake Balkash, Selenga River, etc.) and will be solved with methods similar to those described below.

Because of the absence of a deterministic theory on self-purification processes, the relatively short time series of observations, and a small number of the measured points, the most acceptable method of mathematical modeling is the principle of computerized self-organization (Ivakhnenko 1971, 1975). This principle implies that all the information on the value of coefficients of cause and effect linkages and on the model structure is already available in the field data, and deep insight into the mechanism of self-purification is not required.

The principle of self-organization maintains that with gradual sophistication of the mathematical model structure, the heuristic criteria of selecting the best models ("external supplements") reach their minimum, and further sophistication of the model is useless. Thus, a model of optimum complexity is selected. Most often, a so-called "regularity criterion" is used as the criterion of the selection. This criterion is the value of mean square error determined on a separate test series of points. Some other criteria are also applied.

Complete scanning of models is not always possible when models are nonlinear and when the number of input arguments is large. In these cases, it is more rational to use algorithms of gradual model sophistication which are called algorithms of the Group Method of Data Handling (GMDH). They are based on the principle of multi-series selection of models according to the chosen heuristic criterion. Two types of the GMDH algorithms will be used below: (1) with sequential separation of trends, and (2) a generalized algorithm with selection of projectors. More detailed information on the principle of self-organization, and GMDH algorithms based upon it with their application, can be found in a series of papers by Ivakhnenko (1971, 1972, 1973, 1974, 1975).

Synthesis of a Two-Dimensional Mathematical Model of Dissolved Solids--

Models are written in the form of a self-regressing function (Ivakhnenko et al., 1973).

$$U_{j,t+\Delta t} = f(U_{j,t-K\Delta t},\ldots,U_{j \pm i\Delta x,t}), \qquad (1)$$

$K = 1,\ldots,l;$
$i = 1,\ldots,m.$

It should be noted that the dynamic ratio is not attributed to any definite physical sense. What this implies is a self-evident concept that any value of the field is a function of the proceeding values in time and space. The number of arguments can be unlimited since the self-organization arguments have regularizing properties (Tikhonov, 1963). They are resistant to multicollinearity of variables and are consequently of poor determinancy.

When synthesizing the model of dissolved solids concentration, the expression $U_{j,t+1}$, normalized with respect to the average value of concentration at the j-th point at the next (t+1) moment of time, was adopted as the output value. The following variables "close" in space and time (Figure 1) are used as input arguments:

$U_{j,t}$, $U_{j,t-\Delta t}$, $U_{j+\Delta x,t}$, and $U_{j-\Delta x,t}$, with $\Delta t = 1$ year.

The initial data of the dissolved solids parameter V are shown in Table I.

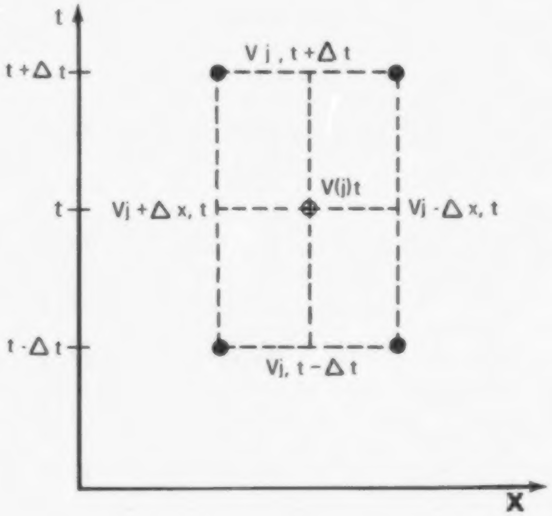

Figure 1: Diagram of the selection of points to synthesize dynamics equations of the (dissolved solids) mineralization conductors and suspended solids.

TABLE I. INITIAL DATA ON DISSOLVED SOLIDS

Point number	$V_{i,t-\Delta t}$	$V_{i+\Delta x,t}$	$V_{i,t}$	$V_{i-\Delta x,t}$	$V_{i,t+\Delta t}$
1	93.3	99.8	97.4	98.5	89.1
2	90.1	98.9	99.8	97.4	97.6
3	88.7	97.8	98.9	99.8	98.1
4	97.4	97.6	89.1	96.1	94.9
5	99.8	98.1	97.6	89.1	97.0
6	98.9	95.7	98.1	97.6	96.2
7	89.1	97.0	94.9	100.2	98.0
8	97.6	96.2	97.0	94.9	77.0
9	98.1	98.7	96.2	97.0	94.0
10	95.0	95.7	93.1	99.8	95.2
11	97.8	96.0	95.7	93.1	97.9
12	100.2	96.4	96.0	85.7	94.7
13	93.1	97.9	95.2	97.1	95.0
14	95.7	94.7	97.9	95.2	93.0
15	96.0	102.2	94.7	97.9	98.0
16	104.5	89.9	94.1	96.0	92.4
17	107.9	88.1	98.9	94.1	90.2
18	103.2	86.4	88.1	89.9	94.6
19	105.8	93.5	93.0	93.0	94.0
20	101.2	87.7	93.5	93.0	90.9
21	104.9	91.6	87.6	93.5	92.5

The synthesis resulted in models of optimum sophistication (as far as the regularity criterion is concerned) in the form of systems of nonlinear difference equations with retarded arguments (Ivakhnenko et al., 1973, 1974):

(a) according to the generalized GMDH algorithm with selection of projectors

$$U_{j,t+1} = 0.0083 - 0.266 \cdot U_{j,t-1} - 0.037 \cdot U_{j+1,t} - 0.0069 - 0.063 \cdot U_{j,t} + 6.395 \cdot U^2_{j,t} \quad (j = 1 \ldots n); \qquad (2)$$

(b) according to the GMDH algorithm with sequential separation of trends

$$U_{j,t+1} = 0.0082 - 0.282 \cdot U_{j,t-1} - 0.0069 - 0.0609 \cdot U_{j,t} + 6.439 \cdot U^2_{j,t} \quad (j = 1 \ldots n) \qquad (3)$$

where $U_{j,t}$ are the values of the variable $V^*_{j,t}$ normalized with respect to the average

$$U_{j,t} = \frac{V_{j,t} - V^*_{j,t}}{V^*_{j,t}};$$ $V^*_{j,t}$ are average values of the concentration in question;

$$V^*_{j,t} = \frac{1}{N} \sum_{i=1}^{N} V_{j,t}(i);$$

$V_{j,t}$ is the concentration at t-moment of time at the j-th point of the field.

The residual variances obtained for the normalized data on the test set are equal to $\delta_1^2 = 0.03540$; $\delta_2^2 = 0.03505$, respectively, which characterizes the accuracy of the models obtained.

Synthesis of Mathematical Model of a Two-Dimensional Field of Suspended Solids--

Equations for suspended solids (Ivakhnenko et al., 1974) have been derived in a similar way (Table II).

$$\begin{aligned} U_{j,t+1} = {} & -0.022 - 0.053 \cdot U_{j,t-1} - 0.0215 \cdot U_{j-1,t} + 0.272 + 0.155 \cdot U_{j-1,t} \\ & - 0.224 \cdot U^2_{j-1,t} + 0.155 - 0.791 \cdot U_{j+1,t} - 0.799 \cdot U^2_{j+1,t} \\ & + 0.52 \cdot U^3_{j+1,t} \quad (j = 1, \ldots, n) \qquad (4) \end{aligned}$$

This model is more sophisticated than the model of dissolved solids, since it contains more nonlinear terms.

Synthesis of the Mathematical Model of a Three-Dimensional Field of Chemical Oxygen Demand (COD)--

The COD model is synthesized for the case of a three-dimensional field; where, as input variables, besides "close" values of this parameter, cartesian coordinates of the measured points (x,y,z) EBEE(3) have also been selected.

It has enabled more detailed investigation of dynamic properties of the mixing zone in different areas; further, it has been possible to obtain

TABLE II. INITIAL DATA ON SUSPENDED SOLIDS

Point number	$V_{i,t-\Delta t}$	$V_{i+\Delta x,t}$	$V_{i,t}$	$V_{i-\Delta x,t}$	$V_{i,t+\Delta t}$
1	4.3	0.0	1.0	0.0	0.0
2	6.8	0.0	0.0	1.0	1.0
3	1.4	0.6	0.0	0.0	0.8
4	1.2	6.8	4.3	2.0	1.0
5	0.0	1.4	6.8	4.3	0.0
6	0.0	5.2	1.4	6.8	0.0
7	1.3	2.12	2.02	4.82	0.8
8	3.3	3.23	2.42	2.02	1.5
9	1.4	2.0	3.23	2.42	1.7
10	1.0	16.4	4.0	4.8	0.5
11	0.6	6.9	16.4	4.0	1.3
12	0.7	2.0	6.9	16.4	1.6
13	0.4	4.0	4.0	1.2	0.3
14	0.0	2.8	4.0	4.0	1.2
15	0.4	2.0	2.8	4.0	1.5
16	0.0	4.0	2.8	4.8	0.3
17	0.0	2.61	4.0	2.8	1.0
18	0.1	2.0	2.61	4.0	1.4
19	0.0	4.4	2.0	2.8	0.3
20	0.6	2.0	4.4	2.0	0.9
21	0.3	2.0	2.0	4.4	1.2
22	0.2	3.6	2.4	4.4	2.0
23	0.7	2.4	3.6	2.4	5.2
24	0.2	2.2	2.4	3.6	3.1

quantitative characteristics of the process under investigation in terms of short- and long-term forecasting.

One hundred observations have been used as input information: 50 points were treated as a training set, the remainder as a test set.

The optimum equation (according to "global" minimum of the regularity criterion or the mean sequare error on the test set) was obtained after the third selection level of the main network with subsequent projection of the remainder according to an algorithm with sequential trend separation. The equation is as follows:

$$U_{i,j,k,t+1} = y_3^j + f_1\ (U_{i,j+1,k,t}) + f_2\ (U_{i-1,j,k,t}), \tag{5}$$

where $y_3^j = 0.0013 + 0.29 \cdot Z_3 + 0.80 \cdot Z_{10}$ is the equation of the third selection level.

The best variables of the second selection level are as follows:

$$Z_3 = 0.002 + 0.402 \cdot y_2 + 0.776 \cdot y_8;$$
$$Z_{10} = 0.006 + 0.822 \cdot y_2 + 0.626 \cdot y_7$$

The best variables of the first selection level are as follows:

$$y_2 = -0.01 + 0.0358 \cdot U_{i,j+1,k,t} - 0.059 \cdot U_{i,j-1,k,t};$$
$$y7 = -0.015 + 0.0272 \cdot U_{i,j+1,k,t} + 0.070 \cdot x;$$
$$y_8 = -0.016 + 0.062 \cdot Z + 0.056 \cdot x$$

During sequential trend separation from the remainder $y - y_3^j$, the following equations have been obtained:

$$f_1(U_{i,j+1,k,t}) = 0.010 + 0.071 \cdot U_{i,j+1,k,t} - 0.084 \cdot U^2_{i,j+1,k,t};$$

$$f_2(U_{i-1,j,k,t}) = -0.016 - 0.25 \cdot U_{i-1,j,k,t} + 0.458 \cdot U^2_{i-1,j,k,t}$$

$$+ 1.34 \cdot U^3_{i-1,j,k,t}$$

Here

$$X = [X_o + (i-1)\ \Delta X - 4.32]\ /4.32;$$
$$Z = [Z_o + (k-1)\ \Delta Z - 3.12]\ /3.12;$$

where X_o, Z_o are initial values of the region BEE(3) $\Delta X, \Delta Z$ are the step length along the spatial coordinates X and Z.

It should be noted, that unlike the preceding models (2), (3), and (4), the function of the pollution source capacity here is not constant but depends on spatial coordinates.

Study of Stability and Other Dynamic Properties of the Processes Concerning the Change in Hydrochemical Parameters on the Basis of the Models Obtained--

Natural waters can be considered as multicomponent equilibrium systems (Vel'ner et al., 1968). For each water body, the equilibrium is determined by the combination of physico-geographical, hydrological, and meteorological conditions and is established over a long period of time. When pollutants enter the water body, the natural equilibrium is shifted and the water body "mobilizes its forces" to combat the distortion of the natural conditions and strives to return the whole system to its stable state.

If the amount of pollutants exceeds "the self-purifying" capacity of a water body, it results in an irreversible distortion of the biological equilibrium and, as a consequence, the degradation of water quality properties (Messineva, 1973).

The effect on water quality can be described by a nonlinear operator A, which enables us to transform the equations (2), (3) or (5) as follows:

$$U(t+1) = A[U(t),\ldots,U(t-1)], \tag{6}$$

where $U(t)$ is a vector characterizing the state of the field at t moment of time.

To understand the character of pollutant transformation, it is necessary to study the dynamic properties of the operator equation (6). First, it is necessary to determine the set of equilibrium points $U_i^* \mathrm{E} U$ where $U_i^* = A[U_i^*,\ldots,U_i^*]$ and to understand the character of each of them. Then it is necessary to determine the stability regions Q_i with respect to the initial conditions

$$U(t)\big|_{t=1} = U_o; \quad U_{(t-1)}\big|_{t=21} = U_1$$

Of some interest also is the dependence of the stability region critical parameters on the change of the A operator structure, e.g., due to change of "the pollution-source capacity function."

Research on the stability of the operator equation (6) in the vicinity of a stationary point U_i carried out by using Lyapunov functions for the linear systems, $U_{(t+1)} = AU_{(t)} + BU_{(t-1)}$ were selected (Ivakhnenko, 1974).

$$Z(Z^k) = \max \{||U_k||, ||U_{k-1}||\} \tag{7}$$

Thus, the research on stability is reduced to verification of the condition

$$\max \{||U_{(n)}||, ||U_{(n-1)}||\} \ y{>}\max \{||U_{(n+2)}||, ||U_{(n+1)}||\}, \tag{8}$$

which is valid only in case $||A|| + ||B|| < 1$

Investigation of the Dynamics of Dissolved Solids Field Model--

If boundary conditions, for example, in the form $U_o = U_i$; $U_n = U_{n+1}$, are united with the systems (2) or (3), the following matrix equation is obtained:

$$U_{n+1} = AU_n + BU_{n-1} + CU_n^2 + b \tag{9}$$

where

$$A = \begin{vmatrix} 0 & z & q & 00 & \dots & 000 \\ 0 & z & q & 00 & \dots & 000 \\ 0 & 0 & z & q0 & \dots & 000 \\ \cdot & \cdot & \cdot & \cdot\cdot & \cdots & \cdots \\ 0 & 0 & 0 & 00 & \dots & zq0 \\ 0 & 0 & 0 & 00 & \dots & 0zq \\ 0 & 0 & 0 & 00 & \dots & 0zq \end{vmatrix} \qquad B = \begin{vmatrix} p & 0 & 0 & \dots & 000 \\ p & 0 & 0 & \dots & 000 \\ 0 & p & 0 & \dots & 000 \\ \cdot & \cdot & \cdot & \cdots & \cdots \\ 0 & 0 & 0 & \dots & p00 \\ 0 & 0 & 0 & \dots & 0p0 \\ 0 & 0 & 0 & \dots & 0p0 \end{vmatrix}$$

$$C = \begin{vmatrix} c & 0 & 0 & .. & \dots & 000 \\ c & 0 & 0 & .. & \dots & 000 \\ 0 & c & 0 & .. & \dots & 000 \\ \cdot & \cdot & \cdot & \cdot\cdot & \cdots & \cdots \\ 0 & 0 & 0 & .. & \dots & c00 \\ 0 & 0 & 0 & .. & \dots & 0c0 \\ 0 & 0 & 0 & .. & \dots & 0c0 \end{vmatrix} \qquad b = \begin{vmatrix} b \\ b \\ . \\ . \\ . \\ . \\ b \end{vmatrix}$$

Here $z = -0.063$; $q = -0.037$; $p = -0.266$; $c = 6.395$, $b = 0.0014$.

Application of the first Lyapunov method has shown that if $2 \max \{||A + 2CU_F||, ||B||\} < 1$ where the matrix norm is defined as $||X|| = \max\limits_j \sum\limits_i |X_{i,j}|$, then the system (9) is stable in a small increment area, i.e. in a linear approximation.

Determination of the ultimate stationary level has shown that all the trajectories of the free motions used by initial deviations, at $t \to \infty$ approach the value

$$Q_j = V_j + V_{j-F} = 94.75 \text{ mg}/\ell$$

The free term in the right part of the equation (9) corresponds to the capacity of a pollution source. Analysis of the equation (9) solution with pollution source capacity $b = \pm 0.0014$ has shown that the process trajectories converge to a stationary level $V_{j,\infty} = 94.75$ mg/ℓ. With initial deviation $V \leq 114$ mg/ℓ, the processes have a damping character, that is, they are stable (Figure 2), with a larger value of the initial deviation, they are unstable. With further change of the pollution source capacity b, the stationary level gradually increases or drifts up to the value $V_{j,\infty} \leq 103$ mg/ℓ (Figure 3-2',2") while the upper limit of the process (of the admissible pollution threshold) is gradually lowered to the same level (Figure 3-1',1" curves).

Coincidence of the dynamic characteristics of the equations (2) and (3) (Figure 3), obtained with the help of different GMDH algorithms, is direct evidence of the validity of the dissolved solids model.

The stability region Q of the system of equations (2) or of its matrix analysis (9) was determined in two-dimensional space $(\Delta V^{(1)}, \Delta V^{(2)})$, i.e., with respect to "disturbances" of the initial point (Figure 4a)

$$V_j(t) = 94 + \Delta V_j^{(1)}(t);$$
$$V_j(t-1) = 94 + \Delta V_j^{(2)}(t) \quad (j = 1,\dots,n).$$

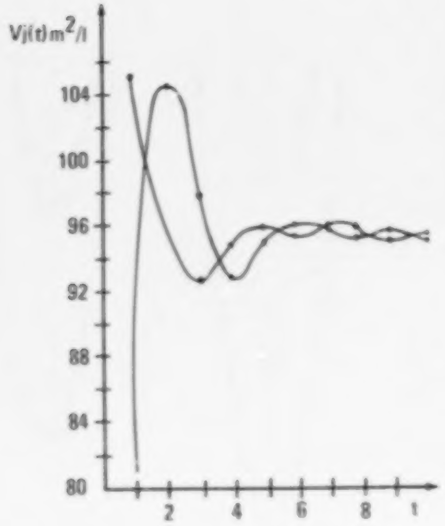

Figure 2. The process of change in dissolved solids' concentration given different initial deviations.

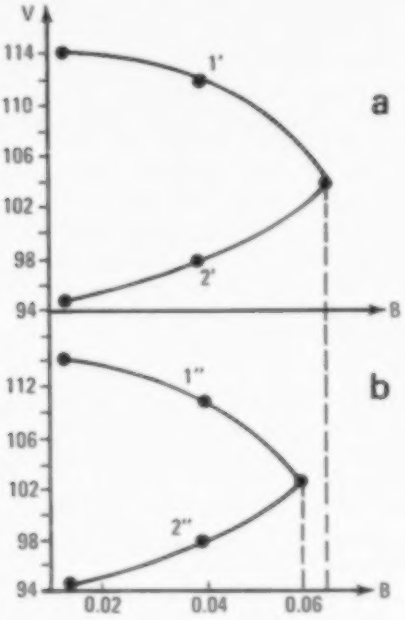

Figure 3. Dependence of the upper limit (1',1") and stationary level of the dissolved solids process (2',2") on the pollution source capacity a) 1',2' according to the generalized GMDH algorithm; b) 1",2" according to the GMDH algorithm with sequential separation of trends.

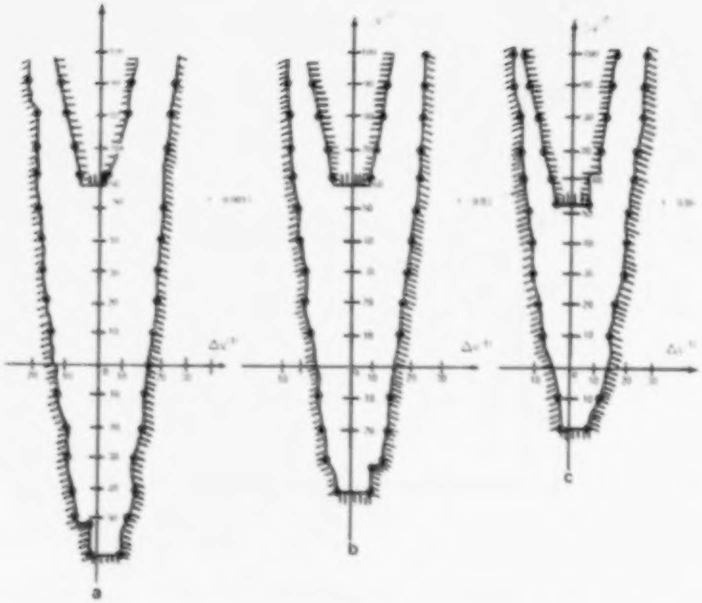

Figure 4. Change in the stability region of the dissolved solids parameter with increase in the pollution source capacity.

The investigations of the stability region limits in the coordinates $\Delta V^{(1)}$ and $\Delta V^{(2)}$ with changing pollution source capacity are of interest. As the value of b increases, the stability region decreases: at values $b \geq 0.07$, it was already impossible to determine (Figure 4). As is clear from the analysis of equation 9, if one of the coordinates exceeds the threshold value, then a process of continuous concentration begins to increase. This process results in the loss of the self-purifying property in a particular locality of the lake. The permissible level of pollution should be somewhat lower than the threshold value, since a locally developed point of eutrophication can gradually expand and cover a much larger zone.

Though it is not possible to directly identify the terms responsible in the equation (9) for self-purification or the terms that reflect multiannual fluctuations of dissolved solids concentration in Lake Baikal, the research shows that the self-purification mechanism does work at low concentrations for dissolved solids.

Study of the Dynamics of Models of Suspended Solids and Chemical Oxygen Demand (COD) Fields--

Numerical experimentation similar to that described above has been carried out for the system of equations (4), which characterize the field of suspended solids. In this case, the stable equilibrium point is equal to 1.5 mg/ℓ. Figure 5 shows how the suspended solids concentration changes in time at a certain initial deviation.

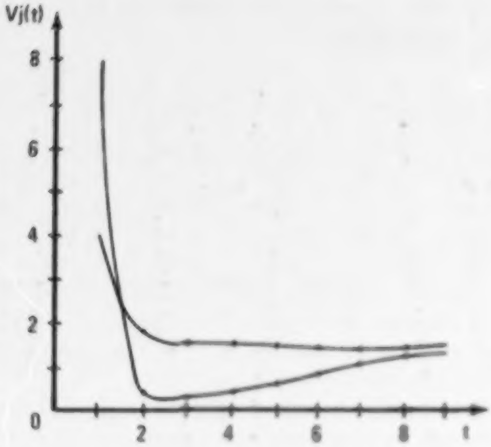

Figure 5. Process of change in the suspended solids concentration.

As far as COD is concerned, the upper limit of the stability region is equal to 7.5 mg/ℓ. The ultimate concentrations of the field in a vertical section (Table III) have been used as a basis for a long-term forecast of CODs. Using the equation 5, my experimental calculations showed that, within the interpolation zone, the process of the COD change is stable.

Conclusion

This paper reviews a method of synthesizing equations that describe dynamics of the main hydrochemical parameter fields. The equations are synthesized on the basis of the self-organization principle for the case of only one pollution source. The first simple dynamic models of the dissolved solids, suspended solids, and COD fields have been obtained. The models are based on the regularity criterion. Dynamic properties of the models have been investigated; a long-term forecast concerning COD was obtained within the interpolation zone of the field.

The regularity criterion, on which the selection of the above mentioned models is based, does not permit distant extrapolation of the mixing zone. To do this, other selection criteria or other regularization methods are required. Further refinement of the mathematical models of mixing zones will be connected with the application of the unbiased criterion, as only unbiased equations can be assumed to be a law describing the zone.

Forecasting the Yield of Melosira Biomass in Lake Baikal by Self-Organization Algorithms*--

Methods of self-organization of forecasting equations ensure the most exact and objective forecast, since the model uses only actual values of the

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TABLE III. CHANGE IN THE STABILIZED LEVEL OF COD AT DEPTH (Z) AND AT (X) DISTANCE FROM SHORE

Z	X									
1	3.68	3.74	3.77	3.82	3.86	3.90	3.94	3.98	4.03	4.07
2	3.70	3.75	3.79	3.83	3.87	3.91	3.96	4.00	4.04	4.08
3	3.71	3.77	3.80	3.85	3.89	3.93	3.97	4.01	4.05	4.10
4	3.73	3.78	3.82	3.86	3.90	3.94	3.99	4.03	4.07	4.11
5	3.74	3.79	3.83	3.87	3.92	3.96	4.00	4.04	4.08	4.12

variables characterizing the process. It is not necessary a priori to set up the equations of the system elements as in the dynamic modeling method.

The following self-organization algorithms will be compared in a specific example of a problem concerning medium-term forecast of Melosira biomass yield in Lake Baikal:

1. forecast according to the harmonic trend (time function) of optimum complexity;
2. multiple (that is, repeating year after year) systematic differential forecast according to "differential equations of regression" synthesized for the "residual," that is, for deviation of the parameter from the harmonic trend mentioned above.

The validity of the forecast is tested on two data sets: the first and second (verification) sets.

The first test set is selected according to the "seventh" (harmonic trend) and to the "third" (differential equations of regression) method of regularization. For the second verification set, data gathered during 1946-1949 are used.

As will be shown below, with the prediction interval $T_y=5$ years, if the "residual" (a multivariable function) is calculated, the precision of the forecast would increase by approximately 50%, as compared with the forecast according to the trend (a time function).

To verify the validity of our forecasting model, we shall use a forecast with time counting "in reverse" (Ivakhnenko, 1971).

Melosira and Cyclotella play a leading role in the biomass of the primary producers of the open water in Lake Baikal. But the plankton biomass succession is not the same in different years. High-yield years are alternated with two or three low-yield ones (Kozhov 1972). This phenomenon can be attributed to a number of factors: temperature conditions, illumination, availability of biogenic elements, and other factors considered as governing abiotic factors. Nevertheless, no singular explanation has been found for the sharp fluctuations of Melosira yield in different years. The first suggestion that the Melosira yield may be governed by environmental factors was first suggested by Kozhov (1965). The idea was further developed by Shimarayev (1971) in a qualitative analysis of the relationship between the changes of the Baikal plankton yield and the hydrometeorological characteristics of the lake.

The paper maintains that the Melosira yield is a random process consisting of two components: a low-frequency and a high-frequency.

The low-frequency component should be treated as a function of certain arguments (their number being small; one or two), and the high-frequency component as a function of other arguments (their number being much larger).

The optimum separation of the process into components with different arguments enables us to define self-organization algorithms, such as the algorithms of the Group Method of Data Handling (GMDH) (Ivakhnenko 1971).

Solving the Problem of the Medium-Range Forecast of the Melosira Biomass-- This paper solves the problem of medium-range forecast of the Melosira average annual biomass (F) by self-organization algorithms. The Melosira biomass is assumed to be a random process that can be represented as a sum of two components: the trend, a time function ($F_T(t)$), and the remainder--a function of the variables of the system $V_1, V_2, \ldots, V_m$

$$F(t) = F_T(t) + f(V_1, V_2, \ldots, V_m).$$

Thus, the low-frequency component of the process will represent a time function--the trend; a high-frequency component will represent a function of the variables of the system.

The selection of the high-frequency components arguments is governed both by the existing general concepts and the available observation data.

The arguments have been composed of the following parameters (data from 1950-1971):

V_1 - average annual precipitation on the lake surface;
V_2 - average yield of Cyclotella minuta biomass during 2nd-4th months;
V_3 - average yield of Cymnodinium biomass during 2nd-4th months;
V_4 - average annual tributary inflow into Lake Baikal;
V_5 - monthly average temperature in April at the depth 0 m;
V_6 - monthly average temperature in April at the depth 50 m;
V_7 - yearly average evaporation from the lake surface;
V_8 - average temperature in the 8th - 9th months at the depth 50 m;
V_9 - amount of heat at the end of the year (determined according to the technique described by Shimarayev (1971);
V_{10} - average annual water discharge from the lake.

The initial data were taken from observations made by the Baikal biological station of the Biological Geographical Institute affiliated with Irkutsk State University (variables F, V_2, V_3, V_5, V_6, V_8), and by the Limnological Institute of the Siberian branch of the USSR Academy of Science (variables V_1, V_4, V_6, V_7, V_9, V_{10}).

The solution of the problem of the Melosira average annual yield medium-range forecast is carried out in the following two stages:

1. separating the process low-frequency component (i.e., the harmonic trend of optimum complexity according to GMDH) and forecasting this component;
2. synthesizing "differential equations of regression" (Ivakhnenko et al., 1973) for "the remainder" (i.e., the deviations of the variable from its

harmonic trend) and forecasting this remainder.

Forecast on the Basis of the Harmonic Trend of Optimum Complexity (in time function)--

In the problems of forecasting random processes that contain latent periodic components, harmonic trends with non-multiple quantization steps, should be separated (Kimura 1913; Serebryannikov and Pervozvanskiy, 1965).

Self-organization algorithms enable us to optimize the problem concerning the selection of the process harmonic components (Koppa and Todua, 1972).

In this particular case the random process is defined by the discrete time series $F_j = f(j\Delta t)$, where Δt is a quantization interval (equal to one year); $j = 1,2,3,\ldots n$ (n is the number of interpolation points). The value F_j in this expression will be approximated by the harmonic series

$$F_T(t) = \bar{F} + \sum_{i=1}^{k} f_i(t),$$

where $f_i(t) = a_{0i} + a_{1i}\sin(\frac{2\pi}{T_i}t) + a_{2i}\cos(\frac{2\pi}{T_i}t)$;

$\bar{F}$ is the random mean $\quad (\bar{F} = \frac{1}{n}\sum_{j=1}^{n} F_j)$;

i is the number of harmonic components of the approximating expression; T is the period; t is time.

The harmonic trends have been separated according to the program (developed by Koppa and Todua, 1972) in the following order. The time series of the data obtained during 1950-1971 (Figure 6) have been split into two successions: a training one and a test one. The points, following the method described by Ivakhnenko et al. (1973) have been separated according to the "seventh" regularization method, i.e., the test succession was comprised of the apexes of the first harmonic of random process (1950, 1955, 1961, 1971). These points formed a training succession.

The criterion indicating cessation of the selection levels of build-up, i.e., the criterion of the complexity of the approximating expression $\hat{F}_m(t)$ is the magnitude of the relative mean square error $\Delta(1)$ on the first test succession

$$\Delta_T(1) = \frac{\sum_{i=1}^{p} (F_i - \hat{F}_i)^2}{\sum_{i=1}^{p} F_i^2},$$

where p is the number of points of the first test succession; F_i is the actual value of the parameter of the test succession points;
$\hat{F}_i$ represents the evaluations of these points.

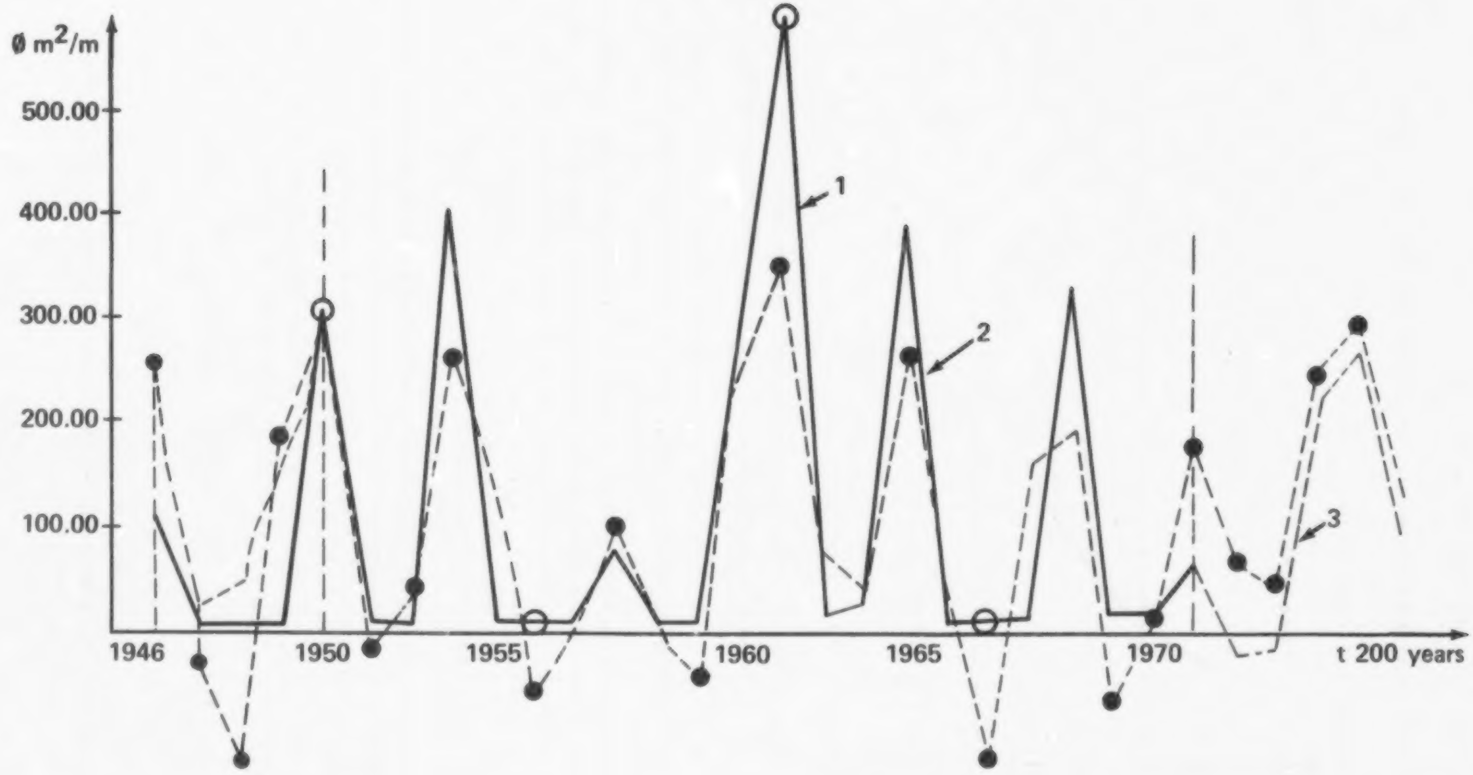

Figure 6. Change in average annual Melosira yield (training and first test succession; forecast T_y=5 years: 1 - actual values; 2 - harmonic trend and forecast on the basis of the trend ($\Delta(1)$ = 0.194); 3 - forecast on the basis of trend and remainder ($\Delta_0^1(1)$ = 0.039).

If the error on the S level of selection is bigger than on the S-1 level, i.e. $\Delta(1)_S > \Delta(1)_{S-1}$ or if the error $\Delta(1)$ decreases negligibly with the increase in the selection levels (the value of the difference $\Delta(1)_S - \Delta(1)_{S-1}$ is an indication of this), then it is necessary to stop at the (S-1)th selection level (Figure 7).

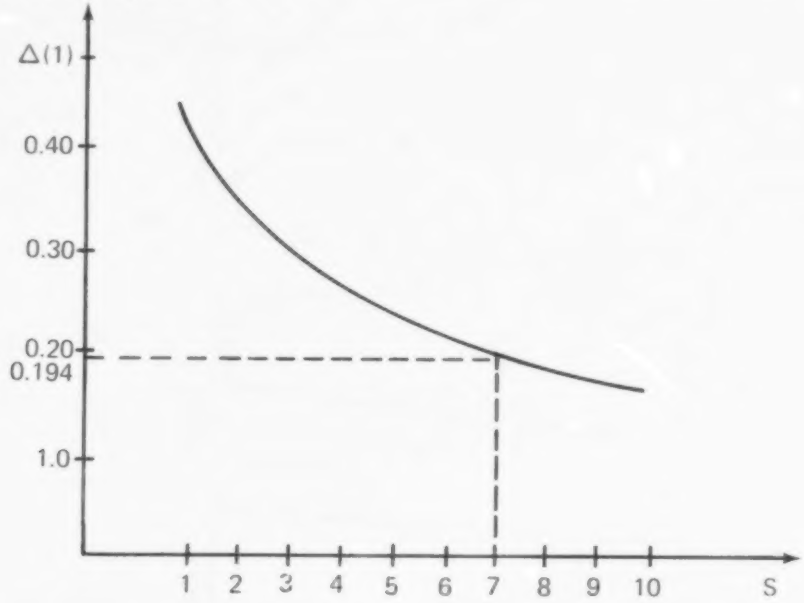

Figure 7. Dependence of the relative mean square error $\Delta(1)$ (on the first test succession) on the number of selection levels S.

After the seventh level, the error $\Delta_T(1) = 0.194$ and its further decrease is insignificant. Thus, it is evidently expedient to stop at the seventh selection level. The harmonic trend of optimum complexity $F_T(t)$ is separated in the figure.

Multiple Systemic Forecast by Differential Equations of Regression--
Differential equations of regression are a specific variety of difference equations that at $\Delta t \to dt^o$ are reduced to common differential equations with continuous argument t in Cauchy's normal form.

The differential equations of regression are synthesized for "the remainder" (Ivakhnenko et al., 1973) i.e., for the deviation of the output parameter of the system from its harmonic trend $F^o(t) = F(t) - F_T(t)$.

In its general form, the differential equation of regression is written in the following way:

$$\frac{\Delta F^o}{\Delta t} = f[F^o_{(0)}, F^o_{(-1)}, \ldots, F^o_{(-1)}, V_{1(0)}, V_{1(-1)}, \ldots, V_{10(-2)}, V_{10(-3)}],$$

where parenthesized indices determine the value of the retardation of the arguments.

In agreement with the algorithm of the multiple differential forecast (Ivanhnenko et al., 1973) as a partial description in the case of medium-range forecast, we recommend selection of "partial" descriptions of the type

$y = a_o + a_1X_i + a_2X_j + a_3X_iX_j$, where $i,j = \overline{1,m}$.

The points were split into a training succession (R=14 points) and a test succession (R1=4 points) using the "third method of regularization" (Ivakhnenko, 1971).

The optimum complexity of the regression differential equation of the remainder is selected on the basis of the mean square error $\Delta^2(1)$ on the first test succession:

$$\Delta^2(1) = \frac{\sum_{i=1}^{P} (\Delta F^o_i - \Delta \hat{F}^o_i)^2}{\sum_{i=1}^{P} (\Delta F^o_i)^2},$$

where p is the number of points in the first test succession,

ΔF^o_i is the actual value of the first difference of the remainder,

$\Delta \hat{F}^o_i$ is the determined value of the first difference of the remainder.

The criterion for the optimum complexity of the equation 6 is the minimum of the error $\Delta^2(1)$, and in our case it is achieved on the third level of selection at $\Delta^2(1) = 0.039$ (Figure 8).

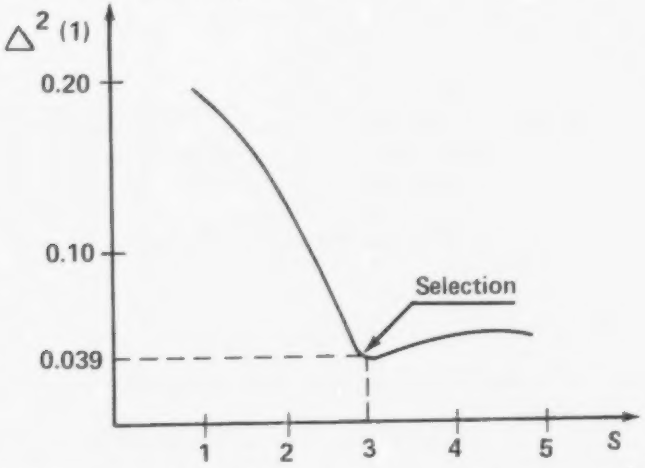

Figure 8. Dependence of the relative mean square error $\Delta^2(1)$ on the first test succession on the number of levels S.

The following equation of the remainder has been obtained by using the program described by Todua and Gulyan (1973).

$$\frac{\Delta F^o}{\Delta t} = 0.47892 + 0.63344\, Y_1 + 0.82953\, Y_2 + 0.02152\, Y_1Y_2, \qquad (1)$$

where $Y_1 = 0.64411 + 0.61788\, X_1 + 0.75660\, X_2 - 0.01058\, X_1X_2$,

$Y_2 = 0.04844 + 0.43567\, X_3 + 0.89378\, X_4 + 0.02866\, X_3X_4$, where $X_1 = 1.47473 + 19.42608\, V_{7(-2)} - 1.07258\, V_{2(-2)} - 25{,}48776\, V_{7(-2)} V_{2(-2)}$,

$X_2 = -0.615839 + 18.09545\, V_{7(-2)} - 0.14528\, V_{3(-1)}$

$- 19.67778\, V_{7(-2)} V_{3(-1)}$,

$X_3 = 1.23069 + 29.11694\, V_{7(-2)} + 0.36028\, V_{3(-2)} + 12.94899\, V_{7(-2)} V_{3(-2)}$,

$X_4 = 0.52410 + 33.66962\, V_{8(-3)} + 17.16956\, V_{8(-2)} + 69.51791\, V_{8(-3)} V_{8(-2)}$

The differential equation of regression (2.1) is written in normalized deviations from the average, that is:

$$V_{i(-j)} = \frac{V_{(a)i(-j)} - V_i}{\overline{V}_i}; \qquad \overline{V}_i = \frac{1}{R + R1} \sum_{i=1}^{R+R1} V_i$$

where i is the variable number; j is the time shift; a is the absolute value of the parameter; R and R1 are the numbers of points in the training and test successions.

It is possible to pass from the trend to absolute values of the deviation in the following way:

$$F^o_{t+1} = \overline{F}^o \left(F^o_{(H)t+1} + 1\right), \qquad F^o_{(H)t+1} = F^o_{(H)t} \pm \Delta F^o,$$

ΔF^o is that formation of the difference increment;
$\overline{F}^o$ is the arithmetic mean of the remainder.

The meaning of the indices is as follows:
(H) is the normalized value of the parameter; t,(t+1) designates the sequence of the parameter values. Calculations based on such complicated equations are possible, only by use of a computer.

In its general form, the structure of the forecast equation (trend + remainder) can be written as follows:

$$\hat{F}(t) = \hat{F}_T(t) + f(V_{2(-2)}, V_{3(-1)}, V_{3(-2)}, V_{7(-2)}, V_{8(-2)}, V_{8(-3)})$$

The algorithm of self-organization has automatically selected the following arguments (except for time):

$V_{2(-2)}$ = average yield of Cyclotella minuta biomass during the 2nd to 4th months two years ago;

$V_{3(-1)}$ = average yield of Gymnodinium biomass during the 2nd to 4th months a year ago;

$V_{3(-2)}$ = same two years ago;

$V_{7(-2)}$ = evaporation from the lake surface two years ago;

$V_{8(-2)}$ = average temperature during the 8th to 9th months at a depth of 50 m two years ago;

$V_{8(-3)}$ = same three years ago.

The first three arguments $V_{2(-2)}$, $V_{3(-1)}$, $V_{3(-2)}$ characterize the role of phytoplankton in the yield of Melosira. The arguments $V_{7(-2)}$, $V_{8(-2)}$, $V_{8(-3)}$

are temperature conditions. Change in the Melosira biomass may depend on temperature conditions, connected in turn with the vertical water exchange (especially at greater depths $H > 50$ m); it is important to note that their influence is manifested every two or three years (which is in agreement with the actual frequency of Melosira biomass yield).

Thus, it follows from the forecast that, as far as Melosira is concerned, 1972 and 1973 should be low-yield years, followed by twin high-yield 1974 and 1975 and low-yield year 1976. Observations in 1972 supported the forecast.

Forecast with Time Counting "In Reverse" to Check the Algorithm Validity-- The accuracy of the forecast was checked on the second test (verification) succession which was composed of the data collected during 1946-1949 (Figure 6). The validity of the forecast algorithms was checked by means of time counting "in reverse" (Ivakhnenko 1975).

The accuracy of the forecast by the harmonic trend with time counting "in reverse" is characterized by the relative mean square:

$$\Delta_T^{(2)} = \frac{\sum_{t=1946}^{1949} (F_t - \overline{F}_t^*)^2}{\sum_{t=1946}^{1949} F_t^2} = 5.5,$$

where F_t is the actual value of the parameter;
$\overline{F}_t^*$ is the evaluation according to the trend.

The equation of the remainder forecast with time counting in reverse is of the following form:

$$\frac{\Delta F^{o*}}{\Delta t} = -0.49860 + 0.39861\ Y_1 + 0.88788\ Y_2 + 0.03276\ Y_1 Y_2$$

where $Y_1 = 0.28378 + 0.22782\ X_1 + 1.06805\ X_2 - 0.15408\ X_1 X_2,$
$Y_2 = -0.84958 + 0.45462\ X_3 + 0.57152\ X_4 + 0.01856\ X_3 X_4$

where $X_1 = 0.07592 - 1.58986\ F^{o*}_{(0)} - 4.04214\ V^*_{1(-2)} + 0.02588\ F^{o*}_{(0)} V^*_{1(-2)}$

$X_2 = 0.02237 - 1.34101\ V^*_{2(0)} + 1.36748\ V^*_{2(-3)} + 1.30351\ V^*_{2(0)} V^*_{2(-3)},$

$X_3 = -0.98033 - 18.408\ V^*_{1(-2)} - 0.28931\ V^*_{8(-1)} + 200.93788\ V^*_{1(-2)} V^*_{8(-1)}$

$X_4 = -0.04640 - 1.68006\ F^{o*}_{(0)} - 9.47947\ V^*_{8(-3)} - 0.32228\ F^{o*}_{(0)} V^*_{8(-3)}$

The superscript * means that the arguments are taken with time counting in reverse.

The accuracy of the forecast on the basis of the trend, taking the remainder into account (i.e. the trend + the remainder), is characterized by the mean square error:

$$\Delta(2) = \frac{\sum\limits_{t=1946}^{1949} (F_t - \hat{F}_t)^2}{\sum\limits_{t=1946}^{1949} F_t^2} = 2.73$$

where F_t is the actual value of the parameter; $\hat{F}_t$ are the estimates on the basis of the trend, with a calculation of the remainder.

Thus, the mean square error on the second test succession has been reduced approximately by 50% if the remainder is taken into account. According to the forecast, low-yield years were 1947 and 1948 and high yields were 1946 and 1949, i.e., there was discrepancy with the actual data only for 1947.

Hence, it can be concluded that the forecast algorithm, described above, is characterized by the trend separation with forecasting the remainder as a function of the system parameters and yield results that are accurate enough for practical requirements.

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APPENDIX I

CORRELATION LINKS OF PHYTOPLANKTON AND ZOOPLANKTON IN LAKE BAIKAL*

The following problem is discussed below. Determine the possible links between the number of zooplankton and phytoplankton on the basis of the measurements carried out at the permanent monitoring station in Bolshiye Koty (Southern Baikal) in 1964-1966.

The number of zooplankton depends upon the number of phytoplankton currently available, but it also depends upon the number of phytoplankton which was available in the past, i.e., there is a certain delay in the algae-zooplankton system. So, it is necessary to find out not only what types of algae have the greatest effect on separate species of zooplankton, but also the "lag" values corresponding to them. These questions can be answered with the help of correlation functions.

$$K_{X_i,X_j(\tau)} = \frac{\sum_{h=1}^{N-\tau} (X_{i,(h+\tau)} - \overline{X}_i)(X_{j,h} - \overline{X}_j)}{\sqrt{\sum_{h=1}^{N-\tau} (X_{i,(h+\tau)} - \overline{X}_i)^2 \sum_{h=1}^{N-\tau} (X_{j,h} - \overline{X}_j)^2}}$$

where $X_{i,(h+\tau)}$ is the time succession of initial data of the i-th species of zooplankton;

$X_{j,h}$ is the time succession of initial data of the j-th species of phytoplankton;

τ is delay time (lag time);

N is the number of measurement intervals.

After calculating the correlation functions, i,e., the values of the correlation coefficient at different values of delay, we can select those values of delay, at which the absolute value of the correlation coefficient is greater than a certain preset value Θ (usually $\Theta \geq 0.6$).

The following species of zooplankton and algae have been selected to construct the correlation functions:

X_1 - Epischura baicalensis Sars
X_2 - Epischura baicalensis nauplii
X_3 - Cyclops kolensis Lill
X_4 - Cyclops kolensis Lill nauplii

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*Authors of the study are Ivakhnenko, A.G., Yu,V. Koppa, and T.V. Makukhiva.

X_5 - Bosmina longirostris (O.F. Mull)
X_6 - Keratella quadrata Mull
X_7 - Kellicottia longispina Kell
X_8 - Filinia terminalis Plate
X_9 - N. grandis Vor
X_{10} - Synchaeta pachypoda Jaschn
X_{11} - Melosira baicalensis
X_{12} - M. islandica
X_{13} - spores of M. islandica
X_{14} - Synedra acus.
X_{15} - Synedra ulna.
X_{16} - Stephanodiscus
X_{17} - Cyclotella baicalensis
X_{18} - Cyclotella minuta
X_{19} - Gymnodinium
X_{20} - Anhistrodesmus

For this example $i = 1,2...,10$; $j = 11,12,...20$. The number of measurement intervals in the available data $N = 180$ (measurementr were carried out every 10 days). When constructing the obtained correlation functions, it was assumed that $\tau = 0,1,2...,73$. Examples of the obtained correlation functions are shown in Figures 9 and 10.

The results of the computations for all the species of zooplankton and phytoplankton are given in Table IV. The positive correlation coefficient shows that zooplankton biomass increases with the increasing number of species; a negative coefficient means that the zooplankton biomass decreases with algae increase. Table IV enables us to draw the following conclusion: that the Epishura baicalensis Sars (X_1) biomass is most severely affected by the quantity of the following species:
X_{17} - 380 days ago;
X_{18} - 310 days ago;
X_{19} - 390 and 350 days ago.
Similar conclusions can also be drawn for other variables.

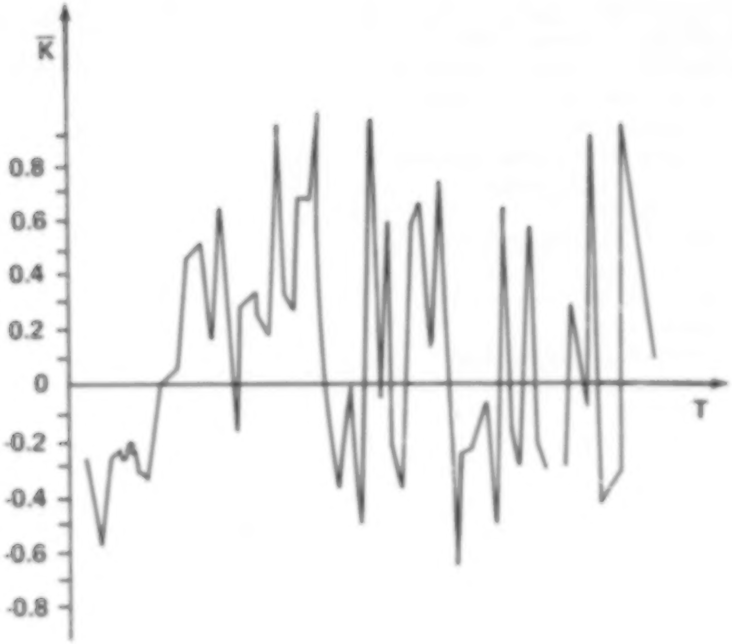

Figure 9: Correlation coefficient of Bosmina longirastris (O.F. Mull)(X_5) with Stephanadiscus (X_{16}).

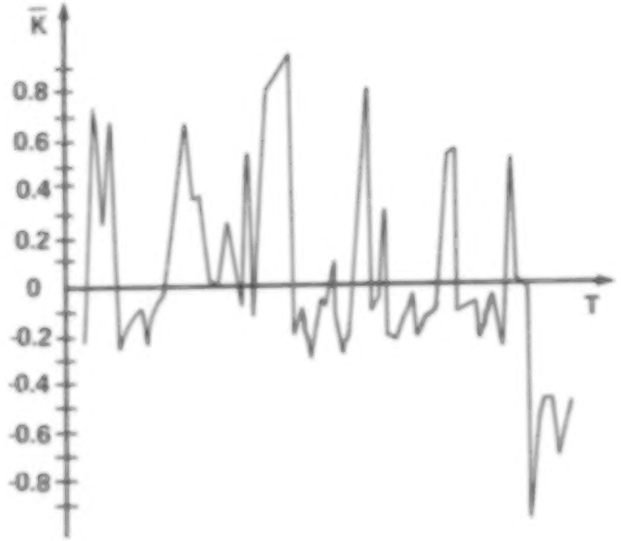

Figure 10: Correlation coefficient of Bosmina longistrostris (O.F. Mull)(X_5) with Synedraulna (X_{15}).

TABLE IV. CORRELATION COEFFICIENTS BETWEEN THE NUMBER OF ALGAE AND DIFFERENT SPECIES OF ZOOPLANKTON*

					Zooplankton					
Algae	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
X_{11}			$\frac{11}{0.659}$	$\frac{11}{0.709}$	$\frac{4}{0.638}$		$\frac{53}{0.691}$	$\frac{53}{0.624}$	$\frac{1}{0.263}$	$\frac{0}{0.815}$
			$\frac{12}{0.860}$	$\frac{12}{0.784}$	$\frac{14}{0.394}$				$\frac{4}{0.756}$	$\frac{1}{0.625}$
			$\frac{13}{0.608}$	$\frac{53}{0.785}$	$\frac{27}{0.779}$				$\frac{11}{0.973}$	$\frac{38}{0.665}$
			$\frac{53}{0.752}$	$\frac{54}{0.799}$	$\frac{29}{0.621}$				$\frac{39}{0.302}$	$\frac{39}{0.782}$
			$\frac{54}{0.948}$		$\frac{70}{0.687}$					
					$\frac{73}{0.379}$					
X_{12}		$\frac{45}{0.661}$	$\frac{11}{0.687}$	$\frac{11}{0.850}$	$\frac{15}{0.657}$	$\frac{53}{0.794}$	$\frac{62}{0.321}$	$\frac{53}{0.770}$	$\frac{2}{0.285}$	$\frac{0}{0.800}$
			$\frac{12}{0.705}$	$\frac{24}{0.676}$					$\frac{3}{0.713}$	$\frac{37}{0.682}$
			$\frac{13}{0.665}$	$\frac{53}{0.680}$						
			$\frac{20}{0.613}$							
			$\frac{53}{0.783}$							
			$\frac{54}{0.707}$							
X_{13}		$\frac{35}{-0.628}$	$\frac{0}{0.680}$	$\frac{10}{0.832}$	$\frac{12}{0.62[illegible]}$	$\frac{53}{0.696}$			$\frac{0}{0.663}$	
			$\frac{1}{0.652}$	$\frac{11}{0.658}$	$\frac{13}{0.331}$					
			$\frac{10}{0.897}$	$\frac{14}{0.797}$						

(continued)

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*The numerator is the lag in measurement interval (10 days). The denominator is the correlation coefficient which exceeds 0.6.

TABLE IV (continued)

	Zooplankton									
Algae	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
X_{13}			$\frac{11}{0.792}$							
			$\frac{14}{0.773}$							
			$\frac{18}{0.632}$							
			$\frac{19}{0.861}$							
			$\frac{33}{0.690}$							
X_{14}		$\frac{95}{0.601}$		$\frac{25}{0.771}$	$\frac{13}{0.618}$	$\frac{18}{0.732}$	$\frac{52}{0.739}$	$\frac{18}{0.757}$		$\frac{38}{0.718}$
				$\frac{67}{0.970}$	$\frac{53}{0.725}$	$\frac{53}{0.716}$		$\frac{53}{0.661}$		
X_{15}			$\frac{13}{0.783}$	$\frac{13}{0.802}$	$\frac{16}{0.705}$	$\frac{55}{0.666}$	$\frac{55}{0.645}$	$\frac{55}{0.770}$	$\frac{4}{0.905}$	$\frac{0}{0.681}$
			$\frac{14}{0.809}$	$\frac{14}{0.672}$					$\frac{5}{0.832}$	$\frac{1}{0.790}$
			$\frac{55}{0.909}$	$\frac{25}{0.702}$					$\frac{11}{0.801}$	$\frac{2}{0.627}$
			$\frac{66}{0.745}$	$\frac{55}{0.830}$						$\frac{38}{0.670}$
X_{16}			$\frac{13}{0.628}$	$\frac{16}{0.643}$	$\frac{18}{0.652}$	$\frac{1}{0.695}$	$\frac{54}{0.303}$	$\frac{56}{0.255}$	$\frac{3}{0.612}$	$\frac{0}{0.673}$
			$\frac{16}{0.746}$	$\frac{25}{0.690}$		$\frac{54}{0.669}$	$\frac{62}{0.768}$		$\frac{4}{0.810}$	$\frac{3}{0.726}$
			$\frac{54}{0.761}$	$\frac{54}{0.770}$		$\frac{55}{0.811}$				$\frac{8}{0.605}$
			$\frac{55}{0.677}$	$\frac{58}{0.644}$		$\frac{58}{0.641}$				$\frac{13}{0.878}$
			$\frac{58}{0.601}$			$\frac{62}{0.737}$				

(continued)

TABLE IV (continued)

	Zooplankton									
Algae	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}
X_{17}	$\frac{38}{0.722}$	$\frac{53}{-0.608}$		$\frac{18}{0.650}$		$\frac{1}{0.600}$	$\frac{1}{0.814}$	$\frac{35}{0.933}$		
						$\frac{36}{0.600}$		$\frac{36}{0.892}$		
						$\frac{-36}{0.710}$				
						$\frac{38}{0.894}$				
X_{18}	$\frac{31}{0.618}$	$\frac{52}{0.628}$		$\frac{70}{0.725}$		$\frac{0}{0.731}$	$\frac{0}{0.684}$	$\frac{32}{0.614}$		
		$\frac{64}{0.683}$		$\frac{73}{0.633}$		$\frac{34}{0.760}$	$\frac{35}{0.784}$	$\frac{34}{0.603}$		
						$\frac{35}{0.867}$	$\frac{36}{0.659}$	$\frac{66}{0.336}$		
						$\frac{36}{0.710}$	$\frac{54}{0.740}$	$\frac{70}{0.869}$		
						$\frac{37}{0.618}$				
						$\frac{70}{0.821}$	$\frac{70}{0.905}$	$\frac{11}{0.776}$		
						$\frac{71}{0.822}$	$\frac{71}{0.759}$			
X_{19}	$\frac{39}{0.887}$	$\frac{9}{0.857}$		$\frac{21}{0.617}$		$\frac{25}{0.675}$	$\frac{21}{0.787}$	$\frac{2}{0.718}$		
	$\frac{53}{0.642}$	$\frac{21}{0.614}$		$\frac{22}{0.805}$		$\frac{51}{0.658}$	$\frac{25}{0.608}$			
				$\frac{25}{0.661}$		$\frac{53}{0.735}$	$\frac{36}{0.659}$			
X_{20}		$\frac{63}{0.625}$	$\frac{13}{0.600}$	$\frac{32}{0.657}$	$\frac{0}{0.782}$	$\frac{66}{0.606}$	$\frac{54}{0.660}$		$\frac{1}{0.695}$	$\frac{20}{0.629}$
							$\frac{51}{0.623}$			

DISCUSSION

ROZHKOV: How are algorithms forecasted when there are few data? Have you tried to establish an equation of the hydrodynamic type as a source equation system to inject purely hydrodynamic research into your work?

IVAKHNENKO: The self-organization method allows us to organize very large surpluses. It is necessary to introduce all known data into the computer. These methods are combined self-organization methods. For example, to construct an economics model of England, we take a determined model with 21 variables. The computer reviews all possible equation variances. The equation form is selected as a partial derivatives equation. There are no unusual terms in these equations.

MORTIMER: Did you allow for absence of motion under the ice?

IVAKHNENKO: The presence of ice does not mean that we fully avoided flow, turbulence, and so forth. In March the ice is still thick on Lake Baikal, and it is possible to freely walk on it. To some degree, this decreases the difficulties which each of us knows.

LEBEDEV: Does your approach eliminate inaccuracies in the regression models? I am referring to the following: 1) the necessity for long series of observations; 2) the impossibility of transferring models from one basin to another; 3) the regression dependency that is destroyed with the appearance of extremely high values for specific pollution indices. Why was a three-dimensional model used for calculating the chemical oxygen demand and a one-dimensional model used for calculating the distribution of suspended substances?

IVAKHNENKO: Our approach eliminates the specified inaccuracies in classic regression models. Long series obtained by using a multiseries model are not necessary. For example, five observation points are sufficient to partially describe a determined member from four coefficients. A complete regression equation can have a million members, constructed along five points. The authenticity of the model is proportional to the square root of the number of points. Twenty points are better than five. But five points are that minimum that allows us to describe the dependency.

It is possible to construct models with 1,000 incoming variables. But man cannot devise equations with a large number of delayed elements. The computer selects all the partial polynomials and finds the optimal density equation. I think it is possible to discover principles according to the nondisplacement criterion and to transfer the data from one water body to another.

THOMANN: The statistical analysis described in the report answers many questions such as how to obtain the maximum quantity of information from field data.

MATVEYEV: Allow me to make several comments on Academician Ivakhnenko's report, since I am a representative of an interested group of specialists working on the Baikal problem. The Hydrochemical Institute, which I represent, is an organization which has been monitoring the condition of the lake for eight years. I agree with Dr. Labzovskiy, it is possible to name around 15 components that could fail in this model. But three components, characteristic for discharges from the pulp and paper industry, were chosen. At the time we were required to forecast the possible influence of man's activities on the water ecosystem of Lake Baikal, this question was an entirely new concern for us both from the standpoint of complexity and components examined. We faced the problem of choosing a model that could forecast with a comparatively small series of observations. The first difficulty which we encountered was to overcome the specialists' opposition to models which required submission of data on frequent observations both in time and space. The second difficulty was the correct understanding and choice of coefficients for equations for the models selected. The continuous dialogue which we carried out with specialists who used different models turned out to be very useful.

The Cybernetics Institute's model allowed us, on the basis of field data gathered for three to five years, to establish a forecast equation for three parameters (components of the environment). A verification by one of the parameters of the forecast showed a similarity to the forecast for one time step; that is, for one year, the divergence was about 9%. From contacts with specialists from the Cybernetics Institute, I have formed an opinion that the interpretation of several coefficients (i.e., the identification of the free term in equations with the source function) is still possible. If so, the importance of this work will grow immeasurably.

My impressions about the symposium follow: the main impression is that the problems of the Soviet and American scientists working on the ecosystem model are practically the same.

MANAGEMENT DECISIONS FOR LAKE SYSTEMS BASED ON A SURVEY OF TROPHIC STATUS, LIMITING NUTRIENTS, AND NUTRIENT LOADINGS

A.F. Bartsch and Jack H. Gakstatter

INTRODUCTION

The National Eutrophication Survey was started by the U.S. Environmental Protection Agency (EPA) in early 1972. Its broad objectives are to identify lakes and reservoirs which receive effluents from municipal sewage treatment facilities and to determine the significance of these point-source nutrient inputs through field investigation. As the program progresses, these objectives were modified by new legislation to include assessment of relationships between drainage area characteristics and nutrient runoff and to develop water quality criteria for nutrients. A number of lakes, not impacted by municipal wastes, were also included in the survey.

The 812 water bodies selected for the continuing study are distributed across the contiguous 48 states as shown in Figure 1.

Information sought for each lake was needed to make management decisions for control of point and non-point sources of phosphorus and other nutrients. An annual nutrient budget was first estimated for each water body with differentiating inputs originating from point and non-point sources. Next, the existing trophic condition was evaluated by sampling, and, finally, an algal assay was performed to determine whether phosphorus, nitrogen, or other elements limited the productivity of the system.

Because a large geographic area was covered in the short period of four years, UH-1H Bell helicopters were used in the sampling program. They were equipped with pontoons so they could land on lakes. They also carried both automated and manually operated water sampling gear to make the appropriate water quality measurements. The sampling teams of limnologists and technicians were supplemented by a mobile, wet analytical laboratory, chemists, electronic specialists, and other staff involved with helicopter maintenance or program coordination. The total staff in the field usually ranged from 12-14 people.

A central headquarters at an airport served as a base for sampling all lakes within a 100-mile radius. When all of the water bodies within the area had been sampled, the support staff moved to a new central location and began sampling on a different set of lakes. In this manner, 150-250 lakes were sampled three times each year, and the survey was completed for all of the 812 lakes in four years.

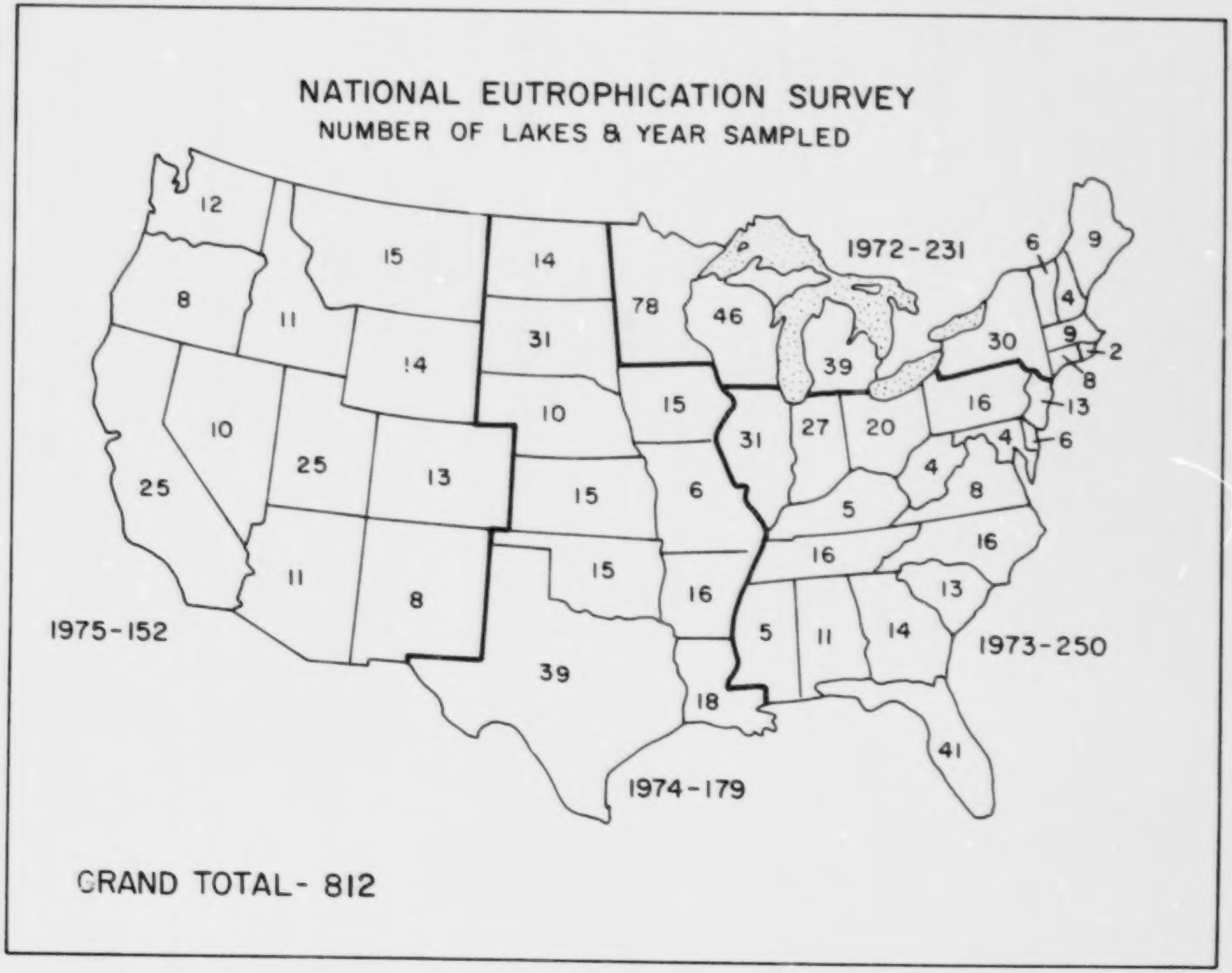
NATIONAL EUTROPHICATION SURVEY
NUMBER OF LAKES & YEAR SAMPLED
1972-231
1973-250
1974-179
1975-152
GRAND TOTAL- 812
12
15
14
78
46
39
30
6
9
4
9
2
8
13
6
4
8
11
31
15
16
27
20
10
15
14
10
25
13
6
31
4
25
15
5
16
16
11
8
15
16
13
5
11
14
39
18
41

Figure 1. Number of lakes and reservoirs sampled in each state and year of sampling by the National Eutrophication Survey.

The quality characteristics measured in each lake are listed in Table I. They are the usual ones needed to assess trophic condition and to provide additional information needed to characterize each lake. Coincidently, samples were collected monthly from the significant tributaries and outlet of each lake. About 4,200 stream sites were sampled nationwide. The tributary sampling was done in each state by National Guard volunteers who received on-site training by EPA to assure acceptable procedures. Immediately after collection, the stream samples were preserved with mercuric chloride and were shipped to the Corvallis Environmental Research Laboratory to be analyzed for nitrogen and phosphorus.

The U.S. Geological Survey provided flow estimates for each of the sampled streams. These data were used in conjunction with concentration values to derive nutrient-loading estimates.

The respective state water pollution control agencies arranged for the monthly collection of effluent samples from municipal sewage treatment plants to determine the nutrients they contribute to the surveyed lakes. This sampling of about 1,000 municipal sewage treatment plants usually was done by the plant operators.

The field aspect of the survey is completed and data analysis is in progress. In the remainder of the paper, I will share some of our findings and those of other investigators as they relate to lake management for eutrophication control.

TABLE I. WATER QUALITY CHARACTERISTICS MEASURED BY THE NATIONAL EUTROPHICATION SURVEY IN LAKES AND RESERVOIRS

Physical-Chemical		Biological	
Alkalinity	Nitrogen:	Algal assay	Algal count and identification
Conductivity*	Ammonia		Chlorophyll a
Dissolved oxygen*	Kjeldahl		
pH*	Nitrate		
Phosphorus:	Secchi depth		
Ortho	Temperature*		
Total			

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* Determined on-site by electronic probes.

Trophic Conditions--Phosphorus Concentration Relationships

Most of the lakes and reservoirs studied during the first year of the survey were eutrophic; only about 20% of them were either oligotrophic or mesotrophic. This was to be expected because the majority selected for study are enriched by effluents from waste treatment facilities. The trophic conditions, therefore, are not typical of all lakes and reservoirs in the United States.

Determining the existing quality of each lake is a matter of both practical and scientific interest. From a control aspect, however, it is probably more significant and productive to slow the eutrophication process in a high quality, but threatened lake, than to restore one in advanced deterioration. As with many environmental endeavors, the practice of prevention is far preferable to remedial efforts. Unfortunately, in this regard, strong public and political pressures for corrective action are usually not imposed until the symptoms of eutrophication become very obvious.

Data from lakes and reservoirs in the northeastern United States were used to examine relationships between total phosphorus concentrations and other aspects of water quality which are indicative of the trophic state. Such relationships, once derived, are useful in establishing water quality criteria for phosphorus as well as in divulging more precisely the characteristics of different trophic conditions. Figure 2, for example, shows the relationship between chlorophyll a and total phosphorus concentrations for a number of phosphorus-limited lakes. The symbols differentiate oligotrophic, mesotrophic, and eutrophic lakes on the basis of nutrient levels, chlorophyll concentrations, Secchi disc depths, and the degree of dissolved oxygen depletion during periods of thermal stratification. Admittedly, these three trophic terms are subjective and difficult to define in absolute terms. Nevertheless, they are widely used by the limnological community and are generally understood to designate three approximations of water quality.

Several observations can be made concerning the relationships in this figure. Lakes classified as oligotrophic did not exceed a median total phosphorus concentration of 10 μg/ℓ or a mean chlorophyll a concentration of 7 μg/ℓ. Median total phosphorus concentrations in the mesotrophic lakes did not exceed 18 μg/ℓ and chlorophyll a did not exceed 12 μg/ℓ. Most eutrophic lakes had median total phosphorus concentrations greater than 18 μg/ℓ.

These lake data also give a reasonable fit with chlorophyll a ranges suggested by others for various trophic levels (Sakamoto, 1966; National Academy of Sciences, 1972; Dobson et al., 1974):

Chlorophyll a (μg/ℓ)

Trophic Level	Sakamoto	Academy	Dobson	Lake Survey
Oligotrophic	0.3-2.5	0-4	0-4.3	0-7
Mesotrophic	1-15	4-10	4.3-8.8	7-12
Eutrophic	5-140	>10	>8.8	>12

If a working guideline for phosphorus concentration control was established empirically on the basis of the data presented in Figure 2, the following would be concluded:

Trophic Level	Chlorophyll a (μg/ℓ)	Total Phosphorus (μg/ℓ)
Oligotrophic	0-7	0-10
Mesotrophic	7-12	10-20
Eutrophic	>12	>20

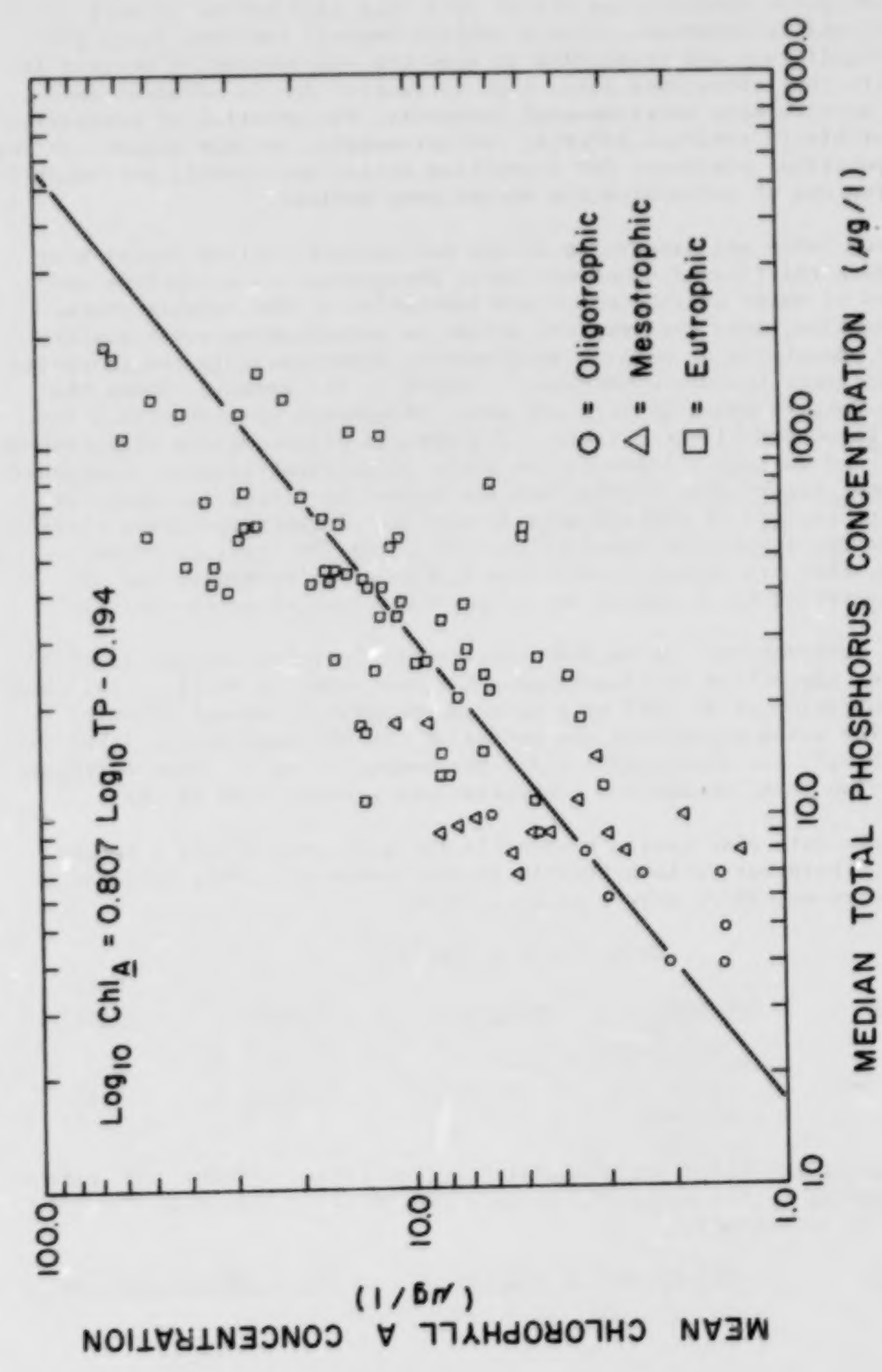

Figure 2. The relationship between chlorophyll A and total phosphorus concentrations in northeastern U.S. lakes and reservoirs included in the survey.

The data presented in Figure 2 also agree with phosphorus guidelines suggested by other investigators. Sawyer (1947) suggested nearly 30 years ago that inorganic phosphorus concentrations at spring overturn should not exceed 10 μg/ℓ if algal blooms were to be avoided. Translated into terms of total phosphorus, Sawyer's values would probably be 15 to 20 μg/ℓ. Bachmann and Jones (1974) recently concluded that total phosphorus concentrations would have to be reduced below 20 μg/ℓ before significant increase in water transparency could be achieved as a result of reduced standing crops of algae. They also presented a phosphorus-chlorophyll relationship which demonstrated that 20 μg/ℓ total phosphorus corresponded to a summer chlorophyll a concentration at about 7 μg/ℓ.

Vollenweider (1968) and Dillon (1975) also concluded that 20 μg/ℓ total phosphorus at spring overturn approximates the transition between mesotrophic and eutrophic waters.

In summary, it becomes obvious that there is substantial evidence to indicate that a concentration of 20 to 30 μg/ℓ total phosphorus in lakes should not be exceeded at any season if eutrophic conditions and attendant nuisance algal blooms are to be avoided. The key to success, however, is capability to determine the degree of phosphorus control necessary within the drainage area to avoid exceeding critical nutrient levels in the lake.

Nutrient Loading--Lake Response Relationships

The relationship between the magnitude of nutrient input to a lake or reservoir and its trophic condition has been known for many years. Specific aspects of this relationship have been probed by several investigators during the past seven years. Although results do not answer all questions, excellent progress has been made in providing improved understanding of nutrient dynamics in the eutrophication process.

Vollenweider (1968) appears to have been the first to examine the relationship between loading and lake response using data from a number of Central and Northern European and North American lakes. Initially, he established an empirical relationship between total phosphorus loading ($g/m^2/yr$), trophic condition expressed as oligotrophic, mesotrophic, or eutrophic, and lake mean depth. Later (1974), he modified the original relationship to account for the fact that lakes with equal mean depths might have different flushing times and, therefore, different nutrient assimilation capacities. His revised model plotted the annual total phosphorus load against the quotient of mean depth in meters divided by the mean hydraulic retention time in years. Vollenweider's work in this area was extremely important, not only because he was the first to investigate the loading-response relationship, but because others became interested in his approach.

Stimulated by Vollenweider's earlier work and by the fact that hydraulic retention time was not adequately accounted for in the model, Dillon (1975) recently published his version of a phosphorus-loading-concentration relationship. He related a factor which included total phosphorus loading, the phosphorus retention coefficient, and the hydraulic flushing time to lake mean depth.

Independently in 1974, Larsen and Mercier of our laboratory also began an analysis of the Vollenweider model and some of its inherent limitations. Their model (in press) relates the theoretical average incoming total phosphorus concentrations to the phosphorus retention coefficients of lakes. The average incoming total phosphorus concentration is defined as the total annual phosphorus load divided by the total hydraulic inflow which is also equivalent to the factor:

$$\frac{L}{\rho . \bar{z}}$$

where, L = annual total phosphorus areal loading ($g/m^2/yr$)
ρ = hydraulic flushing time (exchanges/yr)
$\bar{z}$ = mean depth (m)

Therefore the Larsen and Mercier model incorporates the same parameters as the Dillon model. Both of these models, in effect, predict the total phosphorus concentration of a lake based on its annual phosphorus loading, mean depth, hydraulic retention time, and phosphorus retention coefficient.

Figures 3, 4, and 5 compare the application of Vollenweider's, Dillon's and the Larsen-Mercier models to actual data from 23 lakes studied by the National Eutrophication Survey in the eastern United States. The lakes represent a range of trophic conditions (6 oligotrophic, 9 mesotrophic, 8 eutrophic), mean depths (2.3 to 30.8 meters), mean hydraulic retention times (0.01 to 15.6 years), and total phosphorus loadings (0.03 to 16.94 $g/m^2/yr$).

Figure 3 shows these water bodies plotted against the Vollenweider model. The y-axis represents the annual total phosphorus loading rate ($g/m^2/yr$) and the x-axis represents the mean depth in meters divided by the mean hydraulic retention time in years. All points above the upper curved line are in the eutrophic category, and all points below the lower curved line are in the oligotrophic loading category. Between the two lines is the mesotrophic loading range.

For this set of lakes, the Vollenweider model does not fit very well with observed loadings and trophic conditions. Half of the eutrophic lakes lie within the eutrophic zone of the relationship, while three are in the mesotrophic area and one is in the oligotrophic area.

Only two of the mesotrophic lakes are within the mesotrophic category while the remaining seven are in the oligotrophic category according to the model. All of the oligotrophic water bodies fall into the oligotrophic category as they should.

At least for this group of lakes, the "permissible loading" which corresponds to the transition from oligotrophic to mesotrophic conditions would seem to be a better cutoff point on the Vollenweider relationship than the "dangerous loading" which represents the transition from mesotrophic to eutrophic conditions.

The same lakes are plotted against the Dillon model in Figure 4 in which the eutrophic zone corresponds to the area above the upper line, the mesotrophic zone to the area between the two diagonal lines and the oligo-

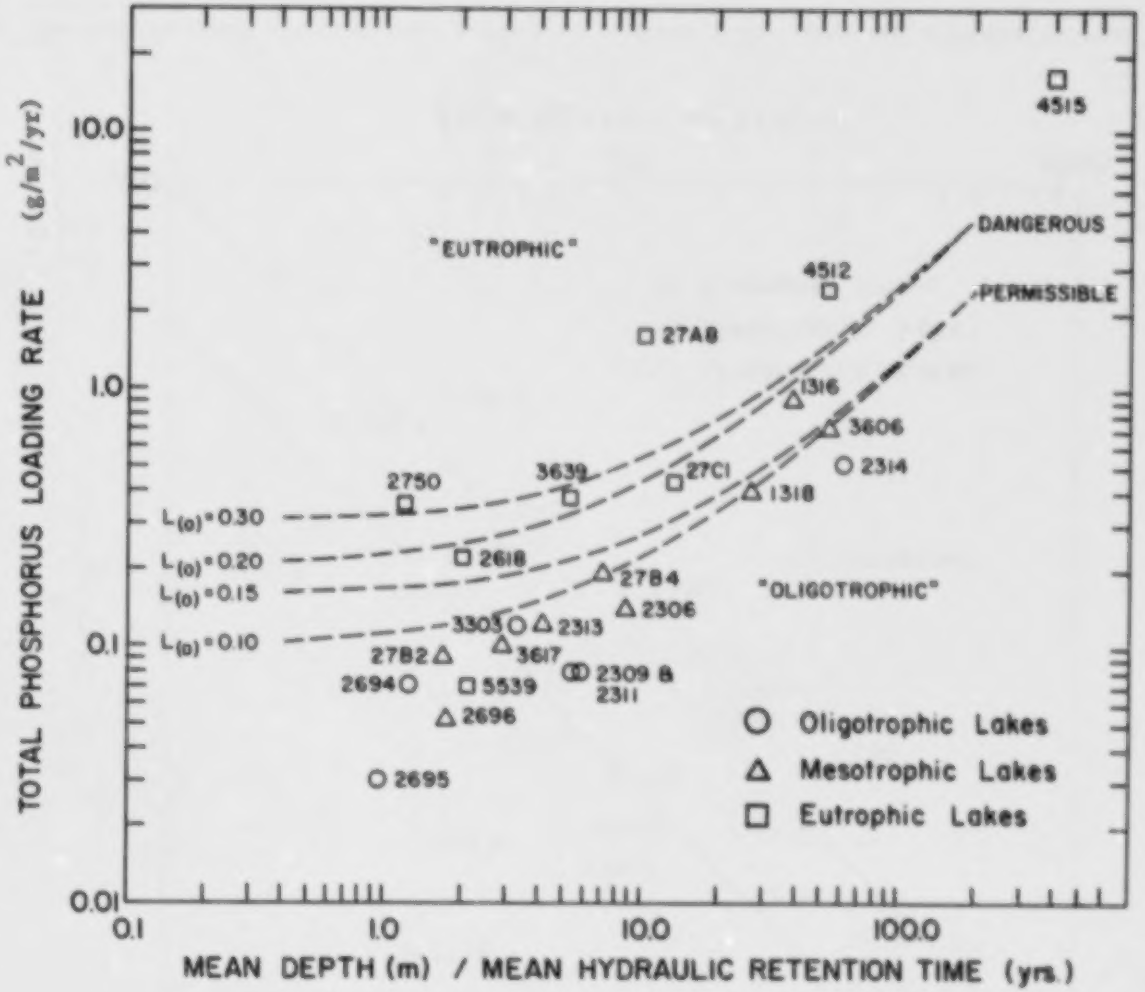

Figure 3. The Vollenweider model applied to a number of eastern U.S. lakes and reservoirs sampled by the survey.

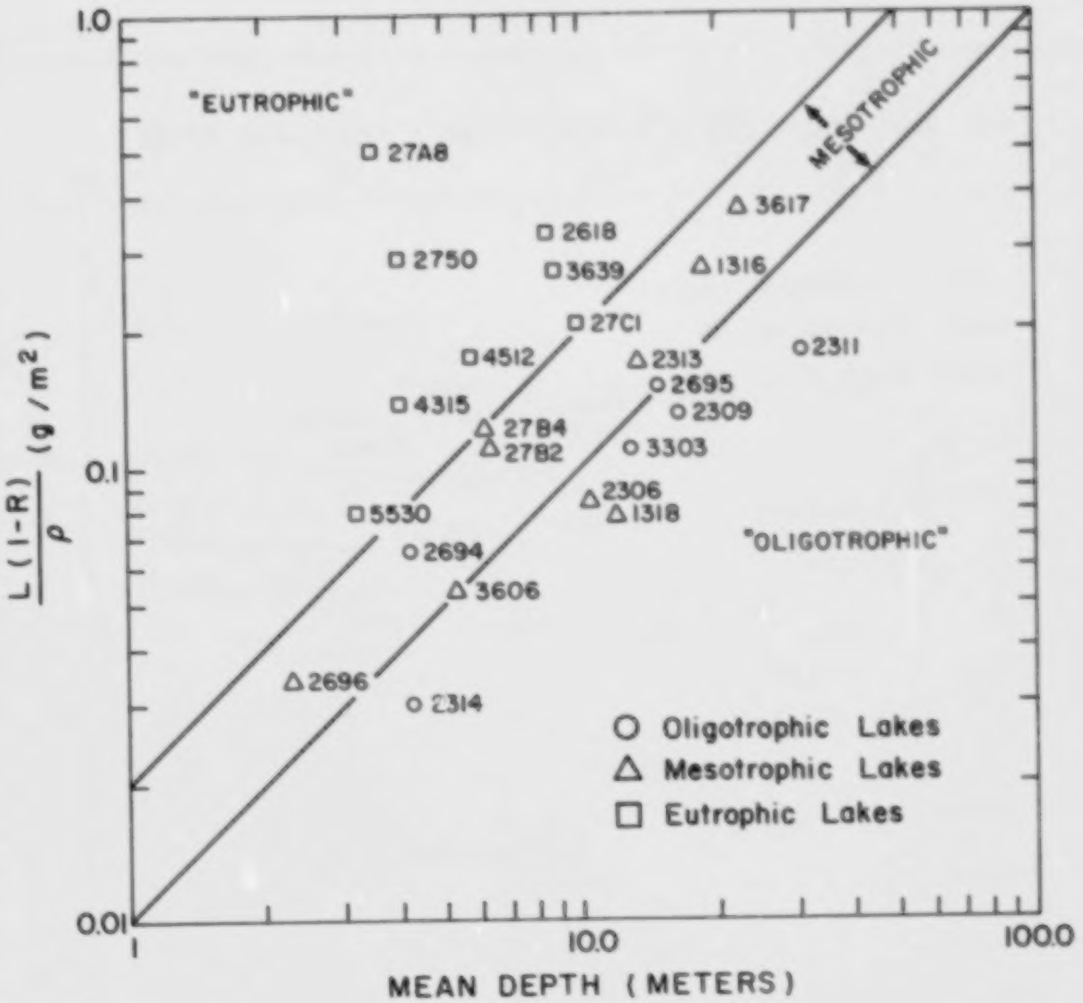

Figure 4. The Dillon model applied to a number of eastern U.S. lakes and reservoirs sampled by the survey.

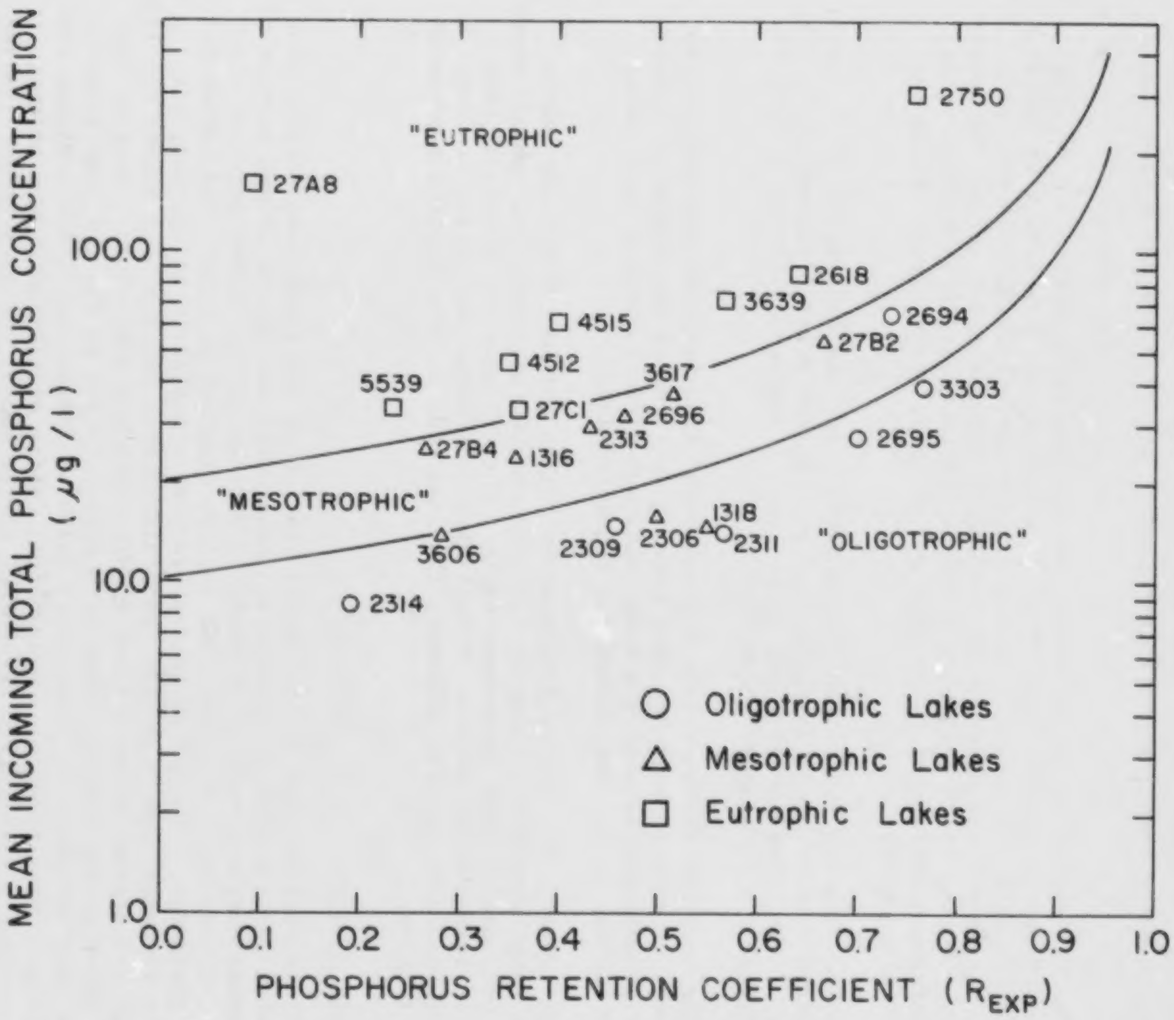

Figure 5. The Larsen and Mercier model applied to a number of eastern U.S. lakes and reservoirs sampled by the survey.

trophic zone to the area below the lower diagonal line. The upper and lower lines correspond to in-lake total phosphorus concentrations of 20 μg/ℓ and 10 μg/ℓ which roughly correspond to the transition between mesotrophic-eutrophic and oligotrophic-mesotrophic conditions. In general, the model well describes the conditions observed by the survey in the 23 lakes. All of the eutrophic lakes fall within the eutrophic area of the graph, while only two of the mesotrophic lakes are out of place. One oligotrophic lake plots within the mesotrophic category, while the rest are where they should be.

The Larsen-Mercier relationship is illustrated by Figure 5. Although their approach is slightly different from Dillon, the net result is nearly identical. With the exception of one oligotrophic lake and two eutrophic lakes, the conditions observed by the survey fit the model well. The upper and lower curved lines in Figure 5 correspond to theoretical in-lake total phosphorus concentrations of 20 μg/ℓ and 10 μg/ℓ, respectively, as in the Dillon relationship.

Since both the latter two models predict in-lake concentrations of total phosphorus, the vertical distance from an observed point, representing a lake, to one of the transitional lines is at least a semi-quantitative measure of the degree of oligotrophy or eutrophy.

The vertical distance from a given point to a transitional line in the Vollenweider relationship has less meaning in terms of the degree of oligotrophy or eutrophy because the model does not directly relate total phosphorus loading to in-lake phosphorus concentrations.

In summary, the models developed by Dillon and Larsen-Mercier, which relate total phosphorus loads to lake phosphorus concentrations, should prove to be very useful lake management tools. The Vollenweider model, at this time, is less precise because it considers only total phosphorus loading without regard to in-lake processes that can reduce the effective phosphorus concentration.

Nutrient Sources

It is obvious from the foregoing that among the first questions of concern when dealing with eutrophication management are magnitude and sources of the annual nutrient input. In general, sources can be either point or non-point in character. Point sources include discrete municipal and industrial waste discharges. Non-point sources include a number of categories, but usually represent runoff within the drainage area which either enters the lake through tributaries or as diffuse runoff along the lake shore. Another non-point input is the nutrient load from precipitation or dry fall-out directly on the lake surface. In areas where cultural influences in the drainage area are minimal, precipitation and dust fall, while of limited actual amount, may represent a significant percentage of the annual nutrient budget (Vollenweider and Dillon, 1974).

Municipal waste treatment plant discharges represent a concentrated point source of phosphorus. It has been estimated that about 1.6 kilograms of phosphorus per person per year are discharged in raw wastewater (Bartsch, 1972). Even where raw wastes are treated before discharge, only from 20 - 30% of the phosphorus usually is removed by the treatment process. As a result, unless a treatment plant was specifically designed to remove phosphorus, most of it moves through the treatment process and is discharged in the effluent.

Effluent data, collected by the survey for 505 municipal waste treatment facilities, gave 1.0 kilogram per year as the median per capita contribution of total phosphorus remaining in treated wastes. The median total phosphorus concentration in the same effluents was 5.9 mg/ℓ. Corresponding figures for total nitrogen were 2.7 kilograms per capita and for median concentration, 15.0 mg/ℓ (EPA, 1974). A wide range of per capita nutrient loads between plants was found to be common, which suggests that actual data, rather than estimates, should be used when important control decisions are involved. The fluctuations are believed to be related primarily to industrial contributions of phosphorus to the wastewater. However, localized domestic practices which either increase or decrease phosphorus contributions to wastewater should not be discounted.

Another significant point to be considered in dealing with municipal wastewater is that effluents contain an abundance of phosphorus in relation to nitrogen. In general, algae require a ratio of inorganic nitrogen to orthophosphorus-P, by weight, in the range of 11:1 to 15:1 for growth (Miller et al., 1973; Porcella et al., 1974). In natural waters, unimpacted by man, the N:P ratio is usually higher than 15:1 which suggests a relative abundance of nitrogen and algal growth limited by the scarcity of phosphorus. In treated municipal waste, however, the average ratio of N:P is about 2.5:1. This then offers an abundance of phosphorus supplied in an available form since more than half of the P is present as dissolved orthophosphate-P (EPA, 1974). In municipal waste effluents, detergents account for about 50% of the total phosphorus in areas where phosphate-based detergents are still used. Although detergent phosphorus certainly is not the sole cause of accelerated eutrophication, substantial reductions in phosphorus loads to some lakes could be achieved by the removal of phosphorus from detergents.

Figure 6 depicts the significance of phosphorus from municipal waste sources to the total annual phosphorus load for 132 surveyed lakes and reservoirs in the eastern United States. The frequency histogram shows that a number of water bodies would have their annual phosphorus loads reduced by 80-100% if the input originating from municipal wastewater were eliminated. The reduction in lake loadings would be half as much if only the detergent fraction of the phosphorus were eliminated. For all of the lakes, the mean phosphorus contribution from domestic wastes was 40% of the total load.

Too often, studies designed to delineate lake problems as a step toward management ignore drainage area characteristics and land use. Such characteristics, including size, geology, climate, land use, and population density, largely determine the quantity of nutrients that enter a lake annually.

It would be extremely enlightening if one could go back to the early 17th Century in the United States and make a lake survey similar to the

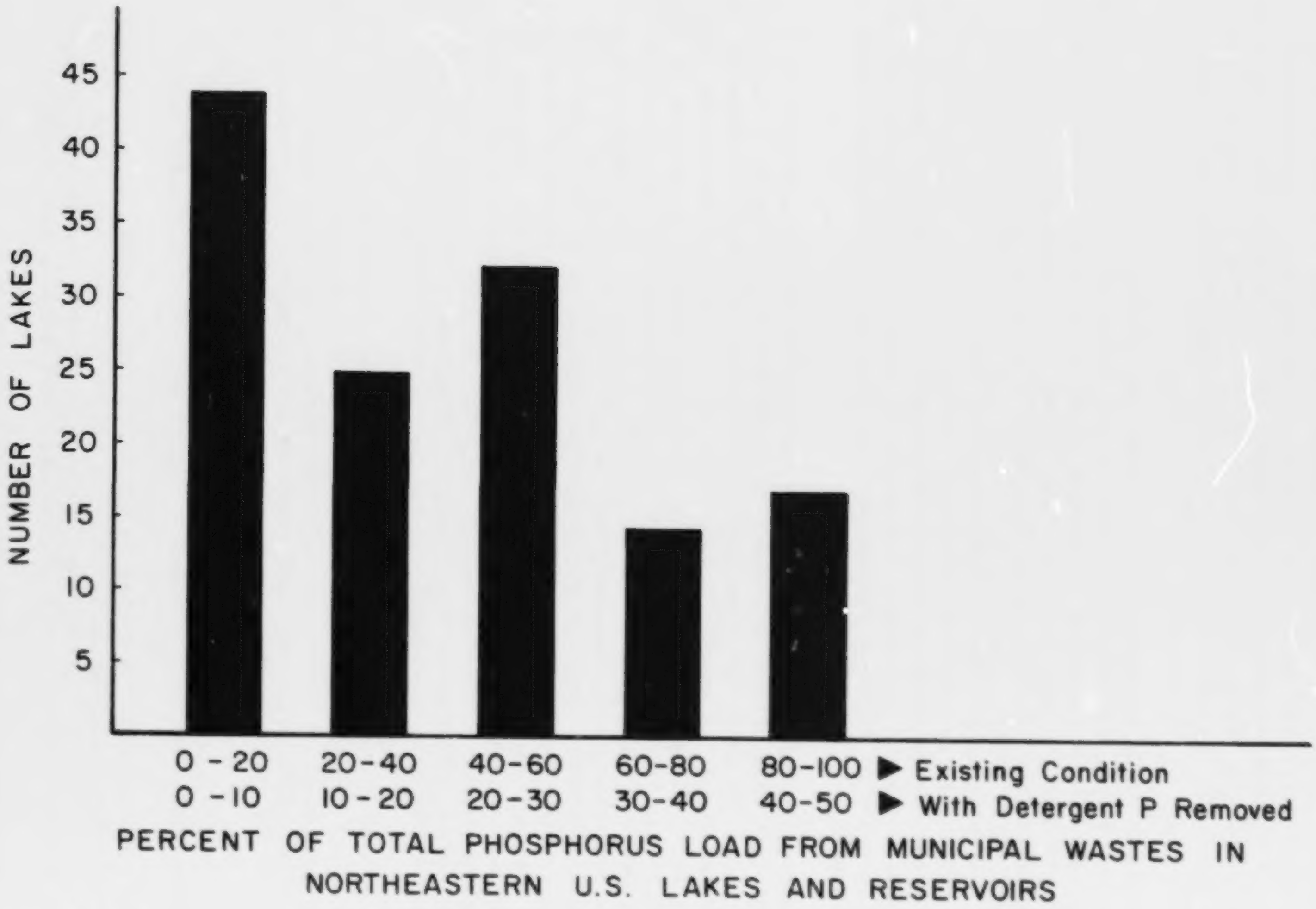

Figure 6. A frequency histogram representing the percent of total annual phosphorus load attributable to municipal wastes for a number of northeastern lakes and reservoirs included in the survey.

present one. Many questions concerning man's impact on lakes could then be easily and accurately answered. It is likely the 17th Century lakes presented a diversity of trophic conditions simply because then, as now, a diversity of geological areas in the United States provided a range of baseline nutrient levels. On such base levels are superimposed the secondary impact that comes from land use patterns within the area. Mean export levels for total phosphorus have been determined for southern Ontario watersheds of different land use types and geological classifications (Dillon and Kirchner, 1974). As shown by Table II, in forested drainage areas of basically igneous origin, only 4.7 kg/km^2 of total phosphorus are exported per year. However, this value increases to 10.2 kg/km^2 when forest and pasture uses are combined. In sedimentary watersheds, total phosphorus exports range from 11.7 kg/km^2 for forest areas to 46 kg/km^2 for agricultural areas, while urban land use contributed an average of 1,050 kg/km^2/year.

In these cases, geological differences, such as igneous vs. sedimentary origin, accounted for a 2.5-fold difference in phosphorus export while land use patterns within one geological type (sedimentary) accounted for a 3.9-fold difference (between forested and agricultural land use types).

TABLE II. MEAN VALUES FOR EXPORT OF TOTAL PHOSPHORUS FROM 43 WATERSHEDS IN SOUTHERN ONTARIO (DILLON AND KIRCHNER, 1974)

Results in kg/km^2/year

Land use	Geological classification	
	Igneous	Sedimentary
Forest	4.7	11.7
Forest and pasture	10.2	23.3
Agriculture	---	46.0
Urban	---	1050.0

As part of the survey, a study is underway to relate drainage area characteristics to nutrient concentrations and loads in streams as a step toward estimating annual lake and reservoir inputs. At this point, the survey evaluations have mainly examined land use rather than geological aspects, although the latter are being considered in future evaluations. Preliminary data for 473 drainage areas on the eastern United States support the view that forested areas contribute the lowest amounts of nutrients to surface waters and agricultural lands contribute the highest. However, some other interesting facts, as well, are indicated in Figure 7 which relates the predominating land use types in the watershed to mean total phosphorus and total nitrogen concentrations in the streams draining the area. The mean total phosphorus concentrations range from 0.014 mg/ℓ in forested drainages to 0.135 mg/ℓ in agricultural areas, nearly a 10-fold difference.

Similar examination shows that mean total nitrogen concentrations range from 0.850 mg/ℓ in forested drainages to 4.17 mg/ℓ in agricultural areas, a 5-fold difference in this case.

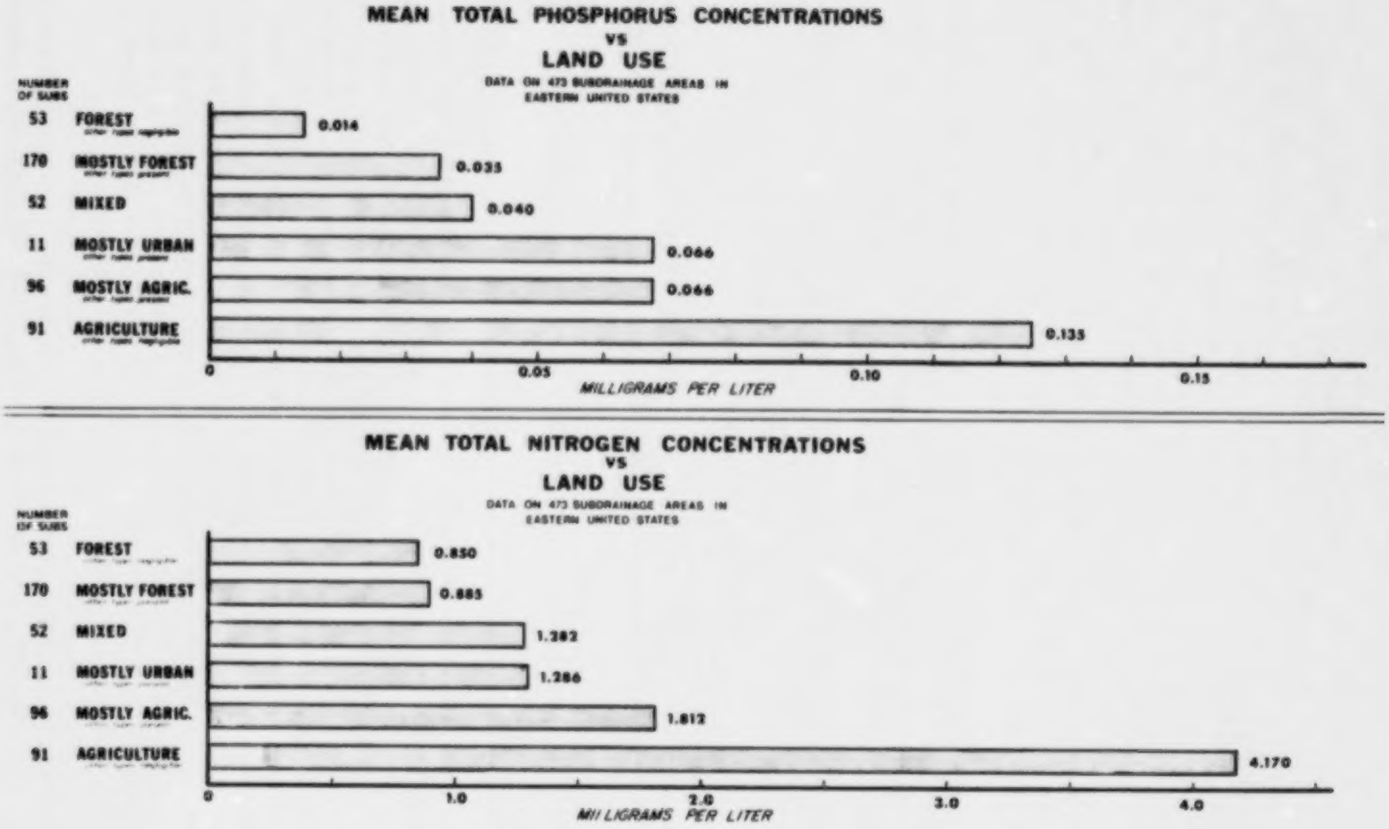

Figure 7. Mean total phosphorus and total nitrogen concentrations in streams draining different land use categories.

If stream flow factors are applied to the above values to calculate annual nutrient loads carried by streams draining the different land use types, the land use impact, while still important, is not as great as concentrations suggest. Figure 8 indicates that 3.7 times as much phosphorus is exported from agricultural watersheds as from forested watersheds while the difference in total nitrogen export is 2.2-fold.

It is evident from the above discussion that both geology and land use of lake drainage areas can play a significant role in determining the quantity of nutrients which are input to a lake. While obviously the basic geology cannot be easily manipulated for management purposes, land use planning efforts must consider the nutrient aspect.

Limited Nutrients

The quest for effective procedures to curb eutrophication has been given increased impetus in many countries. Much has been written during the past few years about the roles of phosphorus, nitrogen, carbon, and trace elements in enhancing or limiting the productivity of aquatic systems. It is apparent that under the right set of circumstances, any of the essential elements might be growth limiting. However, the preponderance of evidence suggest that, in the majority of cases, phosphorus is the element with which lake managers should generally be concerned.

The role of phosphorus in eutrophication and its significance to lake management has been discussed recently by Bartsch (1972). Experimental work by the Fisheries Research Board of Canada (Schindler et al., 1973; Schindler and Fee, 1974), clearly supports previous theories that phosphorus control is of singular importance in slowing lake eutrophication. The U.S. Environmental Protection Agency's current project at Shagawa Lake (Malueg et al., 1973) in Ely, Minnesota, is successfully demonstrating on full scale the capability to reserve the eutrophication process through phosphorus removal from municipal sewage.

Algal assays were performed for each of the surveyed lakes and reservoirs to determine if productivity was limited by phosphorus or nitrogen exhaustion at the time of sampling. The assays were done according to a standard procedure (EPA, 1971) using a Chlorophyte, Selenastrum capricornutum Printz, as the test organism. Assays have now been completed for 623 lakes and the results are summarized in Table III.

That assay results such as these should be evaluated with care has been discussed by Schindler and Lean (1974). Where human impact on both point and non-point sources of nutrients in the watershed is inconsequential, primary production in surface waters generally is controlled by the supply of phosphorus, that is--phosphorus then is the limiting nutrient. Man's perturbations in the watershed, such as establishing waste treatment plants or pursuing agricultural activities, frequently accelerate the influx of phosphorus relative to other nutrients. Thus a lake, once phosphorus-limited in its undisturbed state, may become eutrophic and nitrogen-limited because abundant phosphorus supplied by man's activities far exceed the algal needs. Because the algal assays generally are performed after such changes have

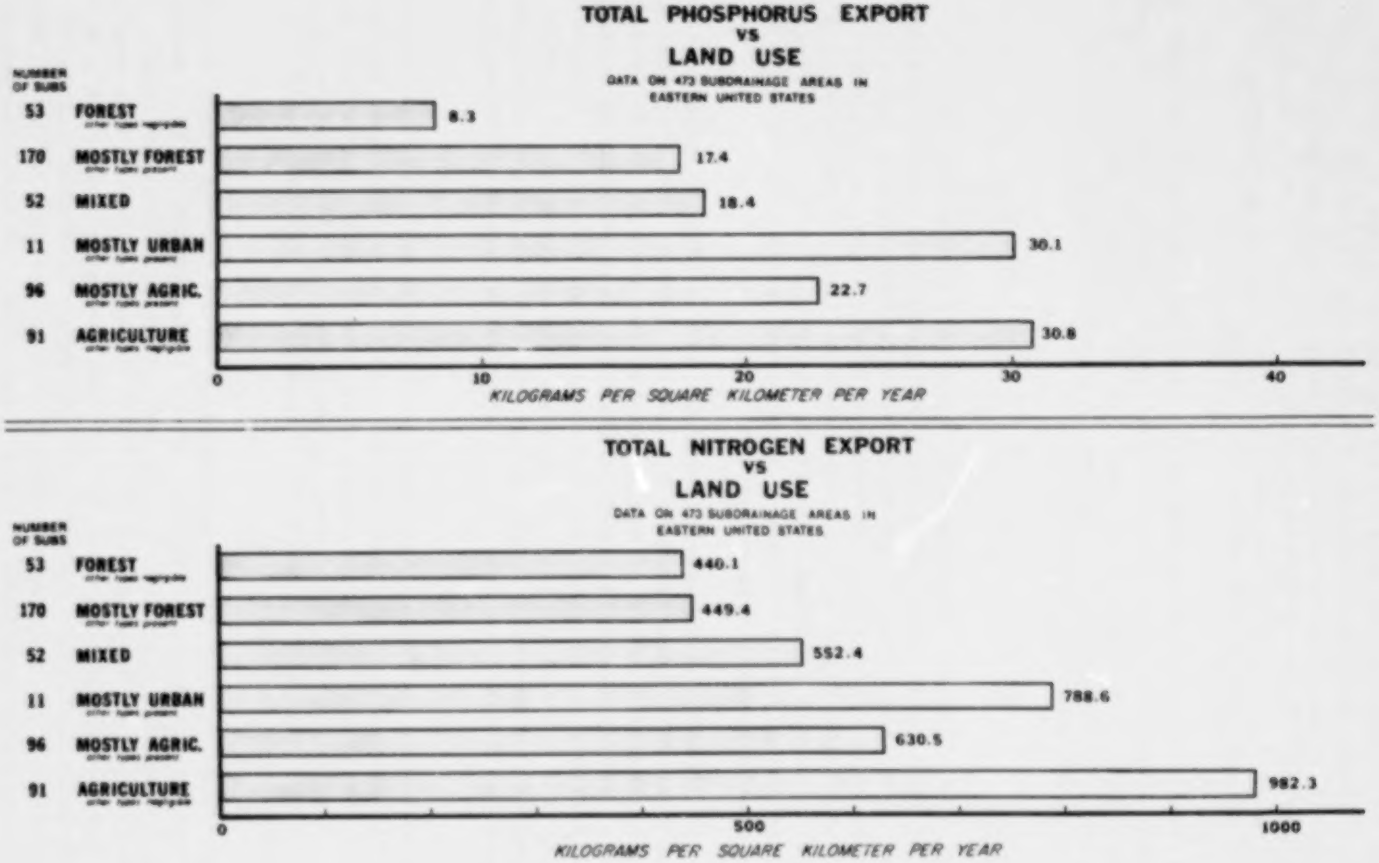

Figure 8. Mean total phosphorus and total nitrogen export by streams draining different land use categories.

occurred, they show that a number of lakes have not reached a nitrogen-limited state. It is important to note that this does not impair the possibility to shift back, through appropriate phosphorus control mechanisms, to a mode in which phosphorus again assumes a limiting nutrient role.

The algal assays revealed that phosphorus was the limiting nutrient in about 67% of the 623 lakes examined, as shown in Table III. This is remarkable when one considers that the majority of the lakes were enriched by municipal effluents. Included in the phosphorus-limited category were lakes in which both phosphorus and nitrogen were present in the optimal ratio for algal growth. There were 28 lakes of this type. These lakes would benefit immediately from phosphorus reductions.

TABLE III. SUMMARY OF ALGAL ASSAY RESULTS FOR LAKES AND RESERVOIRS IN THE 37 STATES EAST OF THE ROCKY MOUNTAINS

Limiting nutrient	Number of lakes	% of all lakes
Phosphorus	417	67
Nitrogen	186	30
Other	20	3
Total	623	100%

About 30% of the water bodies were found to be nitrogen-limited. Many of them could be expected to become phosphorus-limited following sufficient reduction of phosphorus loading. A small percentage (3%) of surveyed water bodies were limited by an element other than phosphorus or nitrogen, such as cobalt, iron, molybdenum, or manganese.

Morphometric Considerations

Data on lake morphometry also are essential to sound management decisions. A deep lake with steep bottom contours is less likely to develop problems with aquatic weeds than a shallow lake where light penetrates through a large percentage of the water volume. For various reasons deep lakes can also absorb greater nutrient loads without serious symptoms. In shallow lakes, there is greater opportunity for nutrient-rich sediments to recycle into the lake biomass through wind action whereas in deeper lakes the nutrients may be removed from the cycle through the sedimentation process. Given two lakes with equal drainage areas and surface areas but different mean depths, the deeper one will have the longer hydraulic flushing time which will permit a greater percentage of the nutrient load to be removed by sedimentation. It also will have a greater volume and therefore a greater diluting effect on the incoming nutrient load.

The ratio of drainage area to lake surface area should also be considered. A high ratio is generally accompanied by a proportionately higher nutrient and hydraulic loading and a proportionately lower hydraulic retention time. For lakes lacking good morphometric data, drainage areas to surface area ratios from better known lakes in the area can be useful in approximating values. For example, Figure 9 relates drainage area-surface area

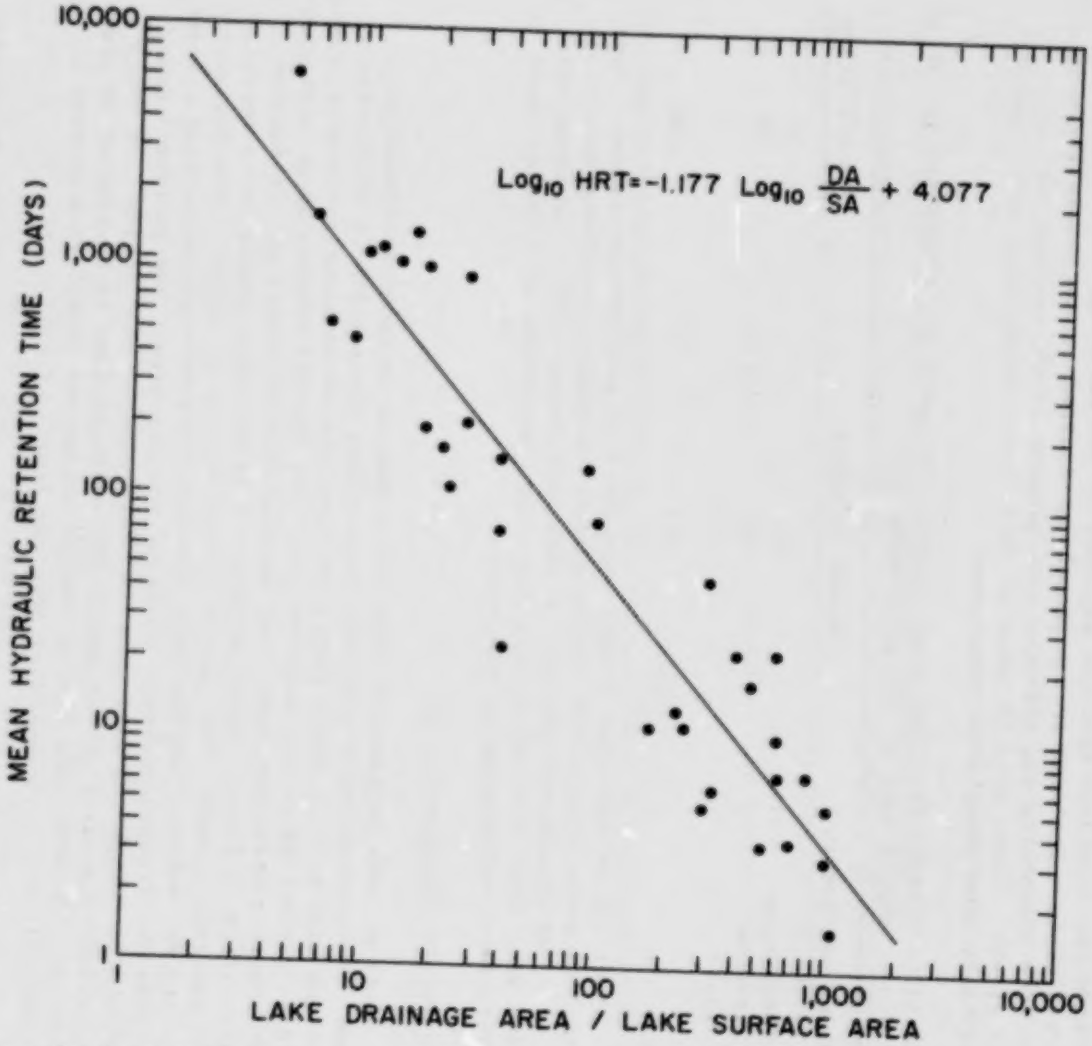

Figure 9. The relationship between mean hydraulic retention time and the drainage to lake surface area ratio for a number of northern U.S. lakes and reservoirs.

ratio to mean hydraulic retention time for several northern U.S. lakes and reservoirs and could be used to approximate the hydraulic retention time of a water body which had not been bathymetrically mapped.

The types of lake restoration techniques which might be applied to a lake with serious eutrophication problems would depend upon lake morphometry to a great extent. For example, one would not choose to employ aeration techniques to a lake which is shallow and well-mixed; whereas in a deep thermally stratified lake, aeration might be considered a potentially useful tool in a restoration program.

Another consideration is the irregularity of the lake shoreline in the form of embayments or peninsulas. Some lakes almost resemble a series of smaller connected lakes rather than a single large one. A good example is Lake Minnetonka, Minnesota, U.S.A., which is so irregular in shape that for study purposes it was divided into 15 nearly separate basins (Megard, 1972). Because most models for predicting lake response to nutrients assume that the system is relatively uniform and completely mixed, shoreline irregularity becomes a factor worthy of serious reflection. Unfortunately, such reflection is often impeded because adequate morphometric and bathymetric information is lacking.

Summary

Several factors have been discussed as important considerations in managing a lake to a desired trophic level. They are summarized below.

1. Total phosphorus concentrations in a lake ranging from 20-30 µg/ℓ or greater are usually associated with eutrophic conditions. These phosphorus concentrations correspond to chlorophyll a concentrations of 9-12 µg/ℓ or more.
2. To determine the extent to which phosphorus must be controlled to obtain the desired trophic condition, one must be able to relate total incoming phosphorus to the lake response. Three general models from the literature were presented which relate total phosphorus loading to lake response.
3. Ratios of inorganic nitrogen to dissolved orthophosphorus-P in surface waters are usually greater than 15 to 1 unless impacted by point-source discharges, particularly municipal wastes. The average N to P ratio of municipal waste is 2.5 to 1, thus an abundance of phosphorus relative to nitrogen is supplied.
4. The National Eutrophication Survey results demonstrate that a number of lakes and reservoirs in the United States receive a substantial part of their total phosphorus load from municipal wastes.
5. Phosphorus is the element with which lake managers will usually be concerned in controlling eutrophication. A survey of 623 lakes in the United States indicated that phosphorus was the limiting nutrient for 67% of these and nitrogen for 30%. Many of the nitrogen-limited lakes could be shifted to a phosphorus-limited mode through appropriate phosphorus control measures.
6. Drainage area geology and land use are important factors that influence the total phosphorus load to a lake. Phosphorus concentrations in tributaries draining agricultural land in the northeastern United States averaged ten times greater than in those draining forested land, while nitrogen concentrations were five times greater.

7. Before lake restoration is attempted, the morphometry of the lake and its drainage area characteristics should be examined in detail because these factors significantly influence lake trophic condition.

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DISCUSSION

LEBEDEV: It was stated in the report that the element which limited algal development was determined. I am interested in knowing how these determinations were carried out and the percentage of cases in which insufficient phosphorus was available for plankton growth?

BARTSCH: We can use many methods to determine the limiting factor. The method we used was jointly developed by EPA, three universities, and private industry. An algae serves as the control organism; water samples are taken and treated by different methods. Of the 812 lakes examined by this method, 67% were limited by phosphorus, 30% by nitrogen, 3% by microorganisms. There is a supplementary method for seawater using diatoms.

BOYCHUK: What do you mean by the "introduction of phosphorus?" Is it a quantity at a given moment in time or the distribution of this quantity in time? When studying the influences of phosphorus, do you consider the time factor?

BARTSCH: We monitor eutrophication by introducing phosphorus. A large phytoplankton bloom is a symptom of eutrophication. For growth, phytoplankton requires a series of elements. If we take away any of these elements, growth stops. One of the problems is slowing down or stopping the bloom by phosphorus retention; we do not introduce phosphorus on purpose.

LEBEDEV: What element limiting the plankton development in water bodies could help determine the direction for modeling processes which take place in water ecosystems? The question on the limiting action of one or another element is not new, and, unfortunately, is complicated. Determination of a deficiency in an element depends on the selection of organisms tested, the length of sample incubation, and type of incubation container. It is possible to say that water bodies under a strong eutrophication influence suffer primarily a deficiency in nitrogen. At this time, a definitive deficiency in phosphorus is observed in lakes where there has been no anthropogenic influence. Our conclusions that nitrogen is the limiting factor where there is strong anthropogenic influence support the results obtained from the mathematical model for forecasting water quality in reservoirs. In the Mozhayshiy reservoir, we obtained good results verifying the limiting action of nitrogen. However, I do not insist on my conception of the limiting action of nitrogen, and I do not negate Dr. Bartsch's conception of the limiting action of phosphorus. The question is too uncertain to construct a model now.

BARTSCH: Let me answer the question about the limiting element. I am not surprised to hear that some lakes are limited by nitrogen, others by phosphorus, and a third group by carbon. From a practical point of view, there is no difference as to what element limits a lake. If the lake is limited by phosphorus, then it is our goal to retard lake eutrophication by means of controlling phosphorus content.

TSYTSARIN: My question is related to the methods for evaluating non-point sources of phosphorus. In Dr. Bartsch's paper, I did not discern a system for collecting samples. During the last 5 to 6 years, our group in the Moscow area labored over this question. We were successful in measuring the flow of phosphorus for determined periods. When evaluating the phosphorus loads, the greatest errors are possible in the spring when 70-80% of the annual flow occurs and in the fall when 10-15% of the flow occurs. During the low-flow period, it is more difficult to evaluate the phosphorus loads because phosphorus transport takes place through the atmosphere. A very high concentration of phosphorus (0.2-0.3 mg/ℓ) is observed in all atmospheric precipitations. The reason for this phenomenon is the terpene-type products that enter the atmosphere during the degradation of vegetation.

BARTSCH: These data are extremely interesting. Sampling water in rivers to determine the phosphorus loss from the drainage basin does not give us accurate results. These values would be characteristic if the sampling day coincides with the flow maximum. Other basins in the United States are studied with a higher accuracy from the flow and chemical points of view.

Regarding the phosphorus in the atmosphere, we admit its existence, but research has not been specifically carried out in this field. Other reports have noted high concentrations of phosphorus in atmospheric precipitations in the U.S. In Oregon large quantities of phosphorus are found far from the ocean. The concept of phosphorus loss from surface films is accepted.

MATHEMATICAL MODELING STRATEGIES APPLIED TO SAGINAW BAY, LAKE HURON

Victor Bierman, William Richardson,
and Tudor T. Davies

INTRODUCTION

The objective of this modeling research is to develop quantitative tools which can be used to supplement intuition and scientific judgment in policy decisions concerning water quality. This research is directed initially toward water quality problems concerning the international waters of the North American Great Lakes. These waters have a very broad range of uses, some of which induce water quality problems and environment degradation. Use of these waters is economically and socially significant to the area, and the consequences of decisions aimed at preserving or even improving water quality affect the well-being of the people in the drainage basin and beyond. Management decisions on major changes in treatment technology and enforcement actions should be based on the results of simulations of the effects of these changes on water quality and the ecosystem.

To apply modeling techniques to the problems concerning the Great Lakes, it is essential to categorize the problem into a series of time and space scales. This series ranges from the short-term transport (on the scale of hours or days) of some hazardous material within a very localized area, to the long-term effects (ranging from years to decades) of control or limitation of nutrients or toxic materials within the drainage basin on the ecosystem of individual lakes or of the whole system of lakes. Two papers in this symposium by Dr. Lick and Dr. Thomann represent both ends of the spectrum, the short-time and small-space scales and the long-term responses of the whole lake.

This paper will attempt to integrate these different approaches to problems of a small area of the Great Lakes which has suffered specific water quality degradation. This study is part of an intensive effort by the U.S. and Canada to study and report on the so-called upper International Great Lakes which are considered to be largely unaffected by cultural degradation but remain in an oligotrophic state in contrast to the lower Great Lakes. Although the main lakes are considered to be largely unaffected, some of the coastal areas and embayments do have significant problems. The Saginaw Bay system is perhaps the most highly degraded.

This study attempts to apply transport models and algal growth modeling concepts to Saginaw Bay to describe prevailing conditions in the bay. The model developed is based on available data on the bay, and the modeling concepts will be verified by using an extensive set of field data collected from a sampling design aimed specifically at this modeling program.

This modeling system will then be used to evaluate the response of the system to a range of management alternatives for enhancing water quality.

SAGINAW BAY--BACKGROUND

Morphology and Geographical Setting

Saginaw Bay is a shallow SW extension of Lake Huron, which is a deep oligotrophic lake. The bay, which has a surface area of 2,960 km^2 and is 21-42 km wide and approximately 82 km long, receives drainage from an area of 21,000 km^2. The bay has two distinct parts, an inner or southern zone which is very shallow with a mean depth of 4.6 m (maximum depth 14.0 m) and an outer bay with a mean depth of 14.6 m ~~and~~ a maximum depth of 40.5 m. The inner bay constitutes 30% of the total baywater volume and is dominated by extensive shallows.

Hydrology

The Saginaw River which has an average flow of 3850 CFS is the major tributary draining 80% of the drainage basin of the bay. The mean annual precipitation of 189.5 cms is evenly distributed throughout the year. Average water levels are controlled by Lake Huron water levels and a predominantly wind-driven circulation system normally influenced by SWS and SW prevailing winds.

Deepwater temperatures remain at about 4°C, while the shallow and surface waters undergo change from season to season. In spring, the total water mass is homothermous at 1-4°C. Shallow waters are warmer, but only in the extreme outer bay does a well-formed thermocline develop in the summer and fall. Consequently, the bay may be considered as vertically well-mixed. Complete ice cover and limited stratification is typical of the bay in winter.

The prevailing circulation is considered counter-clockwise with Lake Huron waters entering from the NE and flowing along the northern bay shore. Saginaw Bay water flows out to Lake Huron along the south shore of the bay. However, this typical circulation pattern is swiftly upset by changing wind conditions.

Land Use

Land use in the drainage basin has a considerable influence on water quality in Saginaw Bay: 50% of the drainage basin is considered to be agricultural, mainly cropland; 35% is forested; only 3.5% is urbanized. This area has significant industrial, chemical, mining, and food processing industries.

Water Uses and Water Quality

The bay and small reaches of the Saginaw River have extensive diverse uses, including mining, commercial and sports fishing, and general recreation. In addition, these waters are used for cooling electrical power plants, disposing of municipal wastes, and producing industrial goods. Some of these uses degrade water quality seriously enough to interfere with municipal water supply, recreation, and the fisheries.

Water quality in Saginaw Bay shows a significant deterioration from Lake Huron water in both physical, chemical, and biological characteristics. High concentrations of dissolved substances that occur in the inner bay are progressively diluted by high quality Lake Huron water in the outer bay. Excessive nutrient concentrations support prolific growths of algae, and other biologic criteria indicate degraded conditions. Although water quality has improved slightly in the past few years, commercial fishing has been curtailed because of a major shift to low value species. This shift occurred because of changes in the food web, changes in habitat, predation, competition from introduced and invading species, and commercial over-fishing.

SAGINAW BAY MODEL STRUCTURE

Objectives

The ultimate goal of this work is to develop a mathematical model that can be used to describe the physical, chemical, and biological processes in Saginaw Bay and to predict the effects of reduced waste loadings. Specifically, the modeling effort will focus on phosphorus, nitrogen, and silicon loadings to the bay and the resultant production of phytoplankton biomass.

Model development is proceeding along a series of pathways which are producing and evaluating alternative submodel designs for the two major submodels: (1) physical circulation (transport and dispersion) and (2) chemical biological processes. To model the temporal and spatial transport of material and nutrient-phytoplankton dynamics, both submodels are essential.

Alternative Transport Schemes

The inadequacy of field measurements in Saginaw Bay requires a mathematical determination of circulation which will be verified with all available field information.

Two basic approaches have been identified:

1. Fine-grid hydrodynamic approach.

This involves a complex mathematical formulation of the physical forces that influence circulation. This approach is outlined by Lick (in this volume). The approach is being pursued for Saginaw Bay as a future resource, particularly to address fine space and time resolutions, but is inapplicable to the present relatively long-time and the broad geographical scale of this study.

2. Coarse-grid, mass balance approach.

A more economical approach to quantifying circulation has been used by

Thomann (1972). This approach utilizes one of the key hydrodynamic principles, mass balance, to trace a material through a system which has been segmented on a relatively coarse-grid. In the case of Saginaw Bay, the most appropriate substance is the chloride inflow from the Saginaw River. Chloride provides a continuous tracer over the entire year, is non-reactive, and can be conveniently measured at its sources and in the bay.

The coarse-grid approach provides sufficient detail to define problems within large time and space scales. This approach is appropriate for this study's consideration of the water quality problems that require definition over time periods of weeks to months and a space scale of 5 to 50 kilometers.

While the chloride mass load and the resulting chloride concentrations are known in the bay, the circulation parameters are unknowns. These parameters are determined through a simple iterative calibration and verification process using a computer solution to the mass balance formulation which requires less than a minute of computer time per run.

Basic Mass Balance Formulation

The basic mass balance equation is formulated as follows:

$$\frac{\partial c}{\partial t} = 0 = \frac{-\partial}{\partial x}(uc) - \frac{\partial}{\partial y}(vc) + \frac{\partial}{\partial x}\left(E_x \frac{\partial c)}{\partial x}\right. + \left(E_y \frac{\partial c)}{\partial y}\right. - K\,(x,y)C \qquad (1)$$

where:

u = velocity in x direction (L)
v = velocity in y direction (L)
E_x = dispersion coefficients in x direction (L^2/T)
E_y = dispersion coefficients in y direction (L^2/T)
c = concentration of substance (M/L^3)
K = decay rate of substance c (1/T)

The approach used to solve equation (1) is described by O'Connor and Thomann (1971) and summarized as follows:

The water body is divided into completely mixed segments, and a mass balance is defined for each by converting equation (1) to the form:

$$V_k \frac{dc}{dt} = 0\ \Sigma_j\ [Q_{kj}(\alpha_{kj}c_k + \beta_{kj}c_j) + E'_{kj}\,(c_j - c_k)] - V_k K_k c_k + W_k \qquad (2)$$

where:

V_k = volume of segment K (L^3)
E'_{kj} = bulk dispersion coefficient - $E_{kj}A_{kj}/\bar{L}$ (L^3/T)
$\bar{L}$ = average length of adjacent sections (L)
A_{kj} = cross sectional area between segments k and j (L^2)
α_{kj} = dimensionaless mixing coefficient; $\beta_{kj} = 1-\alpha_{kj}$
Q_{kj} = advective transport parameter (L^3/T)
W_k = source of c in mass units per time (M/T)

Units: L = length
T = time

M = mass

The parameters α and β are weighing factors which correct cases of adjacent segments of unequal length.

$$\alpha_{kj} \quad \frac{\overline{L}_j}{\overline{L}_j + L_k} \qquad (3)$$

where positive solutions are maintained by the stability criteria:

$$\alpha_{kj} > 1 - \frac{E'_{kj}}{Q_{kj}} \qquad (4)$$

Alternative solutions:

The system of simultaneous equations (one equation for each model segment) can be solved in two ways, steady-state ($\frac{dc}{dt} = 0$) or time variable ($\frac{dc}{dt} \neq 0$).

The steady-state approach is used to investigate average conditions throughout a defined time interval. All model inputs must be defined in terms of averages for this period. Model output, i.e., the concentrations in each model segment, also is defined in terms of a time average.

Model Application to Saginaw Bay

Whether the steady-state or time variable solution is used, the final circulation scheme is determined by using the same iterative calibration and final verification process. As shown in Figure 1, the model formulation requires:

(1) segmentation of the water body,
(2) determination of segment physical characteristics,
(3) measurement of chloride loads,
(4) measurement of chloride concentrations in most segments,
(5) specification of the circulation parameters.

Segmentation--

Specification of a particular segmentation scheme depends on several factors including:

(1) the desired spatial resolution,
(2) water quality gradients,
(3) available research time,
(4) available computer time.

Based on these considerations, Saginaw Bay was divided into 16 segments as shown in Figure 2.

Chloride Loads--

Chloride loads are obtained by field measurements of chloride concentration and of flows at all major sources. For Saginaw Bay, the primary source is the Saginaw River; however, loads from all tributaries and major point sources have been accounted for and are included as input into the appropriate model segment. The average chloride load in 1965 was 2.8 million kg/day

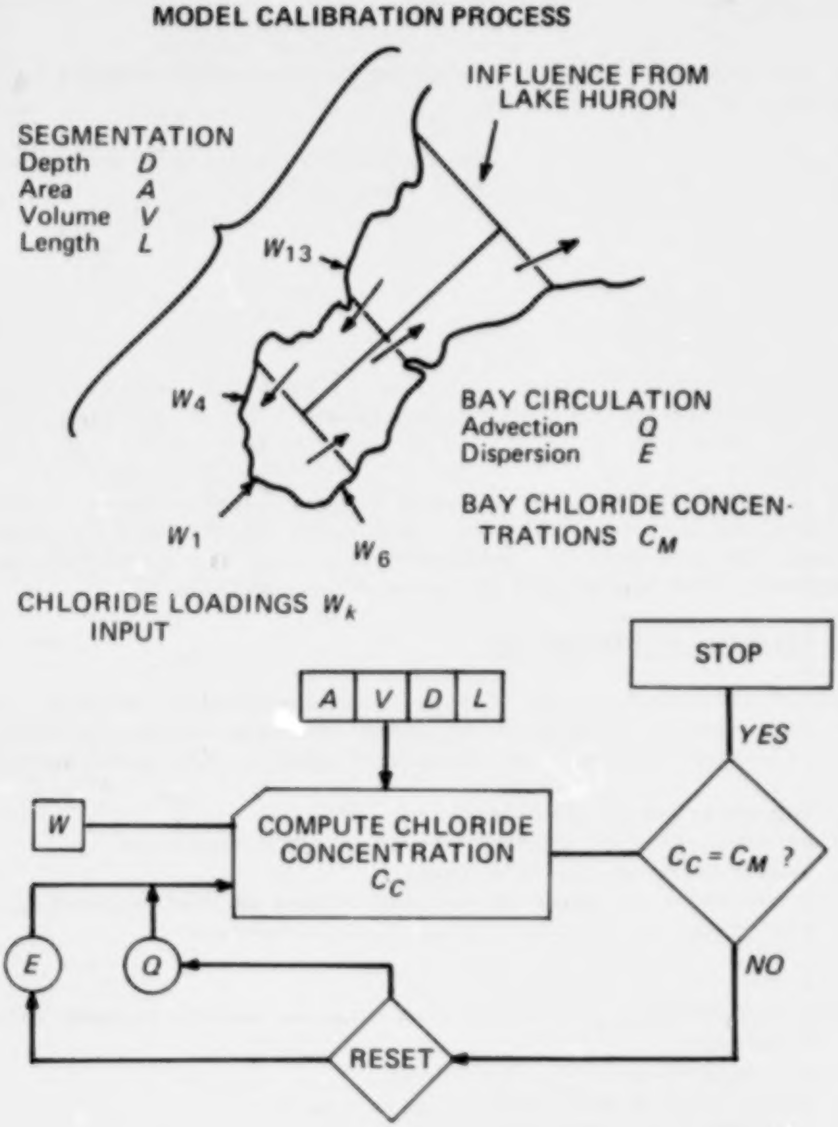
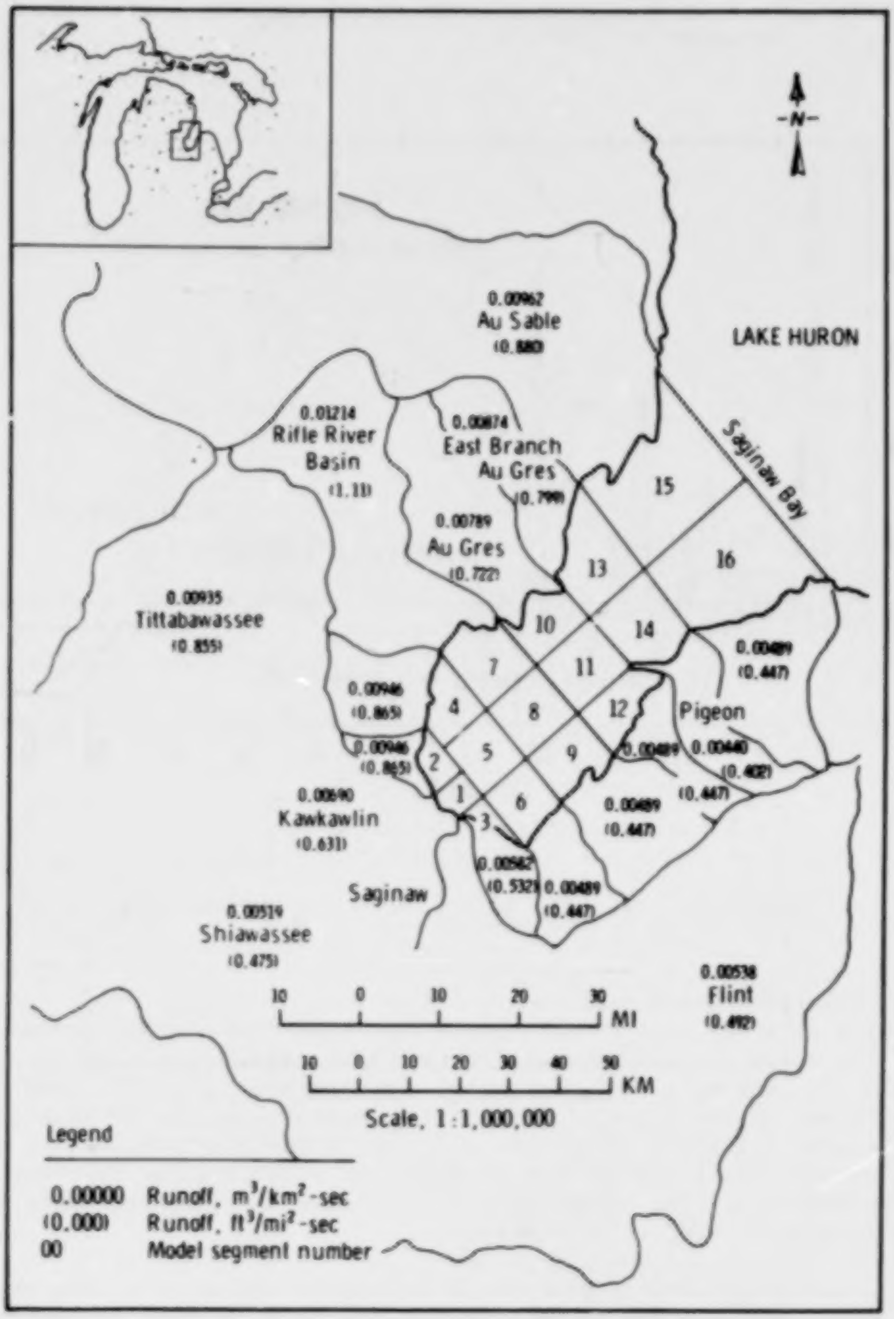

MODEL CALIBRATION PROCESS
INFLUENCE FROM LAKE HURON
SEGMENTATION
Depth D
Area A
Volume V
Length L
W_{13}
W_4
W_1
W_6
BAY CIRCULATION
Advection Q
Dispersion E
BAY CHLORIDE CONCENTRATIONS C_M
CHLORIDE LOADINGS W_k INPUT
STOP
A
V
D
L
YES
W
COMPUTE CHLORIDE CONCENTRATION C_C
$C_C = C_M$?
E
Q
NO
RESET

Figure 1. Model calibration process.



N
LAKE HURON
Saginaw Bay
0.00962
Au Sable
(0.880)
0.01214
Rifle River
Basin
(1.11)
0.00874
East Branch
Au Gres
(0.799)
0.00789
Au Gres
(0.722)
0.00935
Tittabawassee
(0.855)
15
13
16
10
14
7
11
4
8
12
2
5
9
1
6
3
0.00946
(0.865)
0.00946
(0.865)
0.00489
(0.447)
Pigeon
0.00489
0.00440
(0.402)
(0.447)
0.00489
(0.447)
0.00690
Kawkawlin
(0.631)
Saginaw
0.00582
(0.532)
0.00489
(0.447)
0.00519
Shiawassee
(0.475)
0.00538
Flint
(0.492)
10 0 10 20 30 MI
10 0 10 20 30 40 50 KM
Scale, 1:1,000,000
Legend
0.00000 Runoff, m³/km²-sec
(0.000) Runoff, ft³/mi²-sec
00 Model segment number

Figure 2. Saginaw Bay Drainage Basin indicating average 1965 runoff for model segment sub-basins.

(Richardson, 1974). By 1974, the average annual load had been reduced to 1.2 million kg/day, as shown in Figure 3.

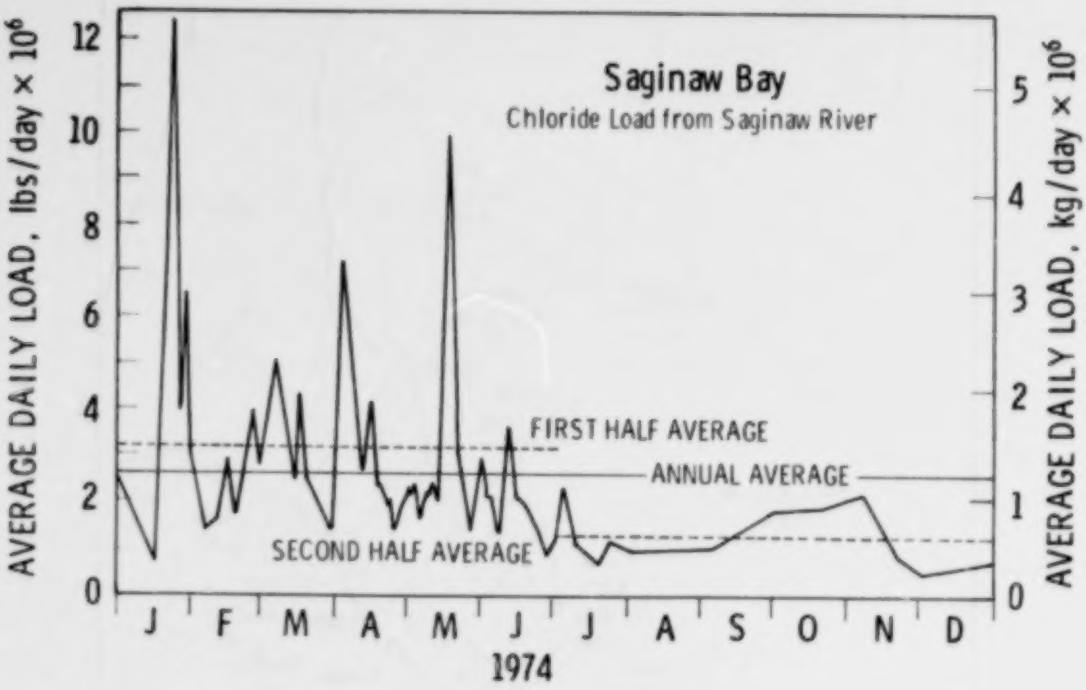


Figure 3. Chloride load in 1974 for Saginaw Bay.

Segmental chloride concentrations--

Results of historical chloride data have been reviewed by Freedman (1974). The specific data sources for this investigation include the 1965 FWPCA chloride data measured at about 20 sampling stations (USDI 1969) and the 1974 Cranbrook Institute of Science data measured at the 59 stations shown in Figure 4 (Smith, 1975). In addition to monthly-to-bimonthly cruise stations in 1974, four stations were sampled at water intakes on a one to three-day frequency. The cruise data are averaged on a segment basis for comparison to calculated concentrations.

The calibration process proceeds with initial estimates of the transport parameters E_{kj} and Q_{kj}, and an initial calculation of segment concentrations is made. The results are compared to the measured segment concentrations. Adjustments of E_{kj} and Q_{kj} are made, and the process is repeated until an acceptable comparison is obtained. Verification of the previously calibrated

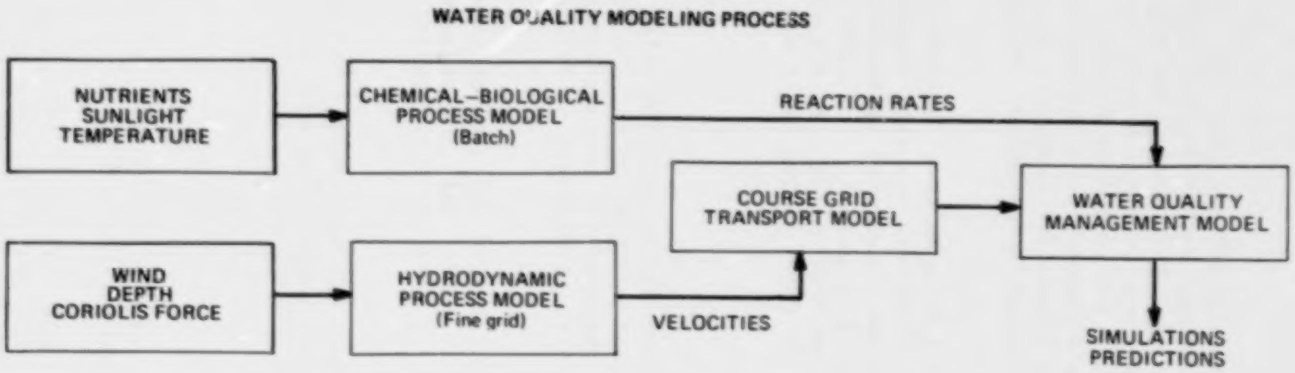

WATER QUALITY MODELING PROCESS
NUTRIENTS
SUNLIGHT
TEMPERATURE
CHEMICAL–BIOLOGICAL
PROCESS MODEL
(Batch)
REACTION RATES
WIND
DEPTH
CORIOLIS FORCE
HYDRODYNAMIC
PROCESS MODEL
(Fine grid)
VELOCITIES
COURSE GRID
TRANSPORT MODEL
WATER QUALITY
MANAGEMENT MODEL
SIMULATIONS
PREDICTIONS

Figure 4. Saginaw Bay 1974 sampling network.

circulation scheme is attained when a new set of load and concentration data is used, as above, and an acceptable comparison is obtained without adjustments.

Steady-State Calibration

This process was first used to test the hypothesis that the circulation in Saginaw Bay could be described by one average annual pattern (Richardson, 1974). The rationale was that circulation is highly random and closely related to meteorological events. These events, however, should average out over a period of time, within the time and space scales of interest to this study (i.e., weeks-to-months and 5-to-50 kilometers). The highly fluctuating and random events would be included in the dispersive transport parameters, E_{kj}. Any prevailing pattern (i.e., net circulation) would be included in the advective parameter Q_{kj}.

The calibration process was performed over a range of values for the transport parameters. The initial advective pattern was that suggested by Ayers (1956). The final scheme (Figure 5) was determined by matching computed and measured concentrations, as shown in Figure 6. For this final calibrated scheme, inflow from Lake Huron was 6400 m^3/sec and dispersion, 5 km^2/day, set at all interfaces, except for those in the outer bay which were set at 15.6 km^2/day.

Steady-State Verification

The verification process simply involves one calculation using the final advective and dispersive scheme from the 1965 calibration and the 1974 chloride load average of 1.2 million kg/day. The chloride concentrations computed in each segment were then compared to those measured in 1975 and the results plotted in Figure 7.

These results indicate that the model predicts the general decrease in chloride concentration one would intuitively expect with the reduced load; however, the spatial comparison significantly differs, most distinctly in the inner bay.

Further analyses conducted by Richardson (1974) verified the steady-state circulation pattern for the summer-fall period when most 1965 data were collected. The spring period, however, appears to be too dynamic for a steady-state analysis (Figures 8a and 8b).

Time Variable Analysis

Using a computer program to solve equation 1 with $\frac{dc}{dt} \neq 0$, a time-variable circulation pattern was calibrated (Richardson, 1974).

This was accomplished using time-variable chloride loads shown in Figure 9. A single circulation pattern used for the entire year with time variable loads resulted in the computed versus measured chloride time series (Figures 12 and 13). The initial (uncalibrated run) reasonably compares to the chloride levels in the summer-fall but was much too low during the spring. This

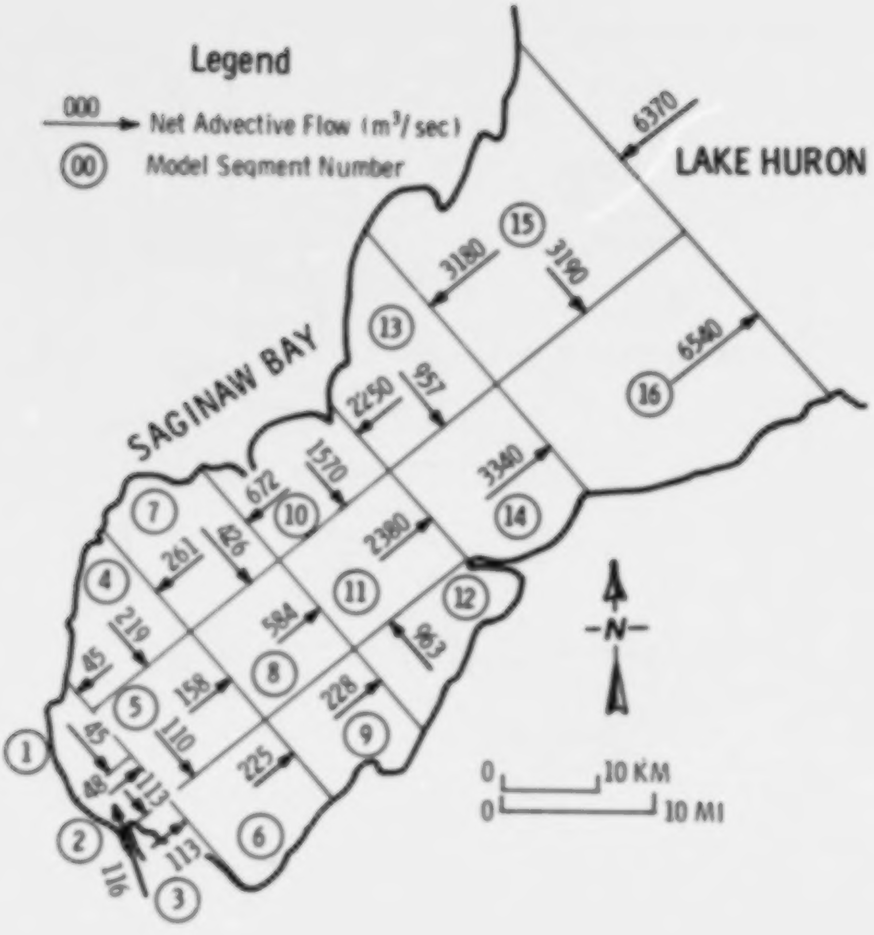

Legend
000 Net Advective Flow (m³/sec)
00 Model Segment Number
LAKE HURON
SAGINAW BAY
6370
3180
3190
6540
2250
957
1570
672
3340
2380
261
426
584
963
219
45
158
228
45
110
225
48
113
113
116
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
N
0
10 KM
0
10 MI

Figure 5. Calibrated average advective transport scheme for Saginaw Bay.

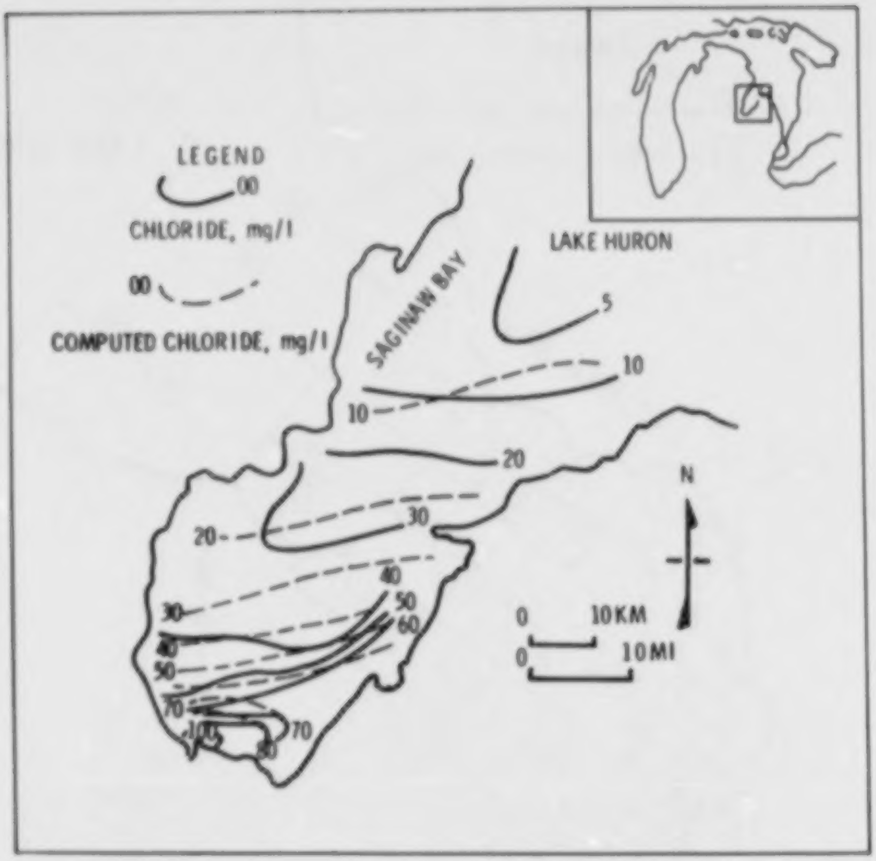

LEGEND
00
CHLORIDE, mg/l
00
COMPUTED CHLORIDE, mg/l
SAGINAW BAY
LAKE HURON
5
10
10
20
20
30
30
40
40
50
50
60
70
70
100
80
N
0
10KM
0
10MI

Figure 6. Saginaw Bay calibrated versus measured 1965 average chloride isopleths.

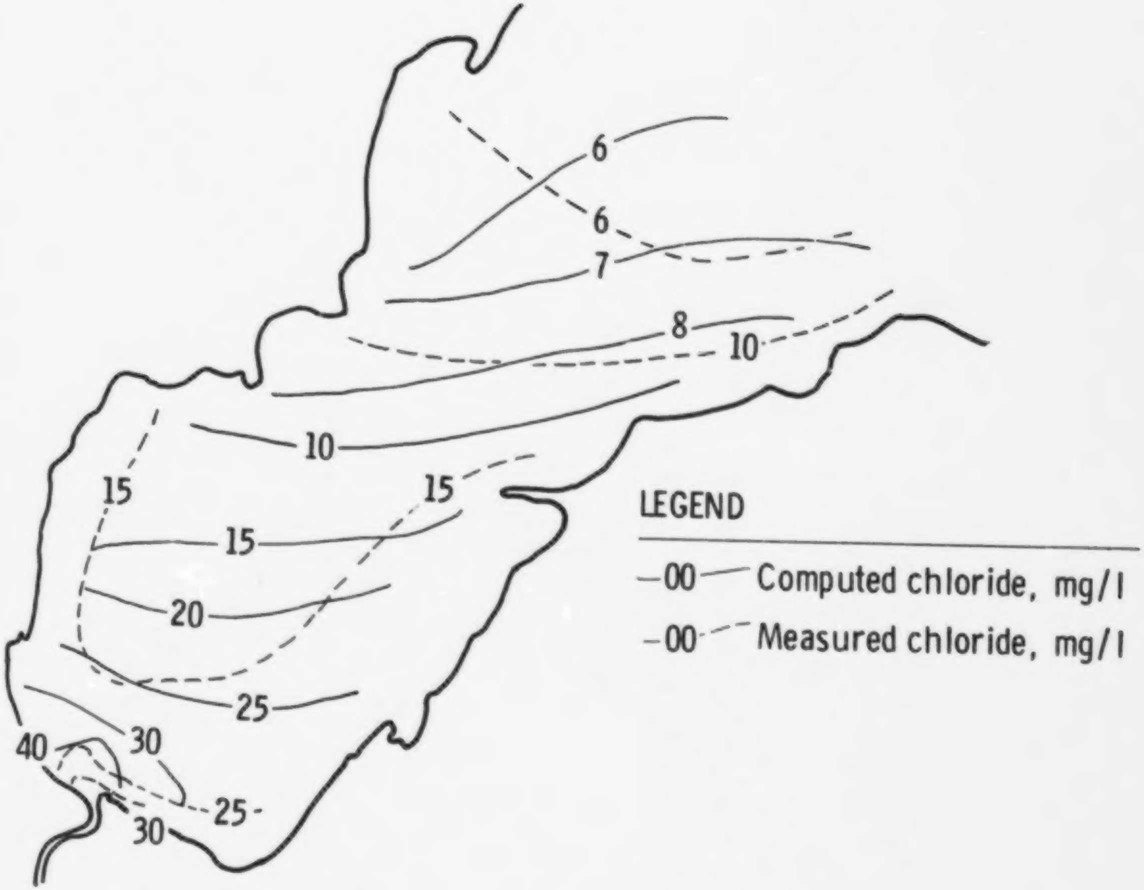

6
6
7
8
10
10
15
15
15
20
25
30
40
25
30
LEGEND
—00— Computed chloride, mg/l
-00- - Measured chloride, mg/l

Figure 7. Computed versus measured chloride isopleths for Saginaw Bay.

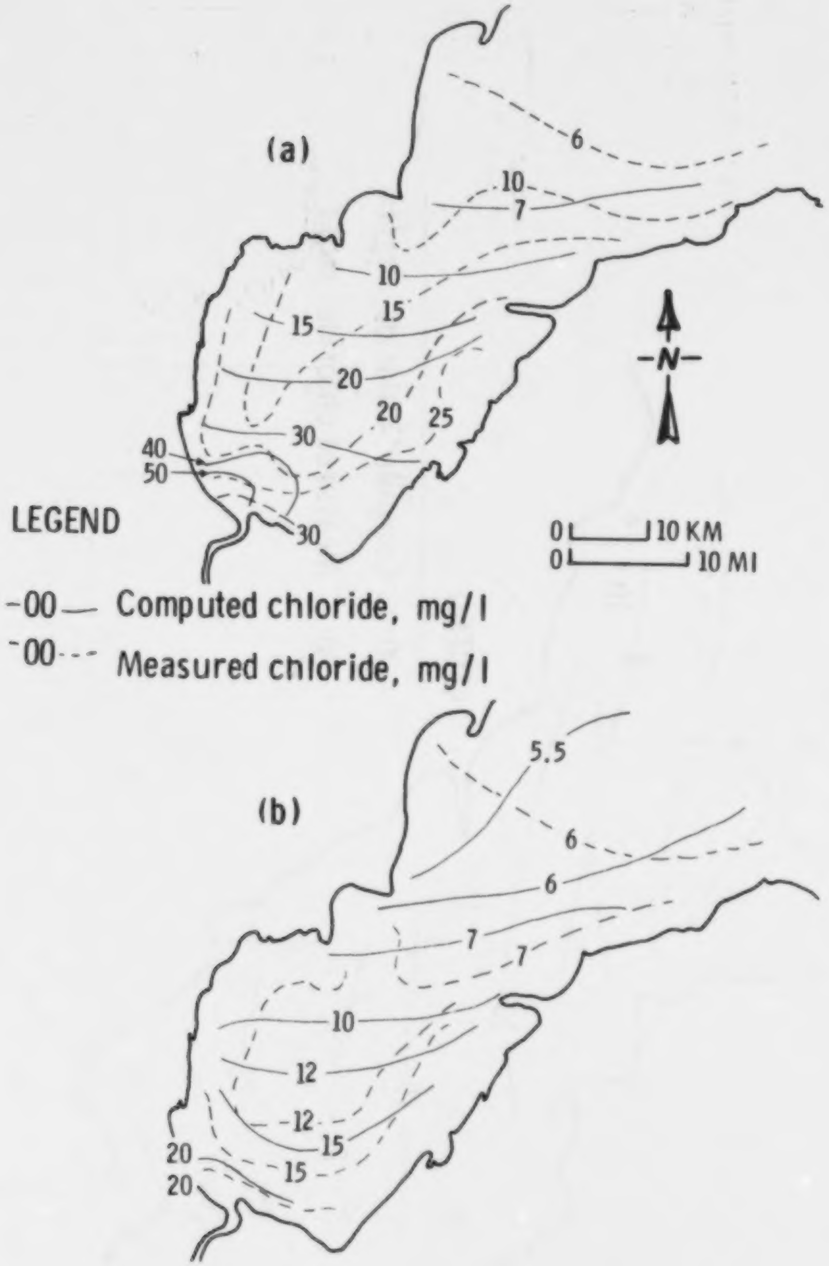

(a)
6
10
7
10
15
15
20
20
25
30
40
50
30
LEGEND
-00— Computed chloride, mg/l
-00--- Measured chloride, mg/l
N
0 10 KM
0 10 MI
(b)
5.5
6
6
7
7
10
12
12
15
15
20
20

Figure 8. Computed versus measured chloride isopleths
(a) March through June 1974
(b) July through November 1974.

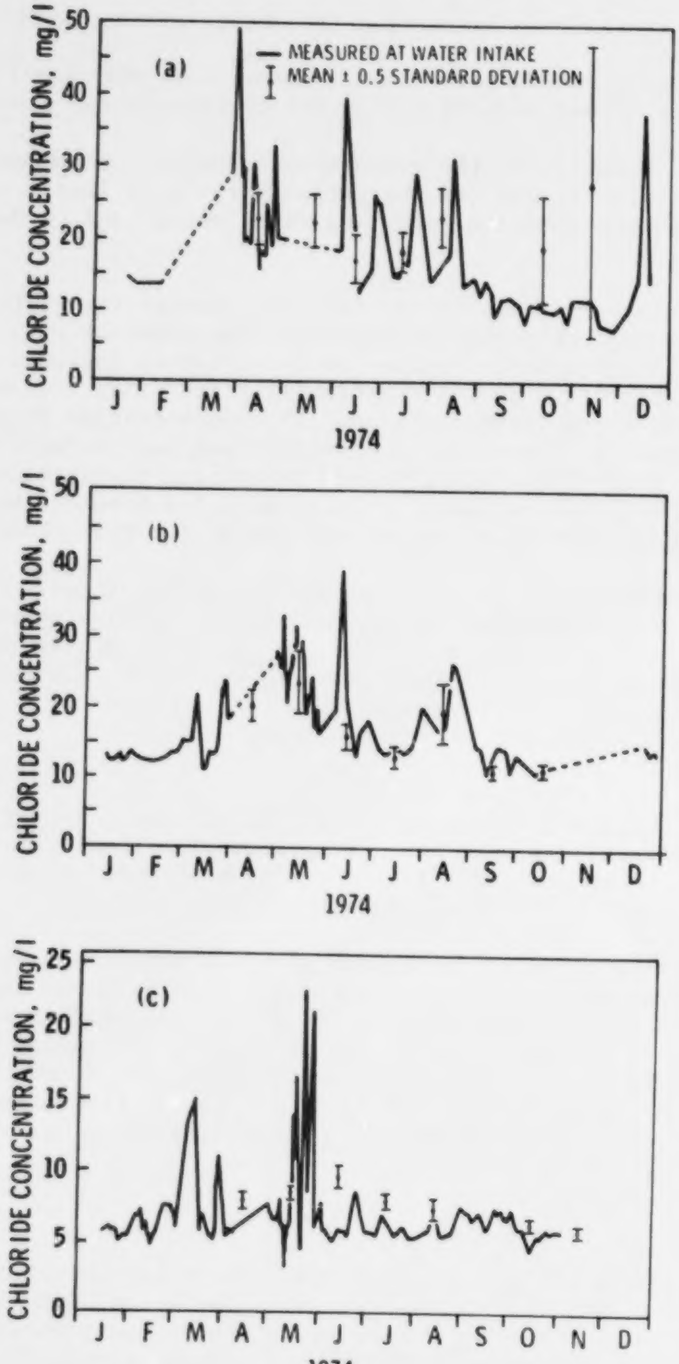


Figure 9. Chloride concentrations measured 1974 in Saginaw Bay model segments (a) segment 2; (b) segment 4; (c) segment 13.

substantiates the conclusions drawn from the steady-state analysis.

Since all chloride loads were accounted for, some additional force must be present during this period to effect a different circulation scheme.

For the next computation, the dispersion transport parameter E_{kj} was reduced at all interfaces by 50% for the period March 1 to June 1. The rationale for this change was that ice cover and floes would tend to dampen the dispersion transport.

This solution shows little change from the initial result for the outer bay segments. However, chloride concentration was substantially reduced in segments near the Saginaw River, while concentration in segments along the southeast shore (segments 3 and 9) substantially increased. In the segments along the northwest shore (segments 4 and 13), concentration decreased primarily in spring, when the dispersion coefficient had been reduced but when loads were high. This effect was contrary to initial expectations. It was apparent that a change was necessary in the advective scheme. However, some rationale or underlying physical reason was sought for this recalibration.

After consideration of all circulation influences, it was decided that only temperature would regularly influence circulation on a seasonal basis. It was suspected that a thermal bar formed at the mouth of the bay during the spring warming period. This hypothesis was reinforced by plotting some of the thermal structure during this period (Figure 10).

Final Calibration

Through the previously described iterative process, a newly calibrated time-variable circulation scheme was determined. The final time-variable model results are shown (Figure 11) with the general indication of the time periods when different circulation regimes are used.

The final spring circulation pattern was used between March 1 and May 20 and is characterized by reductions of advective flow from the outer to inner bay (segment 13 to 10) of approximately 75% of the advective transport, used for the remainder of the year, and of the dispersive transport, by a factor of 50%. From May 20 to June 1, the advective transport was returned to normal and the dispersion increased by a factor of 2. In an attempt to more accurately match chloride levels in the fall, dispersion again was reduced by 50% from October 1 to November 15.

Conclusions

A simple coarse-grid, mass-balance model is an economical and invaluable tool for deducing quantified circulation characteristics of Saginaw Bay, as well as a unique methodology for interpreting other limnological data (i.e., thermal bar effect). The average circulation scheme calibrated with 1965 data (Richardson, 1974) was verified as valid for most of the year, except for spring. Using a time-variable solution to the model equations, it was determined that a 75% reduction of advective transport from outer to inner bay must be included, as well as a 50% reduction in dispersion, to reasonably

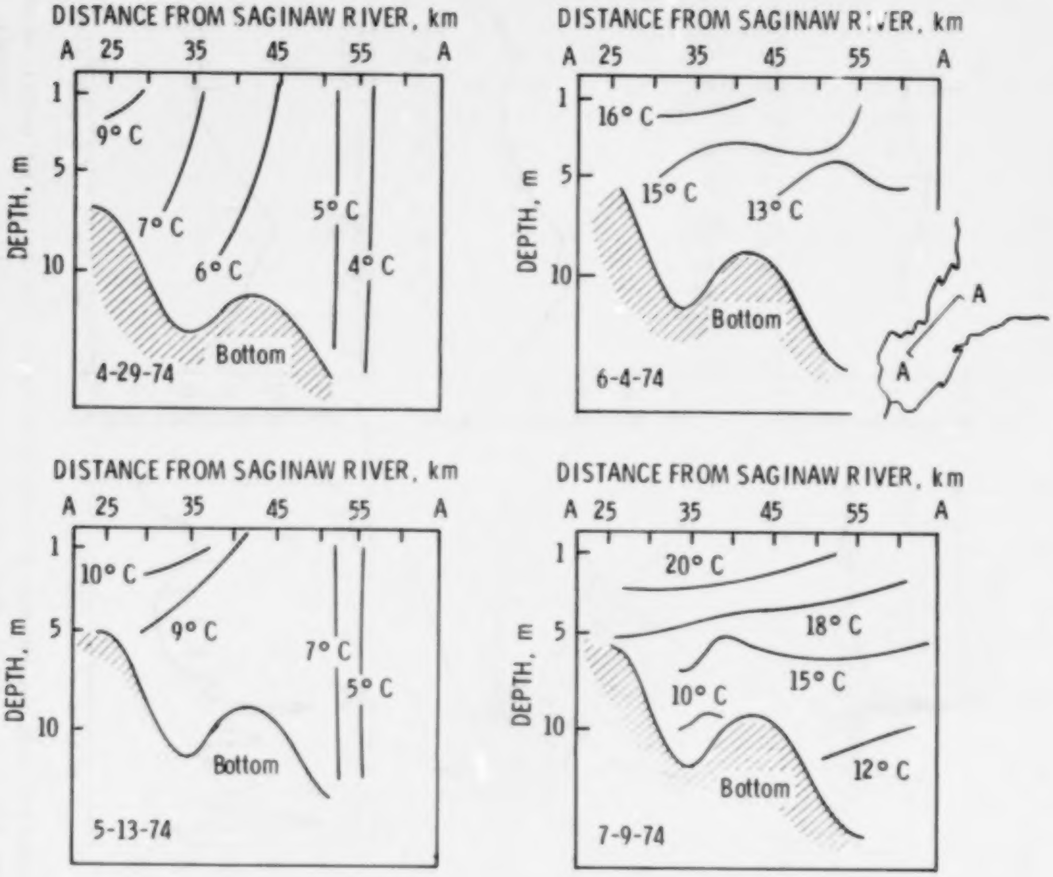


Figure 10. Saginaw Bay temperature profiles along transect A-A.

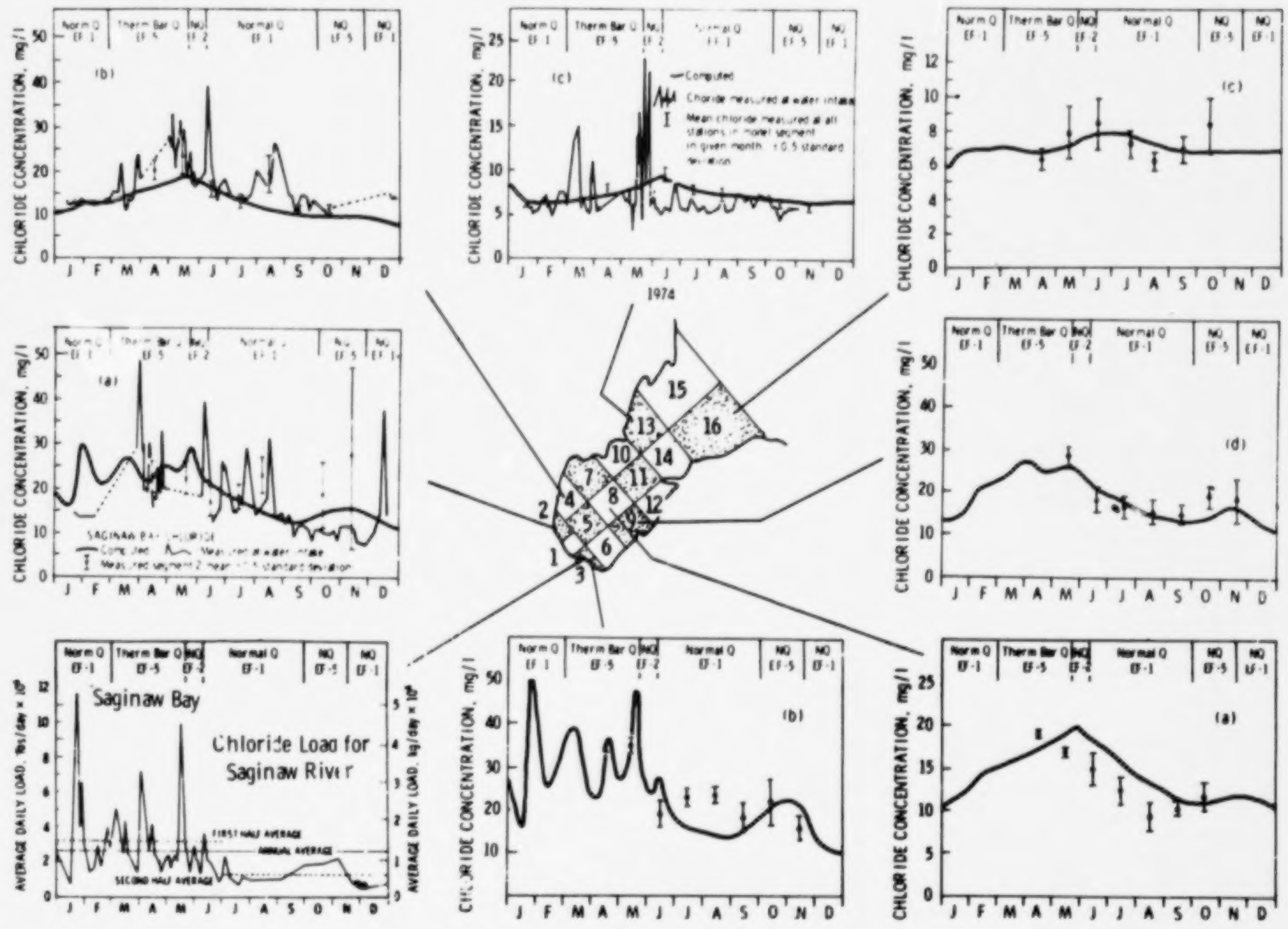


Figure 11. Final computed chloride concentrations vs measured concentrations for selected model segments with indicated circulation schemes (Therm Bar Q - period when advective scheme influenced by thermal bar; normal Q or NQ - period of normal advective scheme; EF-X adjustment factor to dispersion transport).

approximate chloride levels measured in spring. This reduction in transport is indicated by the development of a thermal bar structure which can form a vertical barrier to mass exchange between the nearshore and open lake. A reasonable agreement of computed versus measured chloride concentrations was obtained for most segments throughout the year. The circulation scheme is considered to be calibrated and the necessary verification will be the subject of future research. Further research will also be conducted using a fine-grid hydrodynamic model which should provide more insight into the finer physical processes involved. However, it is concluded that the presently calibrated circulation scheme is adequate to combine with the biological process model to form the first spatially segmented version of a nutrient-phytoplankton water quality management model.

CHEMICAL-BIOLOGICAL PROCESSES

Two basic approaches are being used in this study of Saginaw Bay. The first basically imitates the modeling approaches used for Lake Ontario by Thomann (this volume) who describes phytoplankton biomass as a measure of eutrophication and associated water quality. Further discussion is unnecessary here except to note that the phytoplankton biomass model has spatial definition.

The second approach does not include spatial detail but is a research-oriented process-model with biological and chemical detail. This multispecies phytoplankton model investigates the relative importance of various processes in providing competitive advantages for the development of nuisance algal growth.

MODEL CONCEPTS

The present version of this model evolved from earlier work which dealt initially with microbial substrate uptake kinetics (Verhoff and Sundaresan, 1972; Verhoff et al., 1973) and was expanded to include phytoplankton growth kinetics modeling (Bierman et al., 1973; Bierman, 1974). The compartments in the model are four phytoplankton, two zooplankton, and three nutrients (Figure 12). The four functional groups of phytoplankton were chosen primarily for water quality considerations and for the fact that phosphorus, nitrogen, and silicon are usually considered to be the major nutrients limiting algal growth. Carbon is not considered important in limiting either the growth rates or the maximum sizes of the algal crops in the simulations. Considerable support for this assumption can be found in the recent in situ work by Schindler et al. (1973a, 1973b) in the Canadian Shield and in the recent laboratory work by Goldman et al. (1974). Each nutrient is assumed to exist in two different forms: an available form which the phytoplankton can directly absorb and an unavailable form not directly assimilable. No distinction is made between the dissolved and particulate fractions of the unavailable forms. Two of the phytoplankton groups, diatoms and green algae, are assumed to be grazed by two zooplankton types, differentiated on the basis of their maximum ingestion rates. It is recognized that zooplankton grazing is a complex phenomenon and involves, among other processes, phytoplankton size-selectivity (McNaught, 1975). However, the present version of the model is only intended to investigate gross changes in grazing among the phytoplankton groups.

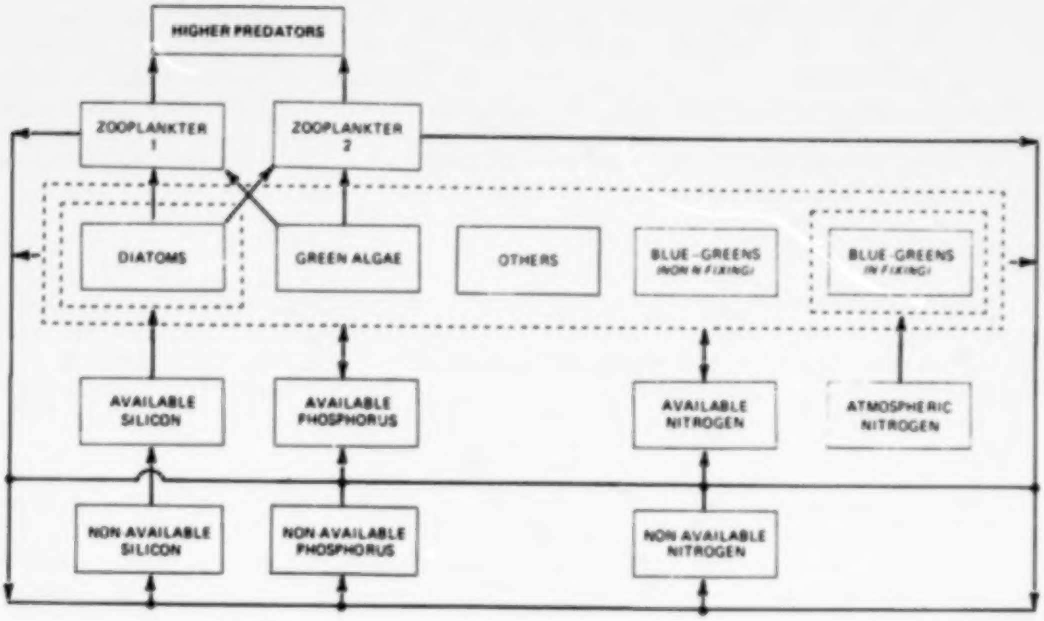
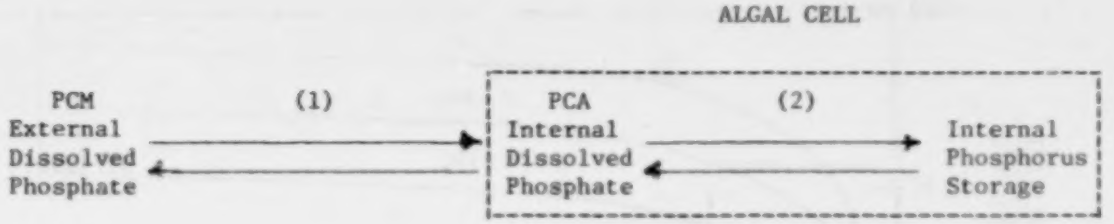

HIGHER PREDATORS
ZOOPLANKTER 1
ZOOPLANKTER 2
DIATOMS
GREEN ALGAE
OTHERS
BLUE-GREENS (NON N FIXING)
BLUE-GREENS (N FIXING)
AVAILABLE SILICON
AVAILABLE PHOSPHORUS
AVAILABLE NITROGEN
ATMOSPHERIC NITROGEN
NON AVAILABLE SILICON
NON AVAILABLE PHOSPHORUS
NON AVAILABLE NITROGEN

Figure 12. Principal compartments of the Saginaw Bay eutrophication model.

A unique feature of the model is that cell growth is considered to be a two-step process involving separate nutrient uptake and cell synthesis mechanisms. The motivation for this "variable stoichiometry" approach is that an increasingly large body of experimental evidence indicates that the mechanisms of nutrient uptake and cell growth are actually quite distinct (Dugdale, 1967; Fuhs, 1969, 1971; Droop, 1973; Azad and Borchardt, 1970; Caperon and Meyer, 1972a, 1972b; Eppley and Thomas, 1969; and Halmann and Stiller, 1974). The model includes carrier-mediated transport of phosphorus and nitrogen using a reaction-diffusion mechanism and possible intermediate storage in excess of a cell's immediate metabolic needs. Specific cell growth rates are assumed to be directly dependent on the intracellular levels of these nutrients, in contrast to the classical Michaelis-Menten approach which relates these rates directly to extracellular dissolved phosphorus.

Net specific phosphorus uptake rate is a response to the balance between extracellular- and intracellular-dissolved phosphorus:



(1) Net specific uptake rate =

$$\text{Maximum uptake rate} \cdot f(T),f(L)\left[\frac{1}{1 + (PK1)(PCA)} - \frac{1}{1 + (PK1)(PCM)}\right]$$

(2) $PCA = (PCAMIN)e^{(P/PO-1)}$

where PK1 = affinity constant between phosphate and an assumed membrane carrier
P = actual total phosphorus per cell
PO = minimum stoichiometric level of total phosphorus per cell
PCAMIN = minimum possible value of PCA
f(T),f(L) = temperature and light reduction factors, respectively.

An identical approach is used for nitrogen uptake kinetics (Bierman, 1974). The quality PK1 has actual physical significance because it is the equilibrium constant for the reaction between phosphate and an assumed membrane carrier molecule in the cell. Such a molecule has been isolated in the bacterium Escherichia coli, and its binding constant with phosphate has been measured (Medveczky and Rosenberg, 1970, 1971). Since insufficient knowledge of complex biochemical processes exists at this time, attempts to develop mechanistic theories of these processes usually lead to empirical assumptions at some point. Equation 2 relates actual intracellular phosphorus concentration, PCA, to a minimum value, PCAMIN, which is assumed to be a small number. This is not a rate equation, but is an empirical device to instantaneously calculate PCA as a function of the cell's total phosphorus storage. Intracellular phosphorus concentration must be determined in some manner so that feedback can be obtained.

A unique value for net specific uptake rate does not exist for a given concentration of phosphorus in solution. Instead, there is a family of values, each corresponding to a different level of intracellular phosphorus (Figure 13). Negative values for net specific uptake rate correspond to the phosphorus leakage back to solution that can occur under certain conditions. If the cells are assumed to be starved, that is, if P = PO, then the net specific uptake rate becomes half-maximum at 30 μg·P/liter in the example in Figure 13. Since the traditional Michaelis-Menten approach to nutrient uptake kinetics does not include a feedback mechanism, Michaelis-Menten kinetics is actually a special case of this theory which assumes the cell's nutritional state is constant. As PCA approaches PCAMIN in Equation 1, the first term in brackets approaches unity and the equation for net specific uptake rate reduces the Michaelis-Menten equation to the familiar hyperbolic form.

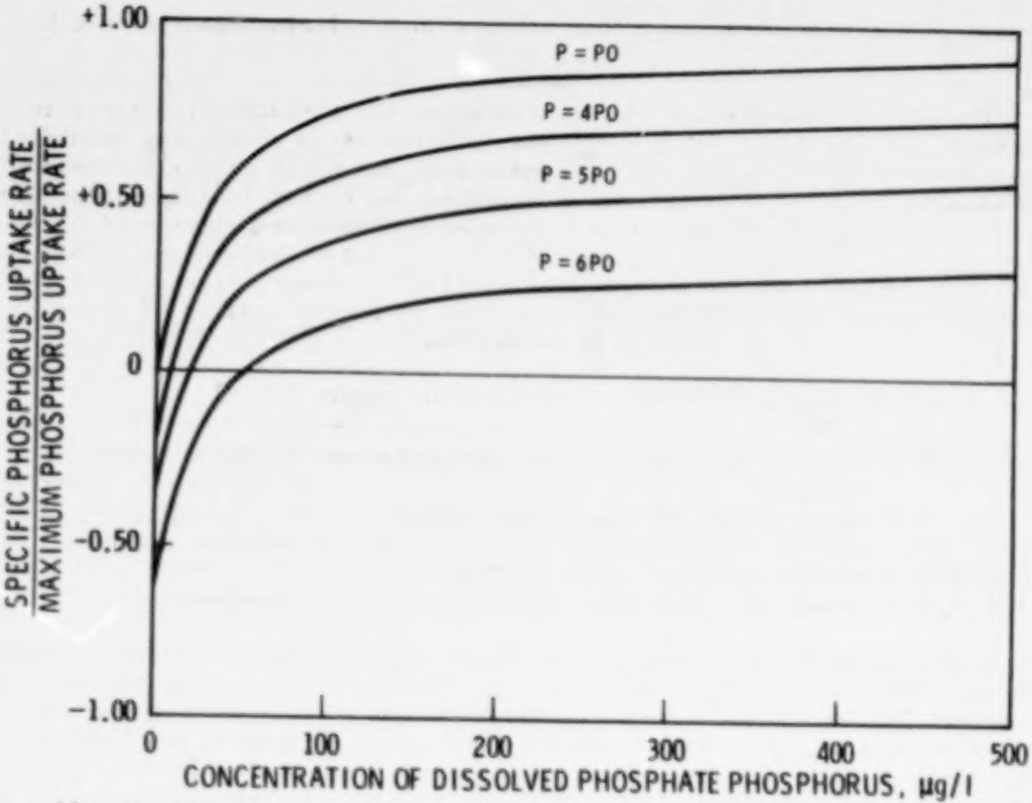


Figure 13. Degree of saturation of the phosphorus uptake system for the blue-green algae in the model as a function of both extracellular and intracellular phosphorus.

The two-step approach presented here is far from a rigorously correct treatment of phytoplankton kinetics. However, it is more realistic and consistent with the experimental evidence than the traditional single-step approach. Further, in contrast to empirical attempts to develop two-step approaches (e.g., Huff et al., 1973), the carrier-mediated mechanism used here is only one application of a more general substrate uptake theory (Verhoff et al., 1973). The latter provides a systematic methodology for investigating alternative and more sophisticated uptake mechanisms which can serve as a framework for future research.

Model Implementation

A major problem in implementing a complex chemical-biological process model is that such models usually contain coefficients for which direct measurements do not exist. It is possible that more than one set of model coefficients could produce an acceptable fit between the model output and a given data set. In the transition from single-class to multi-class models, this problem becomes particularly acute because it is no longer sufficient to ascertain a range of literature values for a given coefficient. Multi-class models necessitate the definition of class distinctions within this range.

Given such circumstances, many coefficients in multi-class models must be estimated.

The primary operational differences among the phytoplankton types in the model are summarized in Table I. Class differences in phosphorus uptake affinities are the least well established. Bush and Welch (1972) and Hammer (1964) have recorded strong circumstantial evidence that indicates blue-green algae have higher phosphorus uptake affinities than other classes of algae. Shapiro (1973) has reported that the phosphorus uptake mechanisms of blue-green algae reach half-saturation at significantly lower extracellular phosphorus concentrations than do the phosphorus uptake mechanisms of green algae over a wide range of environmental conditions.

TABLE I. OPERATIONAL DIFFERENCES AMONG PHYTOPLANKTON TYPES

Characteristic property	Phytoplankton type			
	Diatoms	Greens	Blue-Greens (non n-fixing)	Blue-Greens (n-fixing)
Nutrient requirements	Phosphorus nitrogen silicon	Phosphorus nitrogen	Phosphorus nitrogen	Phosphorus
Relative growth rates under optimum conditions at 25°C	High	Moderately High	Low	Low
Phosphorus uptake affinity	Low	Low	High	High
Sinking rate	High	High	Low	Low
Grazing pressure	High	High	None	None

The first step in the development of a comprehensive water quality model is to apply a spatially simplified version of the model to the system in question. The successful results of this application can then be incorporated in a spatially segmented version. This approach is used for the inner portion of Saginaw Bay which is assumed to be a completely mixed reactor.

Preliminary simulations are presented using only chlorophyll a. The model output describes the chlorophyll a pattern reasonably well in the spring and late summer-fall periods, but there is a significant discrepancy in June (Figure 14). Richardson (1975) has shown two distinct seasonal flow regimes in Saginaw Bay with a turbulent transition period in June. This suggests a discrepancy due in large part to the assumption of steady hydraulic flow between the inner and outer bay.

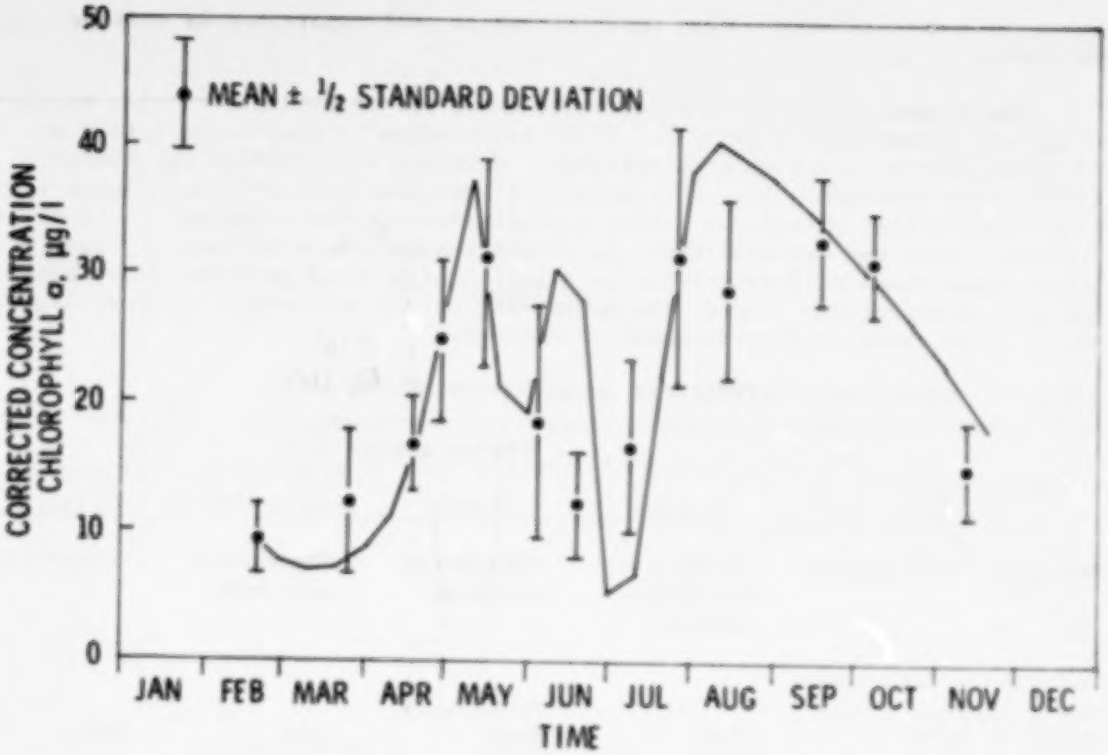


Figure 14. Seasonal chlorophyll a distribution for 1974 in Saginaw Bay, inner portion, as compared to model output (solid line).

Sensitivity Analyses

Approach--

The class composition of the Saginaw Bay model indicates that the early phytoplankton crops are dominated by diatoms and green algae and the broad summer-fall peak is dominated by blue-green algae (Figure 15). This successional pattern has been observed in the inner portion of the bay by Vollenweider et al. (1974). Chartrand (1973) has reported significant late summer crops of *Aphanizomenon*, a filamentous, blue-green algae, in the outer portion of the bay.

The development of multi-class phytoplankton models creates serious research problems due to their comprehensive data requirements. The approach adopted here is to perform sensitivity analyses of blue-green algae to establish the relative importance of several assumed competitive advantages. The coefficients used in the model give the blue-green algae higher phosphorus uptake affinities, lower sinking rates, and freedom from grazing, as compared to the diatoms and green algae. These advantages will be removed individually from the blue-green, and the resulting simulations will be compared to the assumed baseline case (Figure 15). Although the use of recycled phosphorus is not restricted exclusively to blue-green algae, the importance of this process will also be investigated.

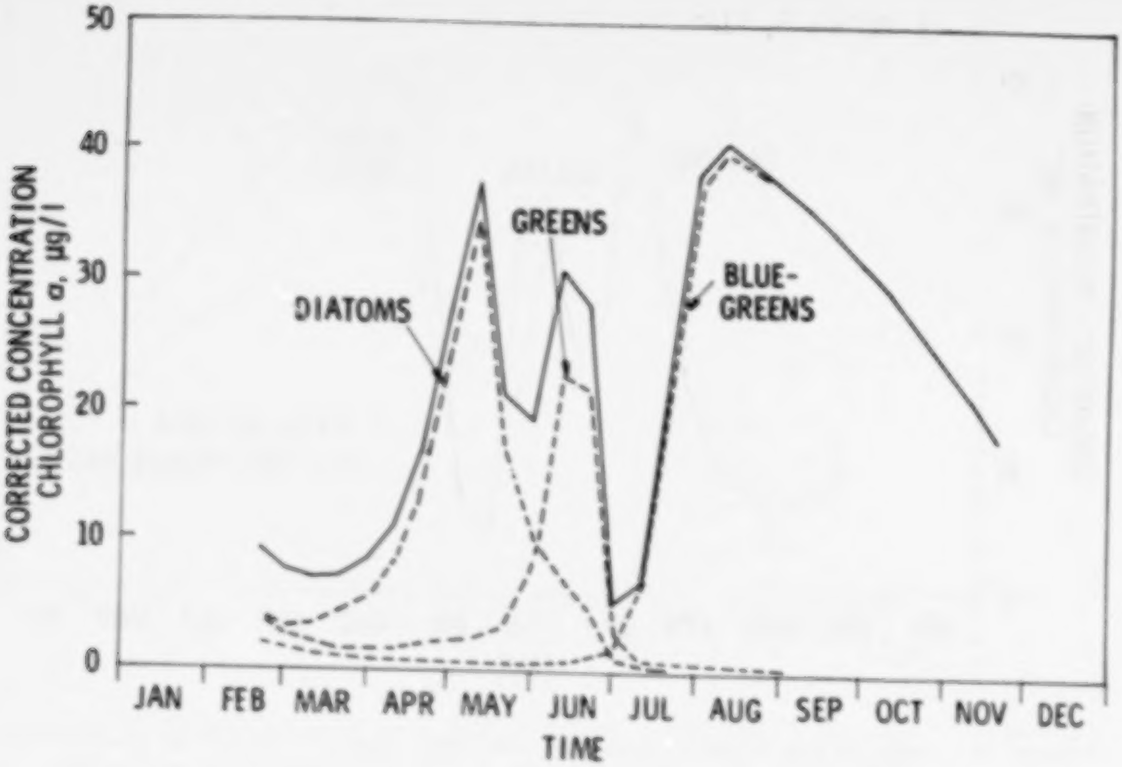


Figure 15. Phytoplankton class composition of model output. Baseline case for sensitivity analyses.

Effects due to class differences in nitrogen dynamics will not be investigated. In developing the model to this point, it is assumed that class differences in nitrogen dynamics are not as important as differences in phosphorus dynamics and that the primary mechanism for nitrogen competition is the ability of some blue-greens to fix atmospheric nitrogen (DePinto et al., 1975).

Results--

Phosphorus recycle occurs in the model as a result of phytoplankton death and zooplankton excretion (Figure 12). Since the phytoplankton cells are assumed to have a variable phosphorus stoichiometry, the recycle mechanism has two components: recycle of the cell's minimum stoichiometric phosphorus level to the unavailable pool and recycle of all cellular phosphorus above this minimum level directly to the available phosphorus pool. Such a two-component nutrient release upon cell death has been reported by Foree et al. (1970). If both phosphorus recycle components are "turned off," the early crops of diatoms and green algae are little affected, but the subsequent blue-green crop shows a significant decrease, compared to the baseline case (Figure 16).

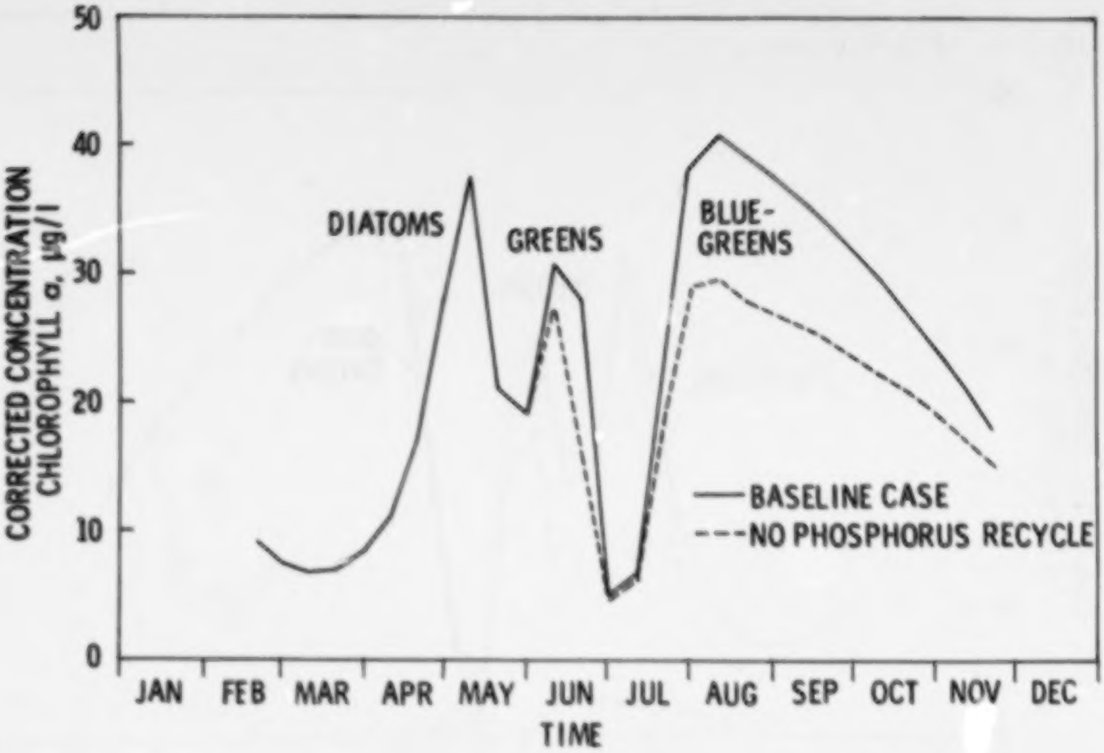


Figure 16. Comparison between baseline case and case for which no phosphorus is recycled upon phytoplankton death or zooplankton excretion.

The choice of coefficients used in the model for the phosphorus uptake mechanisms is based on the assumption that competitive differences primarily result from differences in phosphorus affinities, rather than in maximum phosphorus uptake rates. The actual situation is probably some combination of these factors. Maximum phosphorus uptake rates have simply been chosen in proportion to the minimum stoichiometric requirements of each of the four classes of algae. If the cells are phosphorus-starved, the uptake mechanisms of the diatoms and green algae reach half-saturation at approximately 60 µg·P/liter and the uptake mechanisms of the blue-greens reach half-saturation at approximately 30 µg·P/liter. The phosphorus affinity of the nonnitrogen-fixing blue-greens is set slightly higher than for the nitrogen-fixing blue-greens (Fitzgerald, 1969). This range of values is consistent with the data of Fuhs (1971), Healey (1973), and Rhee (1973).

When the phosphorus affinities of both blue-green algae are reduced so that their phosphorus uptake mechanisms half-saturate at 60 µg·P/liter, it causes a significant reduction in the late summer-fall total crop (Figure 17a) and also changes the class successional pattern. The reduction in the blue-green component of the total crop (Figure 17b) is more significant than apparent from Figure 17a because the diatom-green component develops an extra peak in August (Figure 17c).

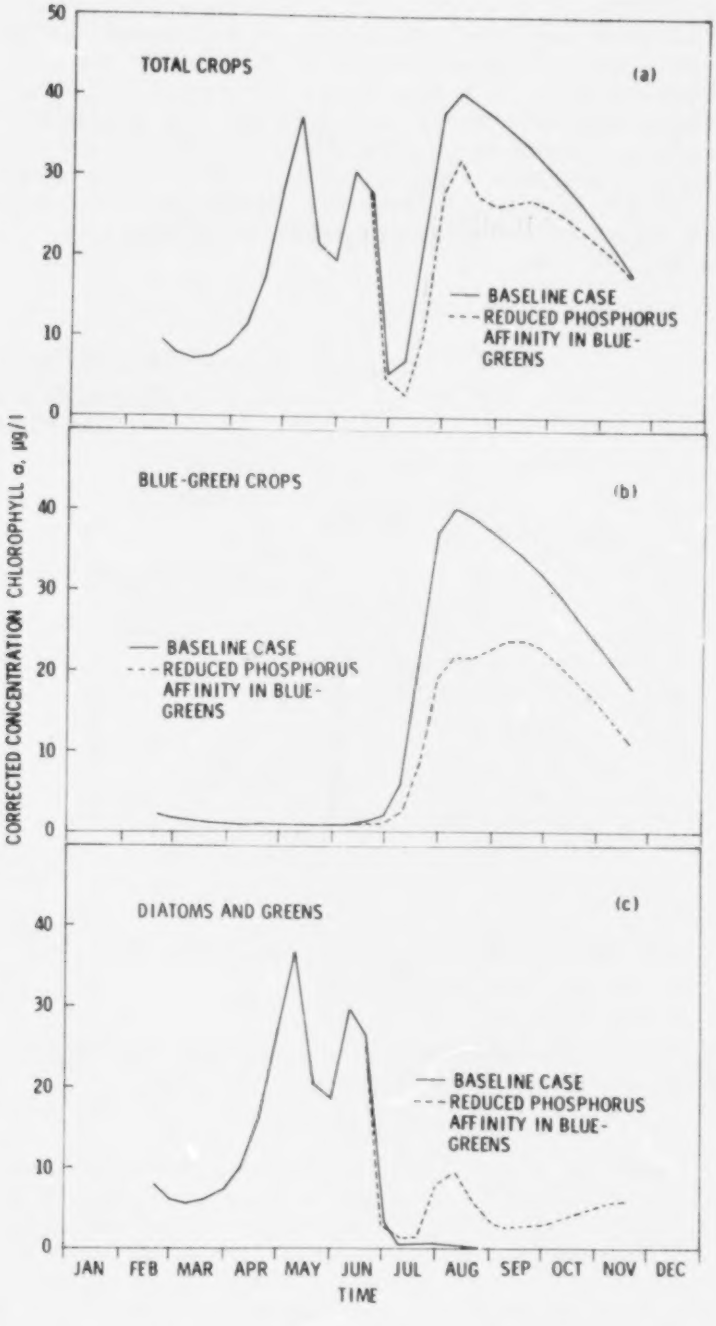


Figure 17. (a) Comparison of total crops between baseline case and case for which phosphorus uptake affinity has been reduced in the blue-green algae; (b) comparison of the blue-green components of the total crops for cases in (a); (c) comparison of the non-blue-green components of the total crops for cases in (a).

The sinking rate for the diatoms and green algae is 0.40 meters/day. This value is consistent with the laboratory data of Smayda (1974) and with the range of sinking rates used by Thomann et al. (1975) for simulations of Lake Ontario phytoplankton. Blue-greens are arbitrarily sunk at 0.15 meters/day. If both of the blue-greens are also sunk at 0.40 meters/day, their contribution to the total crop becomes completely negligible and diatoms and green algae dominate the simulations throughout the growing season (Figure 18). Similarly, if all four algal types are grazed in the same manner as the diatoms and green algae, the blue-green component of the total crop again becomes negligible (Figure 19).

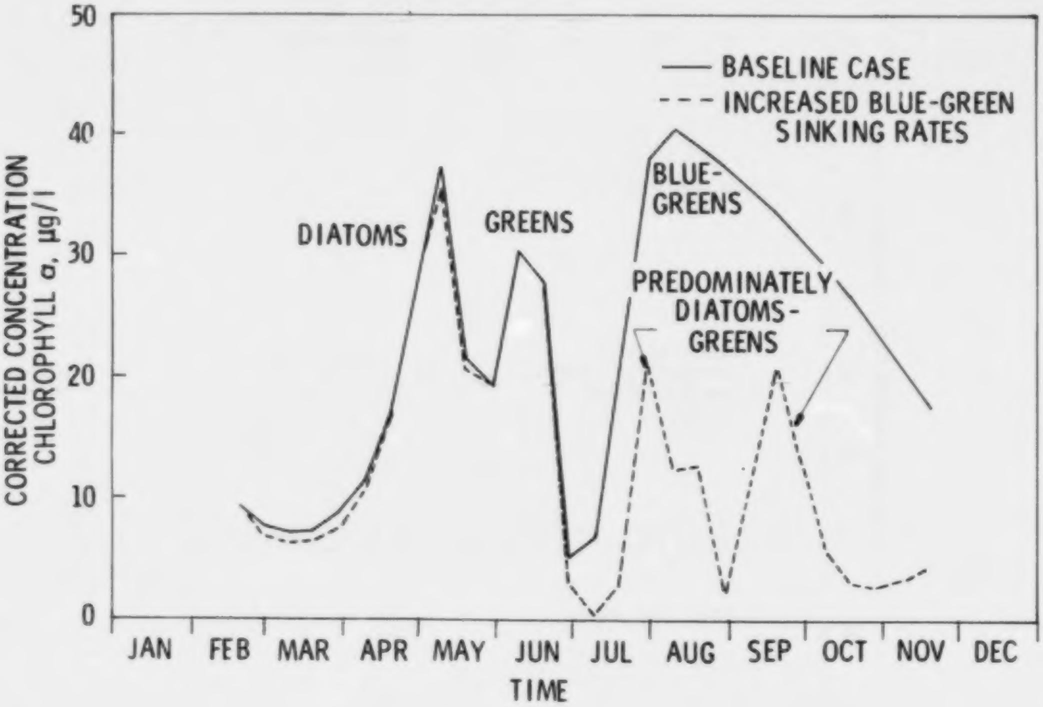


Figure 18. Comparison between baseline case and case for which blue-green sinking rates have been increased.

Discussion--

A general picture emerges: Blue-green algae, although their growth rates are slower than other groups of algae and tend to occur at the end of the typical successional pattern in eutrophic environments, are nonetheless armed with a variety of competitive advantages which can lead to their dominance under conditions of nutrient enrichment. Freedom from grazing and lower sinking rates were the most important of the competitive advantages considered here, although more subtle effects involving phosphorus dynamics were also significant in certain circumstances. The conclusions presented, however, must be qualified by noting that they only apply to the ranges of coefficients and to the particular structure of the model.

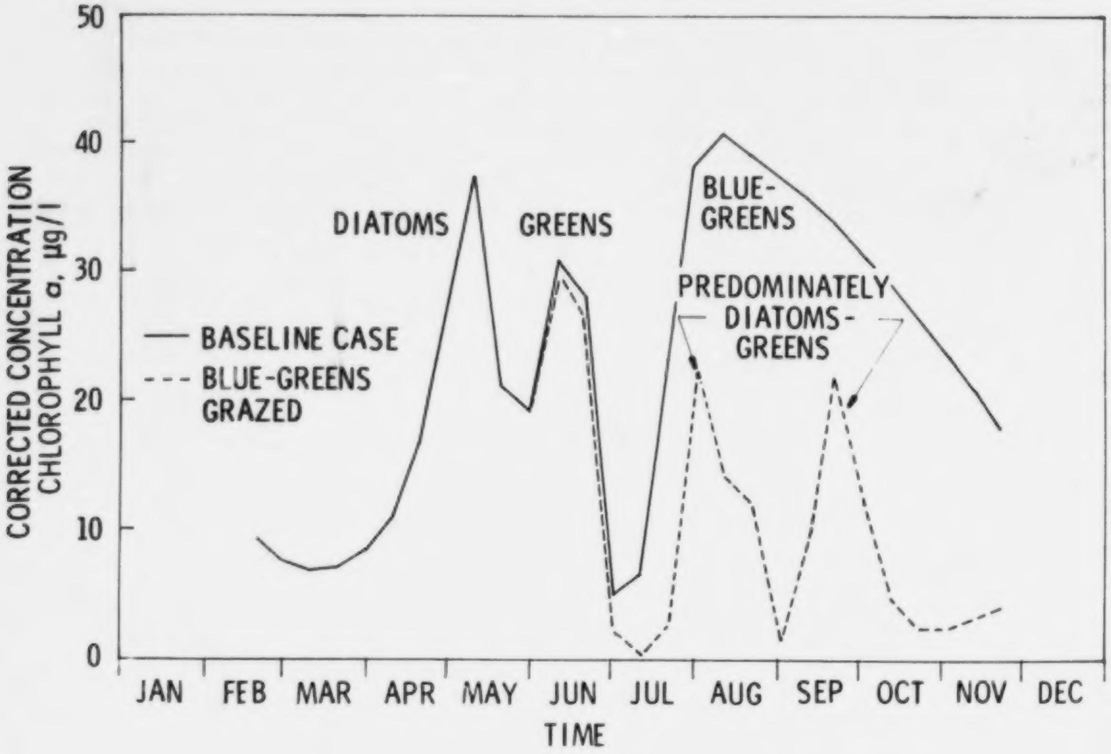


Figure 19. Comparison between baseline case and case for which blue-green algae are subject to the same grazing pressure as the diatoms and green algae.

Phosphorus recycle was shown to be an important factor in the development of late-growing blue-green algae, even in the Saginaw Bay system where there are continuous sources of external nutrient supply. Since little quantitative information is available on the mechanisms of phytoplankton decay, zooplankton excretion, and the role of bacteria in nutrient recycle, this area deserves more research attention. Other studies have also identified the importance of nutrient recycle (Thomann et al., 1975; DePinto et al., 1975). Gunnison and Alexander (1975) have suggested that blue-green algae, as a group, are more susceptible to bacterial decomposition than other algal types. A complete treatment of phytoplankton growth and nutrient recycle may even necessitate the inclusion of heterotrophic activity as a separate state variable in systems models.

The simulations which compared the effects of changes in phosphorus affinities between algal types, even though based on incomplete information, showed that these differences could be important for both total crops and class successional patterns.

Relatively small differences in sinking rates between blue-green algae and non-blue-green algae showed significant effects in the simulations. Modelers frequently think in terms of order-of-magnitude differences, but when blue-green sinking rates were reduced by less than a factor of three in the simulations, previously large blue-green crops were reduced to almost negligible sizes. Unfortunately sinking rates are difficult to measure accurately in the natural environment, but laboratory studies with a variety of algal types under different sets of conditions should at least provide useful comparative data.

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DISCUSSION

DeLUCIA: I would like to have Dr. Davies return to his first statements describing the undesirable phenomena and relate them to biological and chemical characteristics.

DAVIES: The model I am describing is a research tool at this stage of development. It is not yet ready to be used for management purposes; it is not sufficiently understood. In Saginaw Bay, the most apparent problem is the algal succession, which leads to problems of taste and odor in the water supply. Saginaw Bay also has a series of other problems connected with toxic substances, but these are only under study.

MORTIMER: My question relates to the circulation model. What measures were taken to make observations in the open part of the water body?

DAVIES: In 1974, EPA expended considerable resources establishing current measurement stations in a series of transects in the bay. Many measurements were taken throughout the year at the mouth of the bay and in Lake Huron. In conjunction with our studies at Saginaw Bay, we conducted a series of research studies and water quality monitoring studies in the southern part of the lake, and the Canadians studied waters in the northern part.

VASILYEV: In the report, there is a mass balance equation which includes dispersion terms. There is no clarification in the relation of details connected with calculating the dispersion coefficients. Since in this case flows are caused by wind influence, the velocity profiles must be rather complicated. I would like to know how you evaluated the dispersion coefficients in such conditions.

THOMANN: We tried to distinguish averaged transport conditions. The essence of the approach is that when possible, we calculate active circulation to determine the dispersion coefficient.

LOZANSKIY: What practical work has already been done on the basis of models; what can be done in the future to manage water quality; that is, what kind of practical actions will be realized to reach the goal set for water protection? What effect, and I also mean economic effect, can result when the model has been verified?

DAVIES: At present, this model is being examined as a research model and does not yet have a full application. However, the present research will be included in a report to the International Joint Commission, but will not be used until verified to project the bay's response to changes in nutrient loadings.

ZNAMENSKIY: I would like to note two interesting aspects in the model presented by Dr. Davies. First, in the complexity of the model, dynamic chemical and biological parts are considered and the fact that it is a research model does not decrease its value. Second, a good method used in the model, namely use of a conservative substance, makes it possible to calibrate the model.

YEREMENKO: Questions of forecasting flow and questions of forecasting phytoplankton are based practically on those developments reported by Dr. Thomann. Several findings are characteristic of the work. First, the hydrodynamics of flow are not considered in the usual way. Balance ratios are taken, where concentration, dispersion, and convective transport are considered. Since the equations are written factually for a finite sector, we do not have any derivatives here, particularly if this equation is for a stationary flow; we simply have an algebraic equation. Having collected experimental data on the concentration, we can write a necessary system of closed equations and thus determine the coefficients, not yet obtained, which interest us. Hydrodynamics usually give us this data.

In reference to this paper, chlorides are used as an indicator, and they are suitable for determining marked parameters. The fact that not one but four types of phytoplankton are examined here is also a positive note. Likewise, the two-step model is interesting. We understand it now, after a corresponding explanation. The report presented indicates a good future for developing ideas that have been promulgated in the U.S., and can be useful in that area of science in which we are working.

MORTIMER: It is necessary to acknowledge the necessary development of three circulation models for three periods of the year--characterized by stratification in the lake, no stratification in the lake, and a thermocline. Bierman's model imitates the realistic introduction of phosphorus and the fact that all the algae are not examined in one lump, but as subclasses.

THOMANN: I would like to emphasize that these models must be verified. Only when their reliability is established will it be possible to apply the models to a practical situation. We would like to see more diagrams comparing the data obtained with practical results. This is especially important for the transfer to a large number of phytoplankton species. A good data base is necessary in order to test the multitype model.

If there is information on the flow velocity, then we always use it instead of building a hydrodynamic model. The shortage of adequate models for verifying hydrodynamics is disquieting. The basic model structure, when accepted as valid, provides a basis for administrative decisions.

DAVIES: I would like to reinforce Dr. Thomann's statement that verification is the key to acceptance of the program. One of our scientific duties and responsibilities is to decide when to recommend using specific models as aids in decision-making.

DESCRIPTIVE SIMULATION MODEL FOR FORECASTING THE CONDITION OF A WATER ECOSYSTEM

A.B. Gorstko

Highly sophisticated computers are required to forecast the consequences of human activity on aqueous ecological systems. After developing a mathematical model, scientifically valid recommendations can be developed to help prevent these adverse consequences.

The development of a mathematical model depends upon information available on processes occurring in the ecosystem under study. Model selection is based essentially on the volume of accumulated information. For unique water bodies where the ecosystem has been studied for dozens of years (the Azov Sea and Lake Baikal, etc.), the most appropriate model is the so-called descriptive simulation system. These model systems fulfill certain general conditions that must be satisfied by any acceptable model of the ecosystem used for forecasting.

The model must fulfill the following requirements:

1. be derived from a set of submodels individually verified before inclusion in the ecosystem model;
2. include and accommodate the results of field and laboratory experimental work;
3. contain a large number of state variables, their functions (often non-linear) and their limitations;
4. include consideration of the possible extreme values of many parameters;
5. be developed as a complex set of computer programs.

One of the disadvantages of the descriptive model is that it must be reconstructed for each unique objective. Fortunately few unique objectives require the construction of special mathematical models.

The simulation model of the Azov Sea, developed at the Scientific Research Institute of Mechanics and Applied Mathematics, is described as an example.

The Azov Sea is subdivided into seven segments (Figure 1). The ecosystem in each segment is described by a state vector, the components of which are concentrations of different substances in water and the biomass of specific animals and plants. It is assumed that each segment is completely uniform with respect to these parameters. Conditions in the river estuaries and in the Kerchenskiy Strait zone are modeled separately, and are exogenous factors for the Azov Sea. The dimension of the state vector is equal to 120, the distribution of the components is listed below:

1-20 concentration of the chemical elements;
21-22 bacteria biomass;
23-34 biomass of the basic types of phytoplankton;
35-46 biomass of the basic types of zooplankton;
47-61 biomass of the basic types of benthos;
62-113 biomass of the basic types of fish, subdivided into age groups; and
114-120 reserved components.

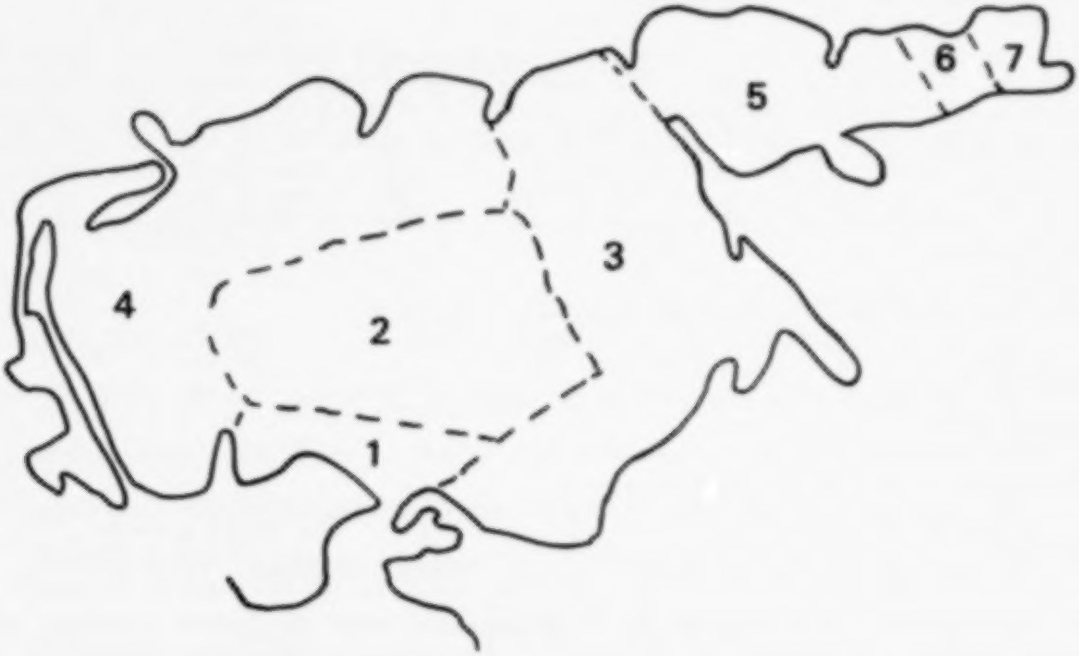


Figure 1. Diagram of the division of the Azov Sea into regions.

The components change with time (the time is assumed to be discrete with a time step of five days) under the influence of various physical, chemical, and biological processes. It is expedient to separate the Azov Sea model into partial models to sufficiently describe similar processes and phenomena. These submodels will be referred to as: 1) mixing, 2) biogenous elements, 3) polluting substances, 4) phytoplankton, 5) zooplankton, 6) benthos, and 7) fish. The fish submodel in fact consists of eight models, one for each main species.

In the mixing model, the volumes of water exchanged during respective five-day intervals between the open-sea segments are calculated according to given external conditions, wind velocity and direction, water exchange with the Black Sea, and the flow of the Don and the Cuban rivers. This overflow determines the exchange of the parameters 1-46.

The biogenous parameter model describes the nitrogen, phosphorus, and silicon cycles, and the processes of decomposition, transfer, removal, and consumption. Temperature and insolation are the very significant parameters affecting this submodel.

The polluting substances submodel determines concentration changes during the five-day time step. In situ decomposition processes and transport into the segment of pollutants are included in the submodel; the controlling factors are similar to those in the preceding submodel.

The phytoplankton, zooplankton, benthos, and fish submodels include feeding and predation, respiration, mortality, reproduction, and migration from the segment. The fish submodels also include the impact of fisheries on the populations.

The processes modeled in each submodel act at different rates under the influence of external factors. Those considered in the Sea of Azov are: 1) flow from the Don River, 2) flow from the Cuban River, 3) water exchange with the Black Sea, 4) insolation, 5) wind velocity and direction, 6) water temperature.

These factors are external to the Azov Sea (not influenced by the sea) and are modeled separately.

In the submodels for mixing, biogenous elements, and polluting substances, the normal mass balance relationships use finite or differential forms. In contrast, the submodels for phytoplankton, zooplankton, benthos, and fish are modeling algorithms based on a complex set of observational data. The data set determines the dynamics of the specific trophic level relative to external and internal variables in the system. These algorithms are developed as computer programs.

The Sea of Azov model operates in the following manner: if the state vector of the ecosystem X_o is known at time T_o, the state vector X_1 can be calculated at T_o+5 days after one time step. It can take one of two possible courses: a consecutive course of the process, or a consecutive and parallel course.

SYSTEM A

1. The external factors for the time interval (T_o,T_1) where $T_1=T_o+5$ days were modeled. For those presumed random factors, this is accomplished by extrapolating respective time series. The following procedure assumes a series of observations of a certain random value $[S(t_i)]$ at the intervals $t_1,t_2,\ldots,t_N$. These values can be approximated by means of the polynomial:

$$X(t) = \sum_{i=0}^{n} a_i t^i + \sum_{j=0}^{m} b_j \cos (2\pi f_j t + \eta_j),$$

where $X(t)$ is the value of the polynomial at the moment t;
a_i are the coefficients of the approximating polynomial;
n is the power of the polynomial; and
b_j is the amplitude of the cosine harmonics with an f_j frequency and an η_j phase.

The polynomial coefficients $X(t)$ are selected from the condition of functional minimum

$$F = \sum_{i=1}^{N} [S(t_i) - X(t_i)]^2$$

Knowing the polynomial $X(t)$, it is possible to approximate the desired value $S(T_1) = S(T_{N+1})$ with the value $X(t_{N+1})$.

Non-random external factors, (e.g., water exchange with the Black Sea) are either calculated, or their forecasted values for the period of (T_o,T_1) are introduced into the model.

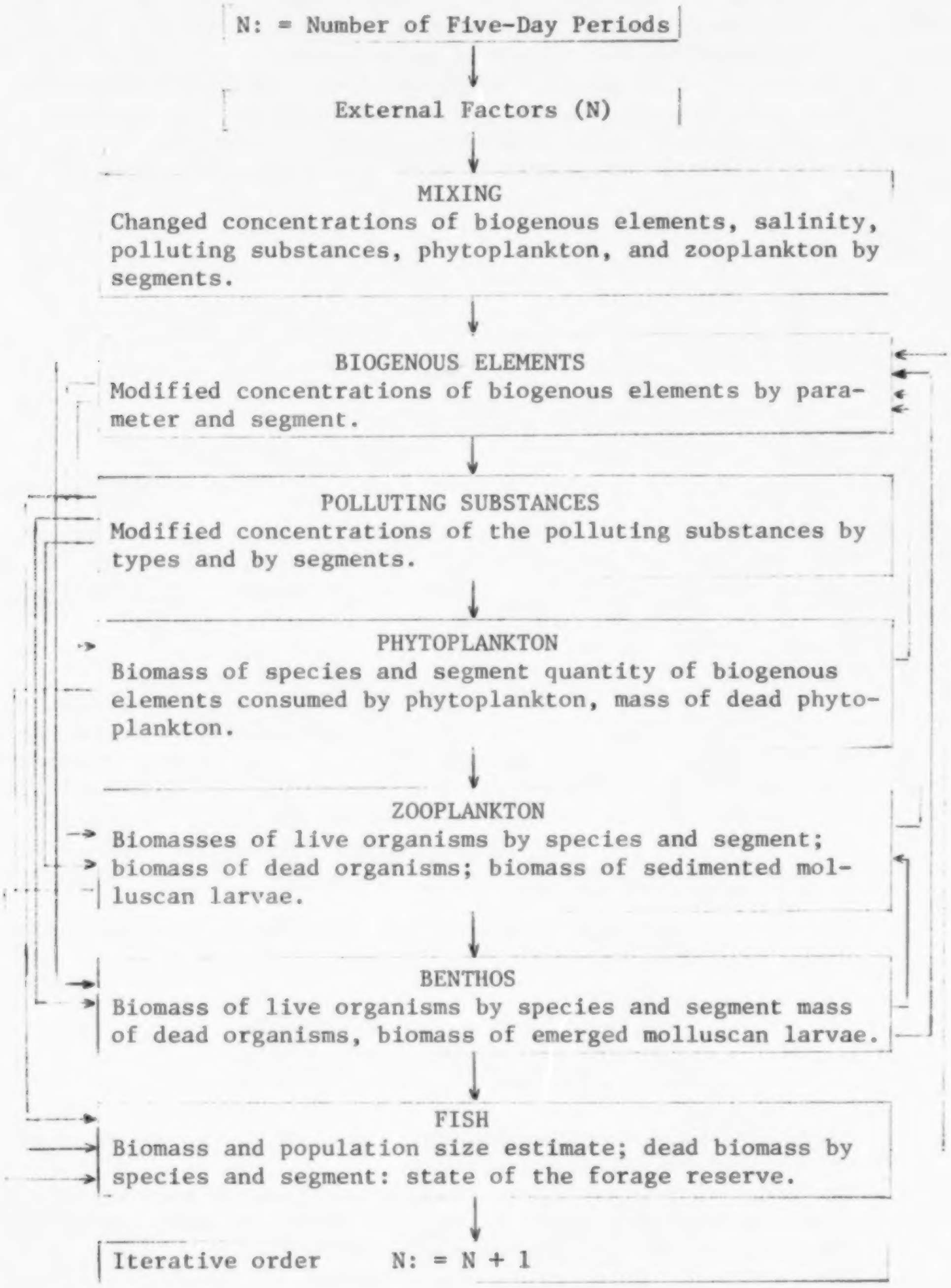
The external factors are introduced into all the submodels, where required.

2. Calculations in the mixing submodel require modification to the first 46 state vector components in each of the seven compartments.

3. Then the state vector X_o in each compartment is transformed by the remaining submodels in the following sequence: biogenous elements < polluting substances < phytoplankton < zooplankton < benthos < fish.

The state vector transformations are carried out in sequence as required by each submodel for the period of (T_o,T_1). The resulting vector X_1 is the system state vector at the time T_1. The order of information exchange between the submodels is important for understanding the multiple functioning of the simulation system (SS), as described in Figure 2.

One of the disadvantages of this system is that it does not replicate actual mechanisms because certain processes in all the submodels take place simultaneously during the (T_o,T_1) time interval. To overcome this difficulty, ordering of the submodels is introduced. Therefore, the Azov Sea contains interacting ingredients pertaining to different time periods. If the time interval is sufficiently small (e.g., about five days), any inaccuracy is insignificant and will not exceed the field measurement error for the parameters. A large interval is not acceptable.



N: = Number of Five-Day Periods

External Factors (N)

MIXING
Changed concentrations of biogenous elements, salinity, polluting substances, phytoplankton, and zooplankton by segments.

BIOGENOUS ELEMENTS
Modified concentrations of biogenous elements by parameter and segment.

POLLUTING SUBSTANCES
Modified concentrations of the polluting substances by types and by segments.

PHYTOPLANKTON
Biomass of species and segment quantity of biogenous elements consumed by phytoplankton, mass of dead phytoplankton.

ZOOPLANKTON
Biomasses of live organisms by species and segment; biomass of dead organisms; biomass of sedimented molluscan larvae.

BENTHOS
Biomass of live organisms by species and segment mass of dead organisms, biomass of emerged molluscan larvae.

FISH
Biomass and population size estimate; dead biomass by species and segment: state of the forage reserve.

Iterative order N: = N + 1

Figure 2: Diagram of information exchange.

SYSTEM B

The external factors and the "mixing submodel" are calculated as described for System A. However, each process submodel is placed in one of two classes, a simultaneous class or a sequential class. The state vector $\hat{X}_o$ is sequentially modified by the output of each submodel calculation for the (T_o,T_1) time step of five days. The state vector is transformed from $\hat{X}_o$ to X_1 by the $Z_1,\ldots Z_z$ processes which act simultaneously. In each process, some parameters increase and others decrease, the exact quantity of change is not known, but the possible dimensional limits are known, depending on a number of factors.

$$\begin{array}{c} X_1^{T_o} \\ \vdots \\ X_i^{T_o} \\ \vdots \\ X_m^{T_o} \end{array} \begin{pmatrix} Z_{11} & \cdots & Z_{ij} & \cdots & Z_{1z} \\ \vdots & & \vdots & & \vdots \\ \vdots & \cdots & [\underline{a}_{ij}(F_{T_o}),\overline{a}_{ij}(F_{T_o})] & \cdots & \vdots \\ \vdots & & \vdots & & \vdots \\ \cdot & \cdots & \cdot & \cdots & [\underline{a}_m(F_{T_o}),\overline{a}_{mz}(F_{T_o})] \end{pmatrix}$$

where F_{T_o} is the vector of factors at time T_o, and $\overline{a}_{ij}$ and $\underline{a}_{ij}$ are the upper and the lower boundaries, respectively. The major difficulty in the modeling is to determine the intensity $P_1\ldots P_z$ of the processes. This difficulty can be overcome by the following algorithm:

1. The F_{T_o} vector is calculated;
2. The values of $\underline{a}_{ij}(F_{T_o})$, and $\overline{a}_{ij}(F_{T_o})$ are determined;
3. If a random value is assumed for $a_{ij}(F_{T_o})$ with a distribution $[\underline{a}_{ij}(F_{T_o}),\overline{a}_{ij}(F_{T_o})]$ it can then be calculated by a Monte-Carlo method;
4. Boundaries of possible changes $X^i_{T_o}$, $\underline{X}^i_{T_o}$, and $\overline{X}^i_{T_o}$ are determined;
5. The respective components of the vector $X^i_{T_o}$ are selected as the initial values of the desired intensities for $P_1,P_2,\ldots,P_z$;
6. In the (a_{ij}) matrix, the positive and negative elements are defined, then the values $\Delta_i=\sum_j a^t_{ij}X^{Kj}_{T_o} + X^{Ki}_{T_o} - \sum_j \overline{a}_{ij}X^{Kj}_{T_o}$ for the obtained values of $P_i\Delta_i \geq 0$);
7. The vectors for each parameter are formulated from real observation $(U_{i1}, U_{i2}, \Theta^1_i,\ldots\Theta^B_i)$ where

   $$U_{i1} = \frac{\Sigma a^t_{ij}\ X^{Kj}_{T_o}}{\Sigma(\overline{a}^t_{ij})X^{Kj}_{T_o}} \qquad U_{i2} = \frac{\Sigma\overline{a}_{ij}\ X^{Kj}_{T_o}}{\Sigma(\overline{a_{ij}})X^{Kj}_{T_o}}$$

   where Θ^K_i is the experimental value for the K-th component of the F_{T_o} vector for the i-th parameter;
8. The parameters are ordered from actual observations to determine the preferential development of related forms, as well as tendencies for the increase or decrease of each parameter;
9. The increment steps for P_i are selected;
10. The values for P_i are refined by the logic steps outlined above in 4 through 9.

11. The process intensity values outside the limits of expected values actually observed are defined by a repetition of the logic steps 5 through 10 outlined above;
12. The values for P_i are selected either as the respective mean values or probability values within the range of acceptable rates; and
13. The X_{T_1} vector is determined.

A scan of the matrix elements will derive (X_{T_1}) as sets of probable states at time T_1. With the modeling approach, a family of projections is necessary to compare with the real system. Statistical analysis of this family of model projections to evaluate the impact of various management strategies on the system is the most important part of the modeling program for the Azov Sea.

Author's Note: The algorithm presented in this paper requires some clarification. There is no method to rigorously validate the model. It is actually an imitation of the real process of feeding and mortality. However, since the processes are treated at the population level and not as individual organisms, it is necessary to apply the iterative procedure described in steps 4 through 10 to determine process rate values which in fact do not contradict accepted biologic concepts.

The most acceptable verification and model refinement can be achieved by testing the algorithm with real data. The basic concepts of this paper will be used on a simulation model for Lake Baikal.

DISCUSSION

ROZHKOV: Is it possible to examine the model as a Marcovian bond? If so, then what is its finite length? Have the divergence effects been studied?

GORTSKO: Yes, it is possible to examine the model as a Marcovian bond. The question about the divergence of forecast lines has not been examined.

THOMANN: Is there any information over a long period of time on the 120 variables which you include in the model? If not, then in my opinion, it does not pay to include them.

GORTSKO: We chose that number of variables as a result of a very lively discussion among specialists--mathematicians, biologists, and others. All variables are guaranteed by information.

IVAKHNENKO: Were the units realized on a digital computer or as analog units?

GORTSKO: On a digital computer.

LOZANSKIY: My question is concerned with the principle of equal stability of the areas. I would like you to specify what criteria were used to realize this principle.

GORTSKO: The principle of equal stability in the ecological system can be evaluated. Biologists know a lot about the water bodies; the Azov Sea, in particular, has been studied by scientists for several years.

IVAKHNENKO: The author's confidence in introducing all 120 variables into the model frightens me; this can complicate the work. The number of variables in the model can be less, since they are all interrelated. Any five out of 120 variables can represent the given system. Belief that all the variables must be introduced into the computer is erroneous.

DeLUCIA: I support the majority of principles presented in the report. My colleague is discovering for himself principles that were discovered earlier in the United States. I think that it is necessary to differentiate the goals of science from the goals of administration. Authenticity in a model is required. We need complex models to describe with accuracy the phenomena studied.

ROZHKOV: The Marcovian bond will be a good approximation in one step. The Marcovian bond requires a transitional probability matrix that is multi-dimensional. We do not comprehend these probabilities very well.

LOUCKS: I think that all the participants in the symposium are interested in developing a model for practical applications. I am confident that the models presented here are useful and will be used in our countries. But the paths used by scientists in various countries to develop models differ.

GORTSKO: In conclusion, I would like to make a few observations. First, regarding the doubts expressed by Professor Ivakhnenko, it is incorrect to assume that we included all the variables. If we had, the variables would not total 120, but 1000. In the future, we plan to use statistical analysis for the results of a model's run.

MATHEMATICAL ECOSYSTEM MODELS AND DESCRIPTION OF THE WATER QUALITY IN WATER BODIES

R.A. Poluektov

INTRODUCTION

Intensified production in the past decades has been characterized by a significant increase in the rate of man's reconstruction of natural communities. The destruction of existing natural linkages and interactions evolved through time, and their recreation, allow the growing population of our planet to provide itself with the quantity of resources necessary for compatible development. But these complex processes of interaction between man and nature have another side. Because we are ignorant of the detailed picture of interaction between numerous animate and inanimate components of a natural community and cannot adequately evaluate the new equilibrium that follows our intervention, we encounter unexpected effects. Consequently, the development of a theory of dynamic behavior, both of the biosphere as a whole, and the biogeocenoses, (i.e. those elementary natural communities that constitute the earth's biosphere), has been pushed into the foreground. The essential problem is to construct quantitative models to reflect, with a certain degree of accuracy, the behavior of existing natural systems and to forecast the impact of man's controlling actions on these systems.

The theory of mathematical modeling of ecosystems has a 50-year history but is only now emerging as an independent branch of science. At present we can distinguish only a cycle of work of a general theoretical character proceeding in two basic directions. V. Volterra (1931) and A.N. Kolmogorov (1972), and a small number of investigators, (Poletayev, 1966; Menshutkin, 1971), were concerned with modeling specific ecosystems. In theory, the problem of the specific processes of interactions among different biological species and environmental factors has not even been considered. In this work, the possible outputs from specially constructed systems of differential equations were studied on a comparatively abstract level. The biological interpretation of results reflects subjective views of the investigator rather than the existing situation. Research on any specific problem should be based on an explicit parametrical notation of the equations of the system being modeled. Subsequent determination of the unknown parameters is based on the available experimental data. Since the experimental points fit sufficiently smooth curves, the type of approximating functions may be chosen arbitrarily by the investigator. Consequently, the number of different models is unnecessarily increased, since the comparison of results related to various models is frequently impossible. Such a situation necessitates a common point of view, or more precisely, a common approach for describing ecosystems. The general outline of such an approach, applicable to the theory of water ecosystems, is summarized below.

The Principles of Ecosystem Modeling

When modeling ecosystems, as any other dynamic system (Kalman et al., 1971), the major problem is choosing the variables to form the state vector. The state vector, understood to be a combination of values, reflects the influence of the system's prehistory on its future development. Since we assume that the ecosystem state vector is finite, the systems being modeled are considered a finite number of compartments, each uniquely characterized by the concentration of its substances or by the quantity of its energy.

The important considerations in research on the dynamics of biological associations are: How does the general number and composition of these associations change with time? What is the equilibrium state of the association after a long period of time? What are the reaction rates and equilibrium concentrations of mineral components in the system? If we tried to describe the fate of each member of the community, the resulting model would evidently be immense. Therefore, we must divide the live organisms into groups, so that members of each group possess an identical or similar set of features that determine their fate in the given community. The above-mentioned basic assumption means that the number of groups in a community is finite. If the feature associated with the survival of the individual is distributed continuously, then the given combination of the individual should be divided into a finite number of classes*.

The assumption that a finite number of compartments are in the system's inorganic and detrital organic parts has been reached in a similar manner. For example, when considering vertical stratification of the biogenic part of the system, it is necessary to divide the water body into a sequence of layers, each with its corresponding average level of concentration. Hence, we have as state variables:

$x(t)$ = n-dimensional vector, the components of which are the numbers (concentrations) or biomasses or a group of members which constitute the biological part of the system in the different compartments, and

$S(t)$ = m-dimensional vector, the components of which are the concentrations of environmental extensive parameters in the same compartments.

Since we are investigating a thermodynamically open system, it is necessary to take into account the input $[x_1(t), S_1(t)]$ and withdrawal $[x_0(t), S_0(t)]$ of both the biological and mineral components, respectively. In addition, it is necessary to consider the dependence of processes taking place in the system on the value of intensive abiotic factors which we denote by $c(t)$. In view of the concepts introduced, the problem of modeling physical-chemical conversions and biological transformations of a substance and energy in the ecosystem is reduced to the description of their movement from one compartment to another. The level of the environmental intensive factors determines the rate reactions occur in the system.

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*We note that the assumption of finiteness of the number of classes is not unnecessarily limiting because it corresponds to the experimental data available to the biologist.

The general system of equations of an ecosystem is:

$$\left.\begin{aligned}\dot{x} &= Y[x,S,c(t),x_1(t),x_0(t)]\\ \dot{S} &= \psi[x,S,c(t),S_1(t),S_0(t)]\end{aligned}\right\} \qquad (1)$$

where Y(...) and ψ(...) are, correspondingly, n-dimensional and m-dimensional functions of the arguments in parentheses. A schematic diagram that represents the processes taking place in the system is shown in Figure 1. The solid-lined arrows in the diagram signify the trophic interactions of the system's elements.

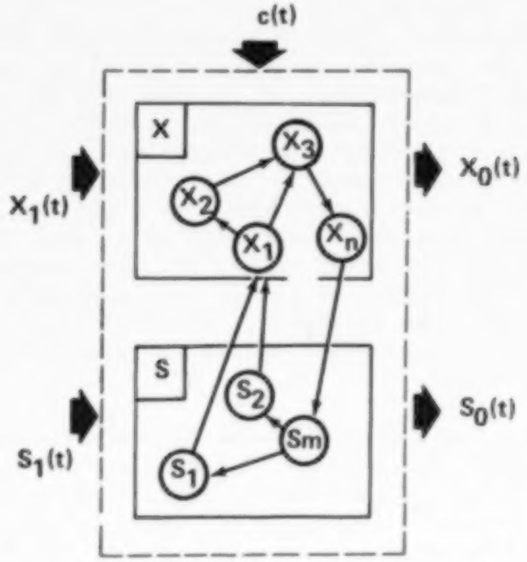

Figure 1. Interaction of biotic and abiotic elements in the ecosystem.

From the material presented, the following stages of constructing a model scheme of the ecosystem are apparent:

1. determination of the combination state variable (functionally homogeneous groups of biological objects, biogeneous, and other factors);
2. determination of chronological (trophic) links and construction of a system diagram; and
3. determination of the dependence of the exchange rates among compartments on the values of extensive factors and the level of effect of the intensive factors.

To clarify the nature of the problems and the difficulties involved, we shall first examine the simplest example: A population of heterotrophic microorganisms of one species develops in a flowing system on a certain substrate. The concentration of the microorganisms is x; the concentration of the substrate is S (here x and S are scaler quantities). The equations of this system (Malek, 1968) are of the form:

$$\left.\begin{aligned}\dot{x} &= \mu(S)x - dx,\\ S &= (S_1-S)d - \tfrac{1}{y}\mu(S)x,\end{aligned}\right\} \qquad (2)$$

where d is the flow rate, $\mu(S)$ is the specific rate of the microorganism's growth, and y is the utilization coefficient. These equations already reflect two main points which also arise in a general case.

1. The concept of specific growth rate. As is evident from equation 2, in the absence of flow (d=0) the relative growth rate of the microbial population is determined by the concentration level of substrate S:

$$\frac{1}{x}\dot{x} = \mu(S)$$

Usually this dependence is evident (as in Figure 2) where different curves correspond to different levels of intensive factors (i.e., temperature). It is essential to note here that numerically the specific growth rate is the value used for constructing the model for a general case (Poluektov 1974). Thus, if $\overline{S}^i$ is the combination of biogenic factors of the i-th group of the community but $\overline{x}^i$ and $\underline{x}^i$ are the combination of organisms adjacent to the i-th group on the lower and higher trophic levels, correspondingly, then,

$$\frac{1}{x_i}\dot{x}_i = \mu_i(\overline{S}^i, \overline{x}^i, \underline{x}^i, x_i) \qquad (3)$$

Here, as a rule, μ_i is a monotonic increasing function of the variables $\overline{S}^i$ and $\overline{x}^i$ and a monotonic decreasing function of the variables $\underline{x}^i$ and its "own" variable x_i. A more detailed investigation of the dependence of the specific growth rate on the concentration of biogenic elements and organics is described by Poluektov et al. (1974).

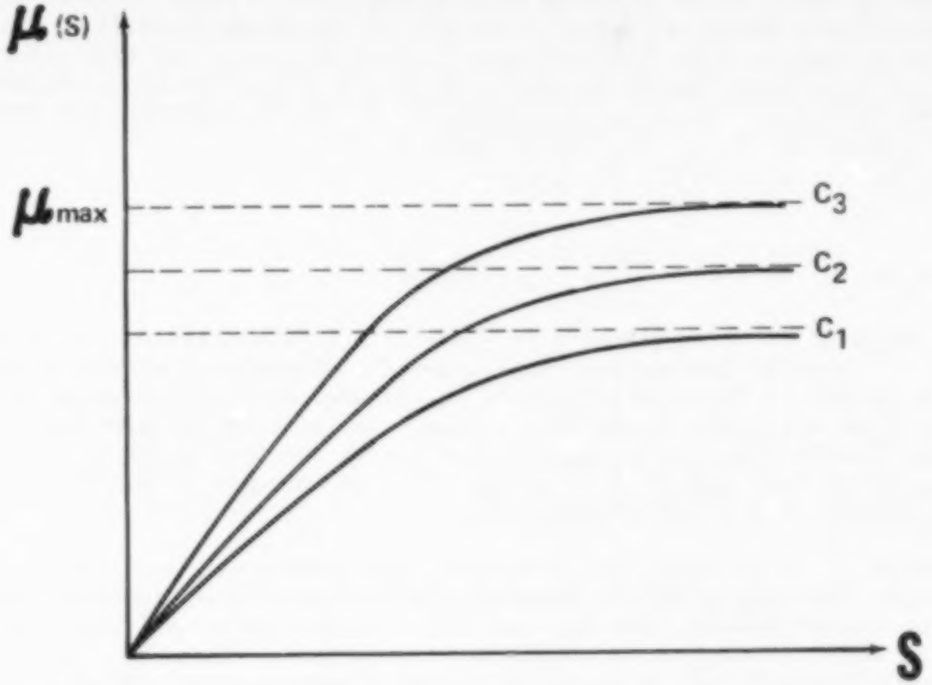

Figure 2. Dependence of specific growth rate on the substrate concentration (S) given different levels of the intensive factors (c).

2. Utilization coefficient. The introduction of this coefficient allows us to "calculate" the consumption of substrate by biomass growth. In the given case, this coefficient has the sense of the partial derivative:

$$y = \frac{\partial x}{\partial S} \approx \frac{\Delta x}{\Delta S}$$

and is equal numerically to the value of increased biomass per unit of consumed substrate. It is evident that the concept of the utilization coefficient (yield coefficient) also has a profound biological sense. Its use allows us to connect the adjacent trophic levels of the ecosystem into a single chain. As experiments demonstrate, the utilization coefficient in a number of cases depends slightly on the values of the intensive and extensive factors.

3. The problem in generalizing the model (2), as in the case of complex trophic interactions, is clarifying the dependence of the specific growth rate of each population modeled on the state parameters of adjacent trophic levels and on the value of the environmental intensive factors. Indeed, the knowledge of the dependence (3) and the utilization coefficient is in principle sufficient to close the system*. In V. Volterra's simplest model (1931), the specific growth rate of prey is a linear decreasing function of the number of predators, whereas the specific growth rate of predators is a linear increasing function of the number of prey. Obviously different schemes of consumption are possible in the presence of a number of substrates consumed by one species (Figure 3a) or a number of species of organisms consumed by the predators of the next trophic level (Figure 3b). One scheme--the completing diagram--is found when, for organisms to live, all components of an allowance must be available. The growth rate is limited by the substrate (or species of "prey") which is in the relative minimum, so that:

$$\mu_i(S_1,\ldots,S_m) = \min_{j=\overline{1,m}} \{f_j(S_j)\}$$

and

$$\mu_j(x_{j-1},\ldots,x_{j-k}) = \min_{l=\overline{1,k}} \{f_{j-l}(x_{j-l})\}$$

where functions $f_{j-l}(x_{j-l})$ and $f_j(x_j)$ are of a form similar to curves in Figure 2. Another extreme case is sequential consumption, when at first one kind of substrate (species of "prey") is consumed until it vanishes from the system, then a second, and so on. A mixed case is possible with various a priori probabilities of consumption of one or another substrate (different "affinity" for the substrate), whereas the real consumption is determined by the available composition of substrates.

Finally, it is necessary to examine some problems of describing energy processes--photosynthesis and chemosynthesis of autotrophs and respiration. Modeling photosynthesis processes are based on the use of the light and carbon dioxide photosynthesis curves, and the calculation of the absorption of photosynthesis radiation by the turbidity of the absorbing water column. It is somewhat more difficult to model respiration processes. Recent studies have established that the respiration of sessile organisms can be divided into two components. One, "existence breathing," is proportional to the accumulated biomass, and the other, "growth breathing," is proportional to the intensity of biosynthesis. In other words it is necessary to write

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*It is assumed that the principles of physical and physical-chemical conversions of energy and matter in the system are known.

$$R_i = ax_i + B\frac{dx_i}{dt} \tag{4}$$

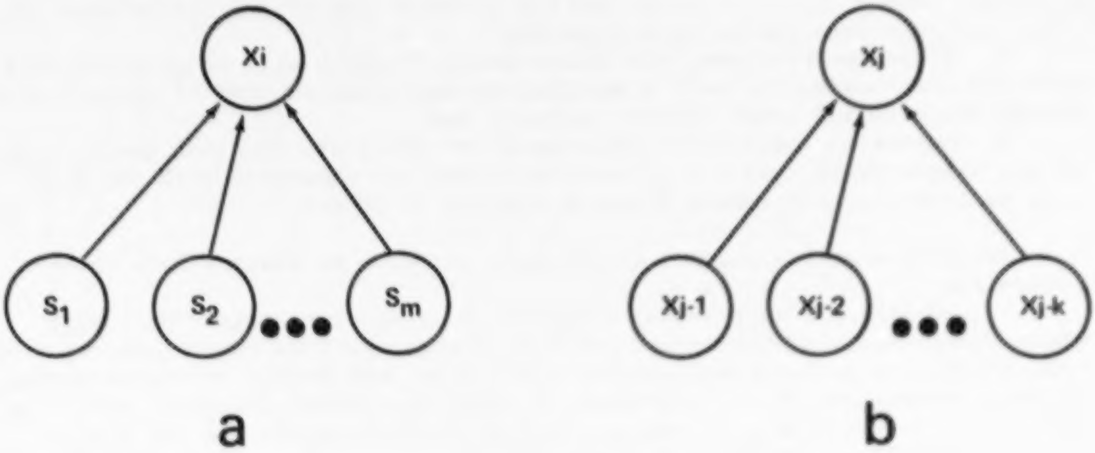

Figure 3. Diagram of the parallel consumption of abiotic (x_i) and biotic (x_j) elements in an ecosystem.

The breath of actively mobile organisms possesses a third component connected with the energy expended on movement. This component is evidently connected with the necessity of pursuing prey or hiding from the predator and should be on the same structure as the specific growth rate $\mu_i(\bar{S}^i, \bar{x}^i, \underline{x}^i, x_i)$. Apparently this breathing component can be connected in the first approximation with the second component of the formula (equation 4).

Modeling Methods

In discussing the modeling of such complex phenomena as the dynamics of water ecosystem development, we, of course, cannot hope to develop a model which adequately reflects the entire assemblage of complex interrelations of the system's elements. A model is a certain view of a phenomenon from a definite standpoint. For research objectives, the most important processes must be described in sufficient detail while less significant processes may be described in general terms or even disregarded. Therefore, before discussing a model of one or another phenomena, modeling objectives must be established.

If problems concerning environmental protection and increased biological productivity of water ecosystems are considered, the models should "conform" to one or several economic management decisions. Therefore, before categorizing the types of models, it would be advisable to classify such decisions.

All the economic management decisions adopted in environmental protection and natural resource management can be divided into three categories, depending on the length of time in which the consequences of these decisions are observed. These are:

1. feasibility and project decisions, the consequences of which will not be revealed for several years. Examples of such decisions are water reservoir-lake construction projects, and the construction of the dam between the Black and Azov Seas now being discussed;
2. planning decisions, the consequences of which will be revealed within one year. An example of such a decision is the establishment of annual catch quotas for fish and other aquatic animals; and
3. operative decisions, consequences of which are revealed during a part of the annual cycle. Basically such decisions are connected with the temporary realization of planning measures covered in item 2.

Further, the models under development can also be divided into three categories:

1. models of interseasonal dynamics of water ecosystems. Basically these models are mass-balance relationships that describe the accumulation or loss of basic material components in the system, the change in the structure of water bodies and their biological productivity, eutrophication, etc. The time step of such models fluctuates from one to two months, to one year;
2. models of seasonal dynamics of water ecosystems used to describe the periodicity of water body bioproductivity within one year. The time step of the model fluctuates from one to several days; and
3. models of diurnal dynamics describing the daily fluctuations of the photosynthetic activity of phytoplankton and other water plants, and the daily cycles of water animals connected with these fluctuations. The time step of the model varies from one to several hours.

Models of the three above-mentioned types can generally be examined as a hierarchical system of models. The coefficients of the interseasonal dynamic models are functions of the outputs of models of diurnal dynamics. An analogy can be drawn between the hierarchy of models and the hierarchy of the degree of turbulence in the atmosphere. The modeling diagram is shown in Figure 4. The basic subsystems of models from each of the three levels can be seen on the left.

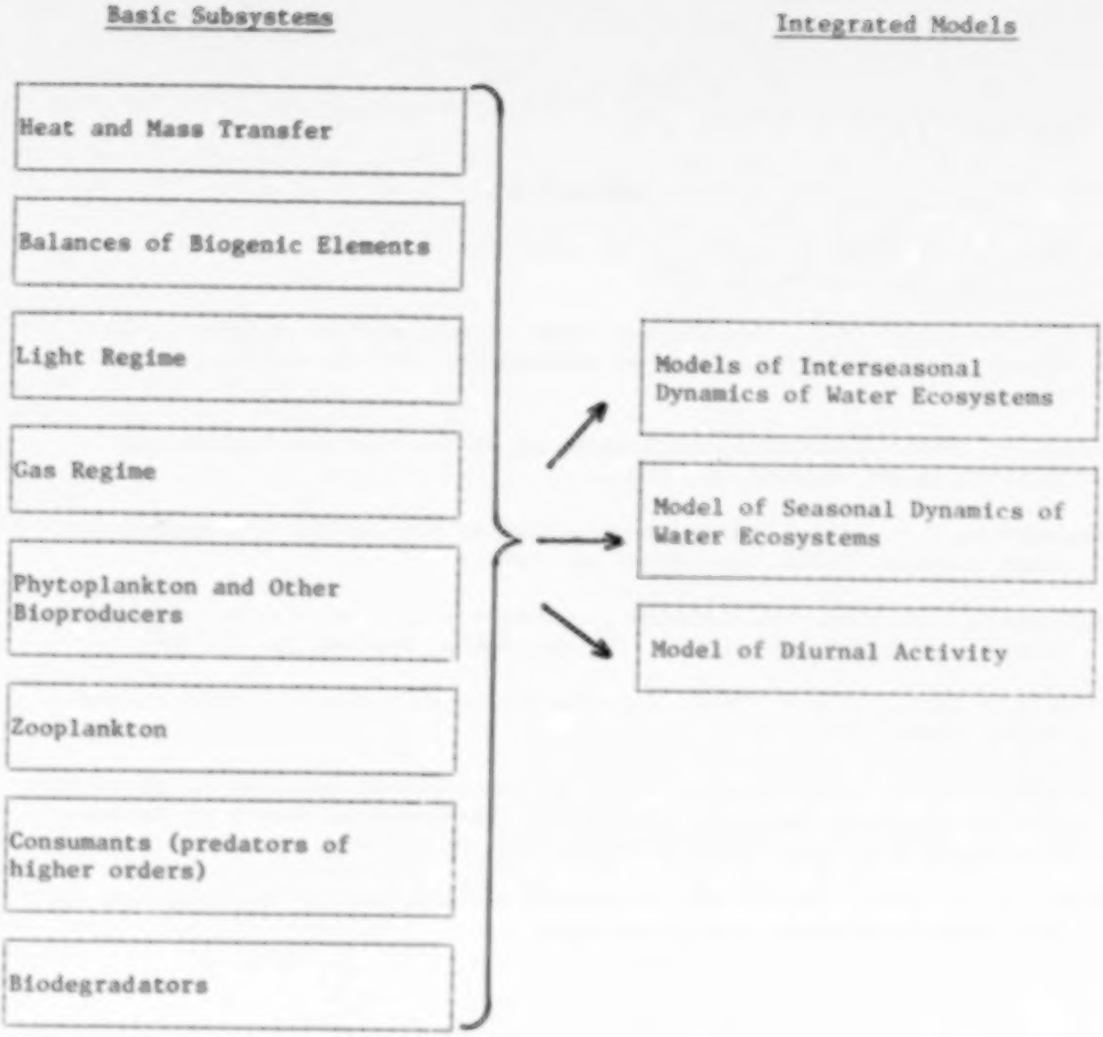
Basic Subsystems
Integrated Models
Heat and Mass Transfer
Balances of Biogenic Elements
Light Regime
Gas Regime
Phytoplankton and Other Bioproducers
Zooplankton
Consumants (predators of higher orders)
Biodegradators
Models of Interseasonal Dynamics of Water Ecosystems
Model of Seasonal Dynamics of Water Ecosystems
Model of Diurnal Activity

Figure 4. Hierarchical diagram of dynamic models of water biogenosis.

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FINAL REPORT
OF THE AMERICAN-SOVIET SYMPOSIUM ON MATHEMATICAL MODELS TO OPTIMIZE WATER QUALITY MANAGEMENT

In accordance with the intergovernmental agreement between the USA and USSR on Cooperation in the Field of Environmental Protection, signed in Moscow on May 23, 1972, the Memorandum of Implementation of the said agreement, signed on September 21, 1972, and the Minutes of the Second Session of the USA-USSR Joint Working Group, September 27, 1974, the American-Soviet Symposium on Mathematical Models to Optimize Water Quality Management was held in Kharkov and Rostov-on-Don, USSR, from December 9 through 16, 1975.

Discussions of the application of mathematical models to optimize water quality of river basins took place in Kharkov December 9-11. The mathematical modeling of water quality in lakes and impoundments was discussed in Rostov-on-Don December 12-16, 1975. Nine summary reports from the USSR, based on 24 reports prepared by Soviet scientists and nine reports of American scientists, dealing with various aspects of the theme of cooperation, were presented and discussed.

USSR specialists from 35 scientific research institutions and representatives of the USSR Ministry for Reclamation and Water Management, the USSR Council of Ministers, Main Office of the Hydrometeorological Service, the USSR and Ukrainian SSR Academies of Sciences, and representatives of higher educational establishments took part in the work of the symposium. American specialists from two research laboratories of the U.S. Environmental Protection Agency and five universities also participated in the symposium.

Scientific problems worked out under the terms of American-Soviet cooperation on projects II-1, "Studies and Modeling of River Basin Pollution," and II-2, "Protection and Management of the Water Quality in Lakes and Estuaries," were considered, along with reports reflecting a general state of development and application of mathematical models that predict and optimize water quality management in the USA and the USSR.

Participants noted with satisfaction signs of progress in the statements of problems and in the development of mathematical models, and observed that the bilateral projects and exchange of scientific information within the framework of the USA-USSR cooperation have facilitated this progress.

In water quality modeling, American specialists have developed and applied deterministic models based on the conservation of mass of various substances significant in the description of physical and biochemical processes. These models were designed for a large number of water bodies and are widely applied to problems.

In the USSR, the above-mentioned approach has been practically applied to plan water protective measures for some water bodies, concurrent with non-deterministic and non-formal methods to afford the advantage of flexibility. The necessity for this approach has been dictated by extreme involution of the processes in aqueous ecosystems caused by human activities. It is assumed that this approach will aid understanding of the processes in water bodies and development of efficient methods of optimal management of large scale ecological and economic systems. It is extremely important to compare applicability of the two approaches to provide practical systems for management of water protective complexes.

Information provided on problems of mathematical modeling is inadequate at present. Therefore, better methods must be developed to mathematically define data collection plans, to acquire field data promptly and inexpensively, and to evaluate water quality. Mathematical models that are based on the principles of conservation of mass, and define physical and biochemical processes involved, have been successfully applied to optimize control of water quality. Water quality control can be improved by the use of more complex models which describe the condition of ecosystems.

To advance development, improvement, and application of mathematical models for water quality control optimization, and also to enhance mutual benefits of American-Soviet cooperation, the participants of the symposium propose the following objectives within the framework of joint cooperation:

1. organize a joint comparative evaluation of mathematical modeling methods;

2. continue to develop improved techniques of water protective programs based on systems analysis of ecological, economic, and social phenomena;

3. initiate projects to develop methods for estimating the stability and resilience of natural aquatic ecosystems;

4. promote joint work on data management and modeling;

5. intensify development of models and algorithms for the economic optimization of water protective measures for river basins; and

6. arrange mutual, long-term exchange of scientists for the development of water quality models in both countries.

Participants in the symposium consider it beneficial to acquaint scientists of both countries with materials of the symposium and request that the USSR Council of Ministers, Main Office of Hydrometeorological Service, and the U.S. Environmental Protection Agency, publish proceedings of the symposium.

The symposium has demonstrated that mutual discussions of questions of mathematical modeling relevant to the problem of water protection are useful and fruitful. Participants propose a symposium to convene on the theme, "Methodology and Practice of Water Protection Activity Planning," in 1979, in the USA.

Further, the participants of the symposium wish to express their gratitude for the high level of organization and for the gracious hospitality shown them during their visit.

This final report was signed in Rostov-on-Don on December 16, 1975, in two copies, Russian and English, both copies being equally valid.

Co-Chairmen of the Symposium

T. DAVIES   V. LOZANSKY

M. PISANO   A. ZENIN

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